# Practice Exercises

**Part A. Directions:** Answer these questions *without* using your calculator.

**1.** $\lim_{x\to 2} \frac{x^2-4}{x^2+4}$ is

**(A)** 1 **(B)** 0 **(C)** $-\frac{1}{2}$ **(D)** −1 **(E)** ∞

**2.** $\lim_{x\to \infty} \frac{4-x^2}{x^2-1}$ is

**(A)** 1 **(B)** 0 **(C)** −4 **(D)** −1 **(E)** ∞

**3.** $\lim_{x\to 3} \frac{x-3}{x^2-2x-3}$ is

**(A)** 0 **(B)** 1 **(C)** $\frac{1}{4}$ **(D)** ∞ **(E)** none of these

**4.** $\lim_{x\to 0} \frac{x}{x}$ is

**(A)** 1 **(B)** 0 **(C)** ∞ **(D)** −1 **(E)** nonexistent

**5.** $\lim_{x\to 2} \frac{x^3-8}{x^2-4}$ is

**(A)** 4 **(B)** 0 **(C)** 1 **(D)** 3 **(E)** ∞

**6.** $\lim_{x\to \infty} \frac{4-x^2}{4x^2-x-2}$ is

**(A)** −2 **(B)** $-\frac{1}{4}$ **(C)** 1 **(D)** 2 **(E)** nonexistent

**7.** $\lim_{x\to -\infty} \frac{5x^3+27}{20x^2+10x+9}$ is

**(A)** −∞ **(B)** −1 **(C)** 0 **(D)** 3 **(E)** ∞

**8.** $\lim_{x\to \infty} \frac{3x^2+27}{x^3-27}$ is

**(A)** 3 **(B)** ∞ **(C)** 1 **(D)** −1 **(E)** 0

**9.** $\lim_{x\to \infty} \frac{2^{-x}}{2^x}$ is

**(A)** −1 **(B)** 1 **(C)** 0 **(D)** ∞ **(E)** none of these

**10.** $\lim_{x\to -\infty} \frac{2^{-x}}{2^x}$ is

**(A)** −1 **(B)** 1 **(C)** 0 **(D)** ∞ **(E)** none of these

11. $\lim_{x\to 0} \frac{\sin 5x}{x}$

(A) $= 0$ (B) $= \frac{1}{5}$ (C) $= 1$ (D) $= 5$ (E) does not exist

12. $\lim_{x\to 0} \frac{\sin 2x}{3x}$

(A) $= 0$ (B) $= \frac{2}{3}$ (C) $= 1$ (D) $= \frac{3}{2}$ (E) does not exist

13. The graph of $y = \arctan x$ has

(A) vertical asymptotes at $x = 0$ and $x = \pi$
(B) horizontal asymptotes at $y = \pm\frac{\pi}{2}$
(C) horizontal asymptotes at $y = 0$ and $y = \pi$
(D) vertical asymptotes at $x = \pm\frac{\pi}{2}$
(E) none of these

14. The graph of $y = \frac{x^2 - 9}{3x - 9}$ has

(A) a vertical asymptote at $x = 3$ (B) a horizontal asymptote at $y = \frac{1}{3}$
(C) a removable discontinuity at $x = 3$ (D) an infinite discontinuity at $x = 3$
(E) none of these

15. $\lim_{x\to 0} \frac{\sin x}{x^2 + 3x}$ is

(A) 1 (B) $\frac{1}{3}$ (C) 3 (D) $\infty$ (E) $\frac{1}{4}$

16. $\lim_{x\to 0} \sin\frac{1}{x}$ is

(A) $\infty$ (B) 1 (C) nonexistent (D) $-1$ (E) none of these

17. Which statement is true about the curve $y = \frac{2x^2 + 4}{2 + 7x - 4x^2}$ ?

(A) The line $x = -\frac{1}{4}$ is a vertical asymptote.
(B) The line $x = 1$ is a vertical asymptote.
(C) The line $y = -\frac{1}{4}$ is a horizontal asymptote.
(D) The graph has no vertical or horizontal asymptote.
(E) The line $y = 2$ is a horizontal asymptote.

