

AP CALCULUS PROBLEM SET 8 CURVE ANALYSIS II

(78-2)

1. Let $f(x) = (1 - x)^2$ for all real numbers x , and let $g(x) = \ln x$ for all $x > 0$. Let $h(x) = (1 - \ln x)^2$.

- (a) Determine whether $h(x)$ is the composition $f(g(x))$ or the composition $g(f(x))$.
- (b) Find $h'(x)$.
- (c) Find $h''(x)$.
- (d) Sketch the graph of h .

(77-2)

2. Consider the function f defined by $f(x) = (x^2 - 1)^3$ for all real numbers x .

- (a) For what values of x is the function increasing?
- (b) Find the x - and y -coordinates of the relative maximum and minimum points. Justify your answer.
- (c) For what values of x is the graph of f concave upward?
- (d) Using the information found in parts (a), (b), and (c), sketch the graph of f on the axes provided.

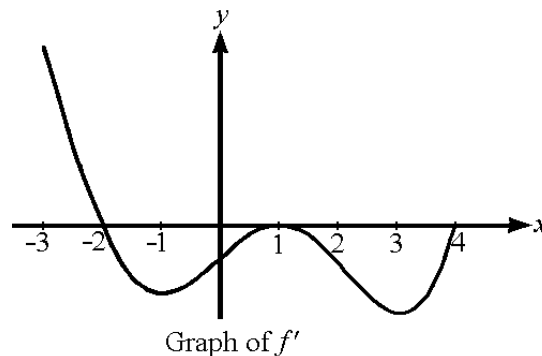
(89BC-3)

3. Consider the function f defined by $f(x) = e^x \cos x$ with domain $[0, 2\pi]$.

- (a) Find the absolute maximum and minimum values of $f(x)$.
- (b) Find the intervals on which f is increasing.
- (c) Find the x -coordinate of each point of inflection of the graph of f .

(2015-5)

4. The figure shows the graph of f' , the derivative of a twice-differentiable function f , on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x -axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12 respectively.



- (a) Find all x -coordinates at which f has a relative maximum. Give a reason for your answer.
- (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.
- (c) Find the x -coordinates of all points of inflection for the graph of f . Give a reason for your answer.
- (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

(2008-6)

5. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.
- Write an equation for the line tangent to the graph of f at $x = e^2$.
 - Find the x -coordinate of the critical point of f . Determine whether this point is a relative minimum, a relative maximum or neither for the function f . Justify your answer.
 - The graph of the function f has exactly one point of inflection. Find the x -coordinate of this point.
 - Find $\lim_{x \rightarrow 0^+} f(x)$

(92BC-4)

6. Let f be a function defined by $f(x) = \begin{cases} 2x - x^2 & \text{for } x \leq 1 \\ x^2 + kx + p & \text{for } x > 1 \end{cases}$
- For what values of k and p will f be continuous and differentiable at $x = 1$?
 - For the values of k and p found in part (a), on what interval or intervals is f increasing?
 - Using the values of k and p found in part (a), find all points of inflection of the graph of f . Support your conclusion.

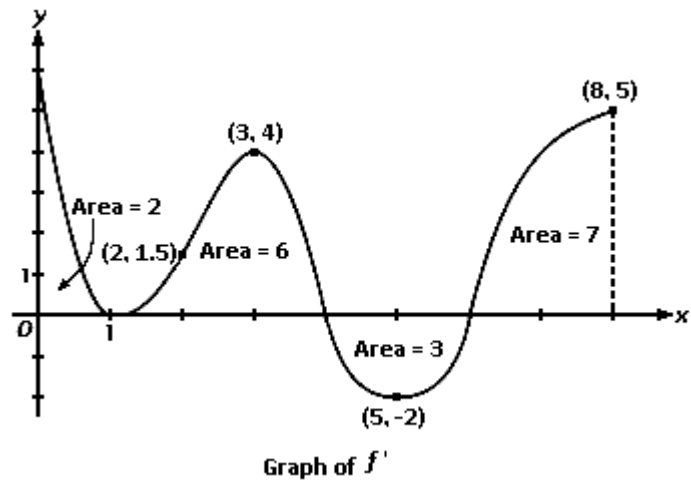
(2001-4)

7. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative h is given by $h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$.
- Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
 - On what intervals, if any, is the graph of h concave up? Justify your answer.
 - Write an equation for the line tangent to the graph of h at $x = 4$.
 - Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

OPTIONAL

(2013-4)

8.



The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the closed interval $0 \leq x \leq 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x -axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

- (a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.
- (b) Determine the absolute minimum value of f on the closed interval $0 \leq x \leq 8$. Justify your answer.
- (c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.
- (d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.