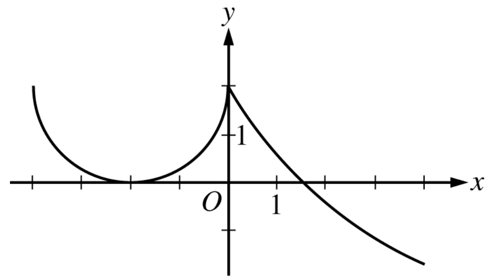
AP CALCULUS PROBLEM SET 11 INTEGRATION III (FUNDAMENTAL THEOREM of CALCULUS)

(2009-6)
1.



Graph of f'

The derivative of a function f is defined by $f'(x)=\begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-x/3}-3 & \text{for } 0 < x \le 4 \end{cases}$.

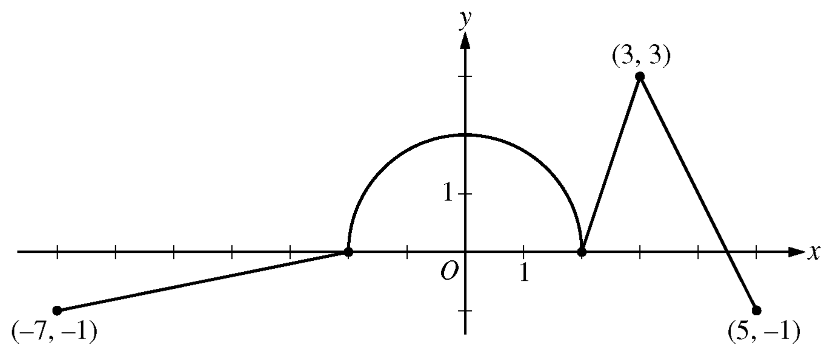
The graph of the continuous function $f'(x)$, shown in the figure above, has x-intercepts at $x=-2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \le x \le 0$ consists of a semi-circle, and $f(0) = 5$.

(a) For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

(b) Find $f(-4)$ and $f(4)$.

(c) For $-4 \le x \le 4$, find the value of x at which f has an absolute maximum. Justify your answer.

(2010-5)
2.



Graph of g'

The function g is defined and differentiable on the closed interval [−7, 5] and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semi-circle and three line segments as shown on the figure above.

(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

(2005-4)

3.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differential except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

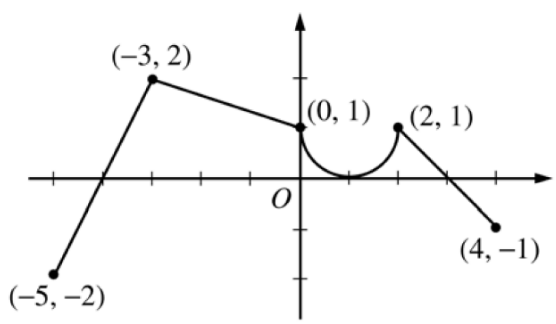
(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

(c) Let g be the function defined by $g(x) = \int_1^x f(t)dt$ on the open interval (0, 4). For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

(2004-5)

4.



Graph of f

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^x f(t)dt$

(a) Find $g(0)$ and $g'(0)$.

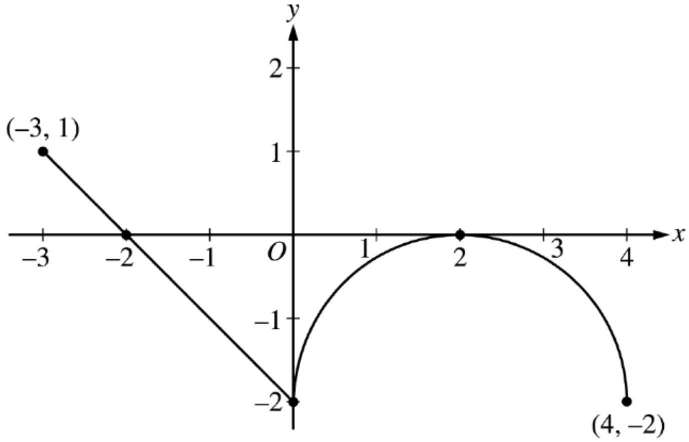
(b) Find all values of x in the open interval (–5, 4) at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval [–5, 4]. Justify your answer.

(d) Find all values of x in the open interval (–5, 4). At which the graph of g has a point of inflection.

(2003-4)

5.



Graph of f'

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle as shown.

(a) On what intervals, if any, is f increasing? Justify your answer.

