

AP CALCULUS PROBLEM SET 5 MOTION

(97-1)

1. A particle moves along the x -axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.
- (a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.
- (b) For what values of t , $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0, 3]$?
- (c) Find total distance travelled by the particle from time $t = 0$ until time $t = 3$.

(93-2)

2. A particle moves on the x -axis so that its position at any time $t > 0$ is given by $x(t) = 2te^{-t}$.
- (a) Find the acceleration of the particle at $t = 0$.
- (b) Find the velocity of the particle when its acceleration is 0.
- (c) Find the total distance travelled by the particle from $t = 0$ to $t = 5$.

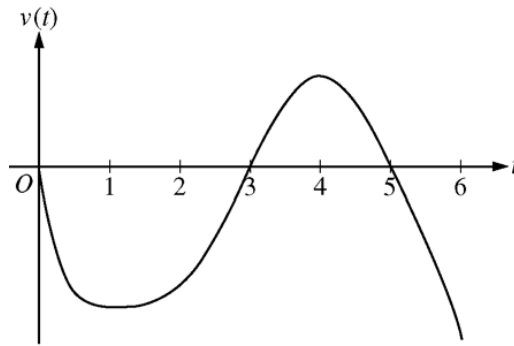
(2012-6)

3. For $0 \leq t \leq 12$, a particle moves along the x -axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi t}{6}\right)$. The particle is at position $x = -2$ at time $t = 0$.
- (a) For $0 \leq t \leq 12$, when is the particle moving to the left?
- (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.
- (c) Find the acceleration of the particle at time t . Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.
- (d) Find the position of the particle at time $t = 4$.

(2010(B)-6)

4. Two particles move along the x -axis. For $0 \leq t \leq 6$, the position of the particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.
- (a) For $0 \leq t \leq 6$, find all times t during which the particle R is moving to the right.
- (b) For $0 \leq t \leq 6$, find all times t during which the two particles travel in opposite directions.
- (c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.
- (d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \leq t \leq 3$.

(2008-4)
5.

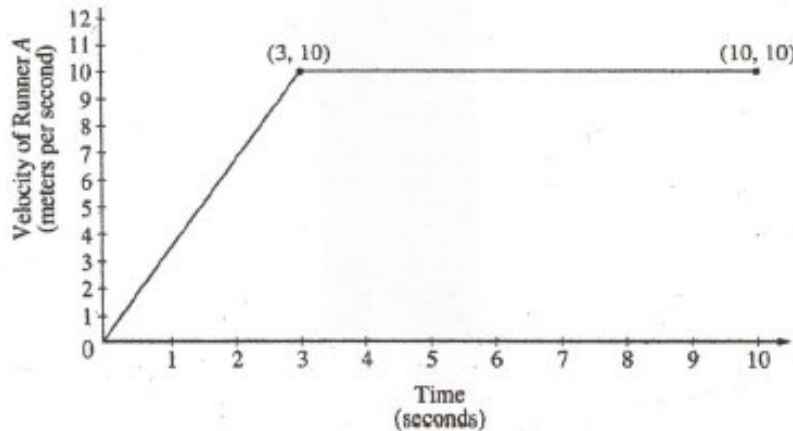


Graph of v

A particle moves along the x -axis so that its velocity at time t , for $0 \leq t \leq 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t -axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

- For $0 \leq t \leq 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
- For how many values of t , where $0 \leq t \leq 6$, is the particle at $x = -8$? Explain your reasoning.
- On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
- During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

(2000-2)
6.

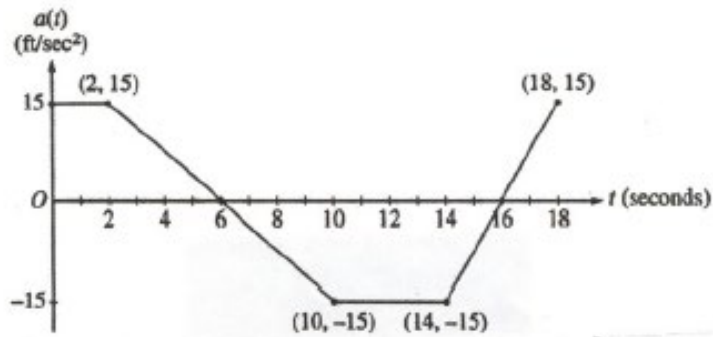


Two runners, A and B , run on a straight racetrack for $0 \leq t \leq 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A . The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \frac{24t}{2t+3}$.

- Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.
- Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.
- Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \leq t \leq 10$ seconds. Indicate units of measure.

OPTIONAL

(2001-3)
7.



A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \leq t \leq 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2 , is the piecewise linear function defined by the graph above.

- Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?
- At what time in the interval $0 \leq t \leq 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?
- On the time interval $0 \leq t \leq 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.
- At what times in the interval $0 \leq t \leq 18$, if any, is the car's velocity equal to zero? Justify your answer.

(99-1)

8. A particle moves along the y -axis with velocity given by $v(t) = t \sin(t^2)$ for $t \geq 0$.

- In which direction (up or down) is the particle moving at time $t = 1.5$? Why?
- Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?
- Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.
- Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

(76BC-1)

9. A particle moves on the x -axis in such a way that its position at time t is given by

$$x = (2t - 1)(t - 1)^2.$$

- At what times t is the particle at rest?
- During what interval of time is the particle moving to the left? Justify your answer.
- At what time during the interval found in (b) is the particle moving most rapidly (that is, the speed is a maximum)? Justify your answer.

(2007-4)

10. A particle moves along the x -axis with position at time t given by $x(t) = e^{-t} \sin t$ for $0 \leq t \leq 2\pi$

- Find the time t at which the particle is furthest to the right.
- Find the value of the constant A for which $x(t)$ satisfies the equation

$$Ax''(t) + x'(t) + x(t) = 0 \quad \text{for } 0 < t < 2\pi.$$