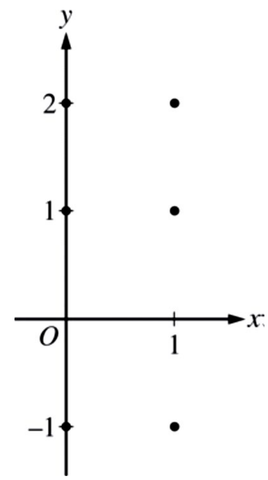
AP CALCULUS PROBLEM SET 14 SLOPE FIELDS

(2015-4)

1. Consider the differential equation $\frac{dy}{dx} = 2x - y$.



(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

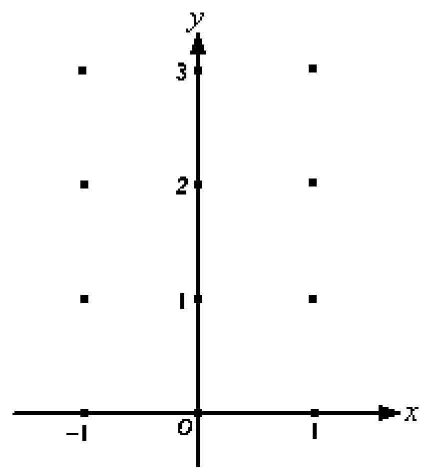
(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y.
Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

(2004-6)

2. Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.



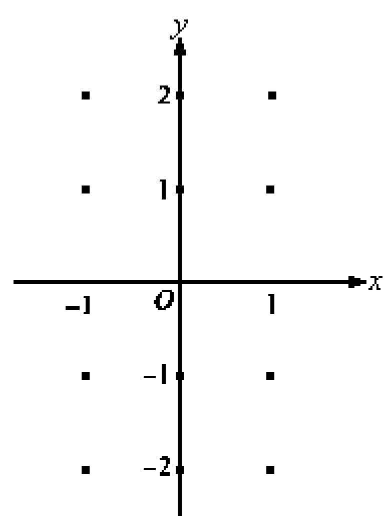
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

(2005-6)

3. Consider the differential equation $\frac{dy}{dx} = -\frac{2xy}{2}$.



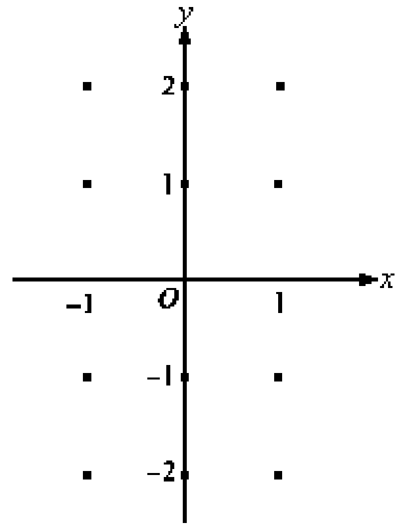
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

(b) Let $y = f(x)$ be the particular solution with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

(2010(B)-5)

4. Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.



(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

(b) While the slope field in part a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$.

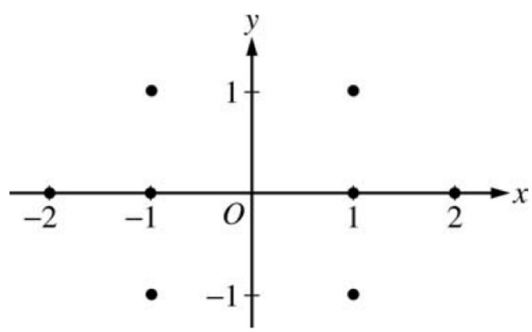
Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

(2006-5)

5. Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

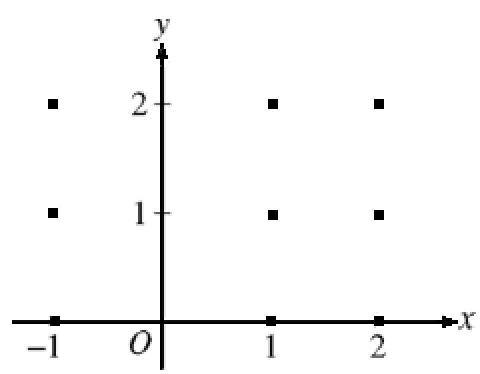
(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



(b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 1$ and state its domain.

(2008-5)

6. Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.



(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

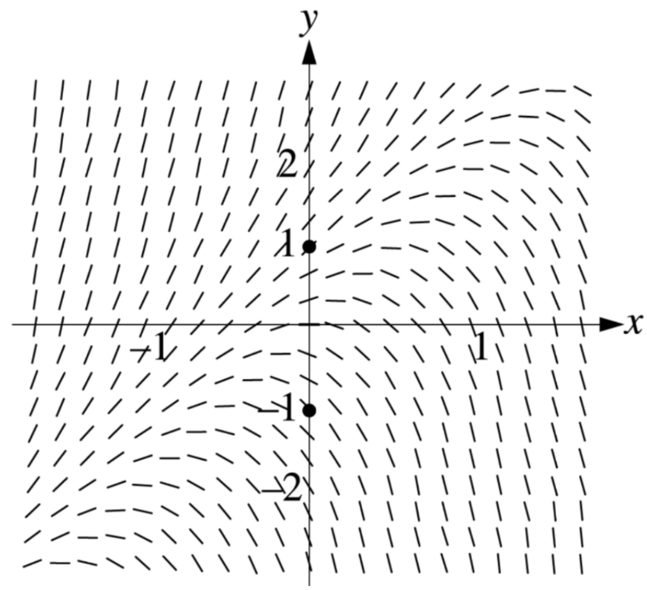
(b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x\to\infty} f(x)$.

OPTIONAL

(BC2002-5)

7. Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.



(a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0,1)$ and sketch the solution curve that passes through the point $(0, -1)$.

(b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$.
**Use Euler's method, starting at $x = 0$ with a step size of 0.1, to approximate $f(0.2)$. Show work that leads to your answer.

(c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

(d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point (0, 0)? If so, is the point a local maximum or a local minimum? Justify your answer.

**not in AP Calculus AB