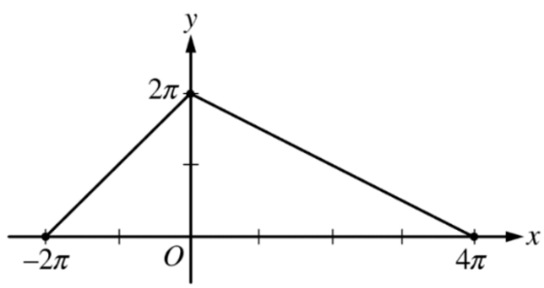
(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point (0, 3)

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers

(2011(B)-6)

6.



Graph of g

Let g be the piecewise linear function defined on $[-2\pi,\ 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

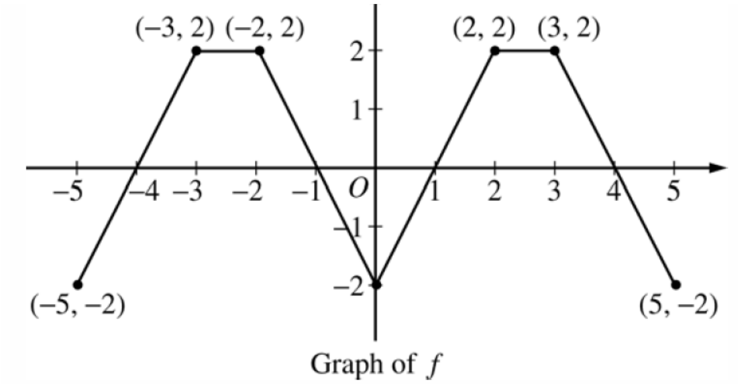
(b) Find all x-values in the open interval $(-2\pi,\ 4\pi)$ for which f has a critical point.

(c) Let $h(x) = \int_0^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

OPTIONAL

(2006-3)

7.



The graph of the function f shown above consists of six line segments.

Let g be the function given by $g(x) = \int_0^x f(t)dt$.

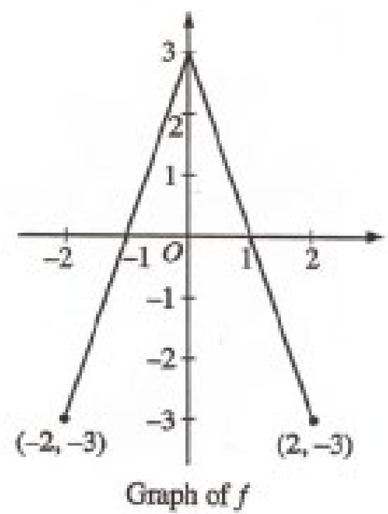
(a) Find $g(4), g'(4), g''(4)$.

(b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.

(c) Suppose f is defined for all real numbers and is periodic with a period of length 5. The graph above shows two periods of f. Given that $g(5) = 2$, find $g(10)$ and write an equation of the line tangent to the graph of g at $x = 108$.

(2002- 4)

8.



The graph of the function f shown above consists of two line segments. Let g be the function given by

$g(x) = \int_0^x f(t)dt$

(a) Find $g(1), g'(1), g''(1)$.

(b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

(c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$

(76-6)

9.

(a) Given $5x^3 + 40 = \int_c^x f(t)dt.$

(i) Find $f(x)$

(ii) Find the value of c.

(b) If $F(x) = \int_x^3 \sqrt{1+t^{16}}\, dt,$ find $F'(x)$.

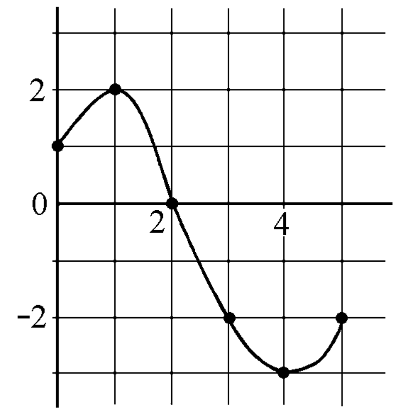
(95BC-6)

10. Let f be a function whose domain is the closed interval [0, 5]. The graph of f is shown. Let $h(x) = \int_0^{\frac{x}{2}+3} f(t)dt.$

(a) Find the domain of h.

(b) Find $h'(2)$.

(c) At what x is $h(x)$ a minimum? Show the analysis that leads to your conclusion.



Graph of f

(2014-5)

11.

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function h is defined by $h(x) = \ln f(x)$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)dx$.