AP CALCULUS PROBLEM SET 7 CURVE ANALYSIS I (MAX/MIN)

(91-4)

1. Let f be the function given by $f(x)=\dfrac{|x|-2}{x-2}$.

(a) Find all the zeros of *f*.

(b) Find $f'(1)$.

(c) Find $f'(-1)$.

(d) Find the range of *f*.

(82-6)

2. A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 metres and its volume is 36 cubic metres. If building the tank costs \$10 per square metre for the base and \$5 per square metre for the sides, what is the cost of the least expensive tank?

(95-1)

3. Let f be the function given by $f(x)=\dfrac{2x}{\sqrt{x^2+x+1}}$

(a) Find the domain of *f*. Justify your answer.

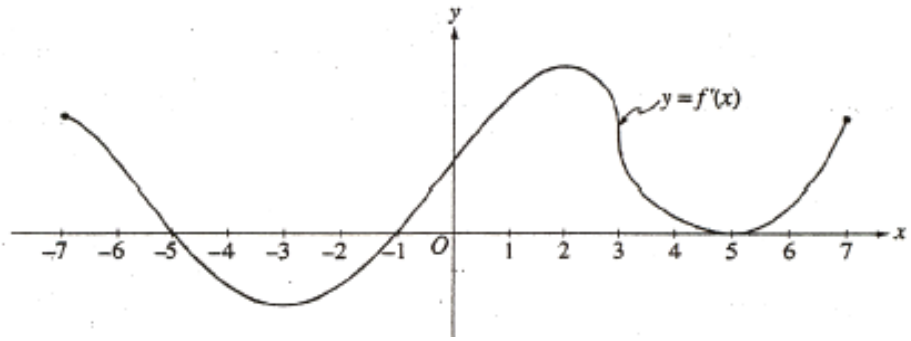
(b) Sketch the graph of f in a viewing window $[-5,5],[-3,3]$.

(c) Write an equation for each horizontal asymptote of the graph of f.

(d) Find the range of *f*. Use $f'(x)$ to justify your answer. Note: $f'(x)=\dfrac{x+2}{(x^2+x+1)^{\frac{3}{2}}}$

(2000-3)

4.



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

(a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

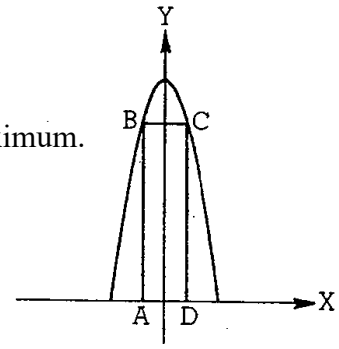
(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

(80-2)

5. A rectangle ABCD with sides parallel to the coordinate axes is inscribed in the region enclosed by the graph of $y = -4x^2 + 4$ and the x-axis as shown.



(a) Find the x and y-coordinates of C so that the area of rectangle ABCD is a maximum.

(b) The point C moves along the curve with its x coordinate increasing at the constant rate of 2 units per second. Find the rate of change of the area of rectangle ABCD when $x = \frac{1}{2}$.

(90-5)

6. Let f be the function defined by $f(x) = sin^2 x - sin\, x$ for $0 \le x \le \frac{3\pi}{2}$.

(a) Find the x-intercepts of the graph of f.

(b) Find the intervals on which f is increasing.

(c) Find the absolute maximum value and the absolute minimum value of f. Justify the answer.

(92-3)

7. Let f be the function given by $f(x) = ln\left|\frac{x}{1+x^2}\right|$.

(a) Find the domain of f.

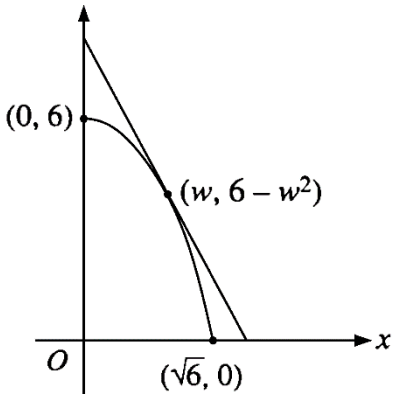
(b) Determine whether f is an even function, an odd function, or neither. Justify your conclusion.

(c) At what values of x does f have a relative maximum or a relative minimum? For each such x, use the first derivative test to determine whether $f(x)$ is a relative maximum or a relative minimum.

(d) Find the range of f.

(94BC-4)

8. Let $f(x) = 6 - x^2$. For $0 < w < \sqrt{6}$, let $A(w)$ be the area of the triangle formed by the coordinate axes and the line tangent to the graph of f at the point $(w,\ 6 - w^2)$.



(a) Find $A(1)$

(b) For what value of w is $A(w)$ a minimum?

(2004-4)

9. Consider the curve defined by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local minimum, a local maximum, or neither at the point P? Justify your answer.