AP CALCULUS PROBLEM SET 4 DERIVATIVES IV: *e, ln*

(90-2)

1. Let f be the function given by $f(x) = \ln \dfrac{x}{x-1}$

(a) What is the domain of f?

(b) Find the value of the derivative of f at $x = -1$.

(c) Write an expression for $f^{-1}(x)$, where $f^{-1}(x)$ denotes the inverse function of f.

(78-2)

2. Let $f(x) = (1 - x)^2$ for all real numbers x, and let $g(x) = \ln x$ for all $x > 0$. Let $h(x) = (1 - \ln x)^2$.

(a) Determine whether $h(x)$ is the composition $f(g(x))$ or the composition $g(f(x))$.

(b) Find $h'(x)$.

(c) Find $h''(x)$.

(d) Sketch the graph of h. (See Problem Set 8, Curve Analysis II)

(81-4)

3. Let f be the function defined by $f(x) = 5^{\sqrt{2x^2-1}}$

(a) Is f an even or odd function? Justify your answer.

(b) Find the domain of f.

(c) Find the range of f.

(d) Find $f'(x)$.

(83-1)

4. Let f be the function defined by $f(x) = -2 + \ln\left(x^2\right)$.

(a) For what real numbers x is f defined?

(b) Find the zeros of f.

(c) Write an equation for the line tangent to the graph of f *at* $x = 1$.

(75BC-7)

5.

(a) For what value of m is the line $y = mx$ tangent to the graph of $y = \ln x$?

(b) Prove that the graph of $y = \ln x$ lies entirely below the graph of the line found in (a).

(c) Use the results of (b) to show that $e^x \geq x^e$ for $x > 0$.

(2011-6)

6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0 \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

**(c) Find the average value of f on the interval $[-1, 1]$.