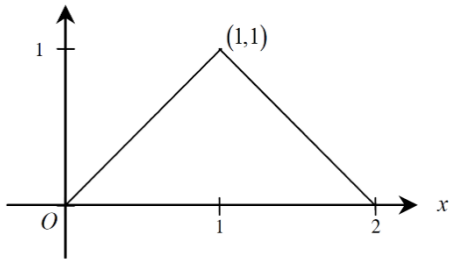
AP CALCULUS PROBLEM SET 9 INTEGRATION I (ANTIDERIVATIVES)

(93-5)

1.



The figure above shows the graph of f', the derivative of f. The domain of f is the set of all x such that $0 < x < 2$.

(a) Write an expression for $f'(x)$ in terms of x.

(b) Given that $f(1) = 0$, write an expression for $f(x)$ in terms of x.

(b) Sketch the graph of $y = f(x)$.

(74-4)

2. Let f be a function defined for all $x > -5$, and having the following properties.

(i) $f''(x) = \dfrac{1}{3\sqrt{x+5}}$ for all x in the domain of f.

(ii) The line tangent to the graph of f at $(4, 2)$ has an angle of inclination of 45°.

Find an expression for $f(x)$.

(88-6)

3. Let f be a differentiable function, defined for all real numbers x, with the following properties:

(i) $f'(x) = ax^2 + bx$

(ii) $f'(1) = 6$ and $f''(1) = 18$

(iii) $\int_1^2 f(x)dx = 18$

Find $f(x)$. Show your work.

(89-BC1)

4. Let f be a function such that $f''(x) = 6x + 8$.

(a) Find $f(x)$ if the graph of f is tangent to the line $3x - 2$ at the point $(0, -2)$.

(b) Find the average value of $f(x)$ on the closed interval $[-1, 1]$.

(2011(B)-4)

5. Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4-x)x^{-3}$ for $x > 0$.

(a) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

(b) Find all intervals on which the graph of f is concave down. Justify your answer.

(c) Given that $f(1) = 2$, determine the function f.

(2003-6)

6. Let f be the function defined by $f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5-x & \text{for } 3 < x \le 5 \end{cases}$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

(c) Suppose the function g is defined by $g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx+2 & \text{for } 3 < x \le 5, \end{cases}$ where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

(1990BC-6)

7. Let f and g be differentiable functions with the following properties:

(i) $g(x) = A - f(x)$ where A is a constant

(ii) $\int_1^2 f(x)dx = \int_2^3 g(x)dx$

(iii) $\int_2^3 f(x)dx = -3A$

(a) Find $\int_1^3 f(x)dx$ in terms of A.

(b) Find the average value of $g(x)$ in terms of A, over the interval [1, 3].

(c) Find the value of k if $\int_0^1 f(x+1)dx = kA$.