18. $\lim_{x\to\infty} \frac{2x^2 + 1}{(2 - x)(2 + x)}$ is

(A) $-4$ (B) $-2$ (C) 1 (D) 2 (E) nonexistent

19. $\lim_{x\to 0}\frac{|x|}{x}$ is

(A) 0 (B) nonexistent (C) 1 (D) −1 (E) none of these

20. $\lim_{x\to\infty} x\sin\frac{1}{x}$ is

(A) 0 (B) $\infty$ (C) nonexistent (D) −1 (E) 1

21. $\lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi-x}$ is

(A) 1 (B) 0 (C) $\infty$ (D) nonexistent (E) none of these

22. Let $f(x)=\begin{cases}\frac{x^2-1}{x-1} & \text{if } x\neq 1\\ 4 & \text{if } x=1.\end{cases}$

Which of the following statements is (are) true?

I. $\lim_{x\to 1} f(x)$ exists

II. $f(1)$ exists

III. $f$ is continuous at $x = 1$

(A) I only (B) II only (C) I and II
(D) none of them (E) all of them

23. If $\begin{cases} f(x)=\frac{x^2-x}{2x} \text{ for } x\neq 0,\\ f(0)=k,\end{cases}$

and if $f$ is continuous at $x = 0$, then $k =$

(A) −1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

24. Suppose $\begin{cases} f(x)=\frac{3x(x-1)}{x^2-3x+2} \text{ for } x\neq 1,2,\\ f(1)=-3,\\ f(2)=4.\end{cases}$

Then $f(x)$ is continuous

(A) except at $x = 1$ (B) except at $x = 2$ (C) except at $x = 1$ or 2
(D) except at $x = 0$, 1, or 2 (E) at each real number

25. The graph of $f(x)=\frac{4}{x^2-1}$ has

(A) one vertical asymptote, at $x = 1$
(B) the $y$-axis as vertical asymptote
(C) the $x$-axis as horizontal asymptote and $x = \pm 1$ as vertical asymptotes
(D) two vertical asymptotes, at $x = \pm 1$, but no horizontal asymptote
(E) no asymptote

**26.** The graph of $y=\dfrac{2x^2+2x+3}{4x^2-4x}$ has

**(A)** a horizontal asymptote at $y = +\frac{1}{2}$ but no vertical asymptote
**(B)** no horizontal asymptote but two vertical asymptotes, at $x = 0$ and $x = 1$
**(C)** a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = 0$ and $x = 1$
**(D)** a horizontal asymptote at $x = 2$ but no vertical asymptote
**(E)** a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = \pm 1$

**27.** Let $f(x)=\begin{cases}\dfrac{x^2+x}{x} & \text{if } x \neq 0\\ 1 & \text{if } x = 0\end{cases}$.

Which of the following statements is (are) true?

I. $f(0)$ exists
II. $\lim_{x \to 0} f(x)$ exists
III. $f$ is continuous at $x = 0$

**(A)** I only **(B)** II only **(C)** I and II only
**(D)** all of them **(E)** none of them

**Part B. Directions:** Some of the following questions require the use of a graphing calculator.

**28.** If $[x]$ is the greatest integer not greater than $x$, then $\lim_{x \to 1/2} [x]$ is

**(A)** $\frac{1}{2}$ **(B)** 1 **(C)** nonexistent **(D)** 0 **(E)** none of these

**29.** (With the same notation) $\lim_{x \to -2} [x]$ is

**(A)** $-3$ **(B)** $-2$ **(C)** $-1$ **(D)** 0 **(E)** none of these

**30.** $\lim_{x \to \infty} \sin x$

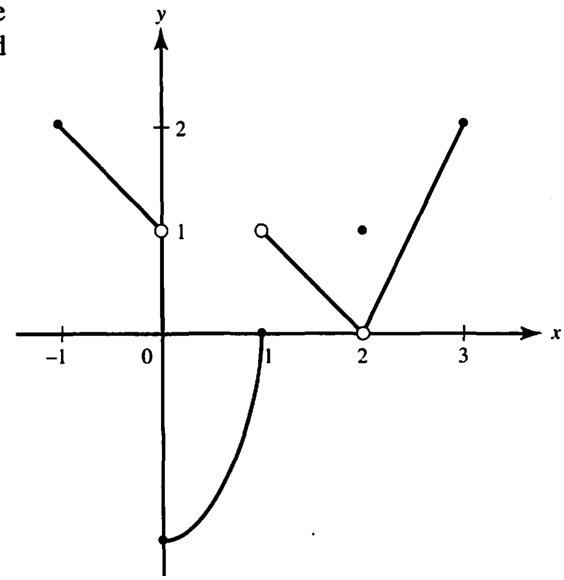
**(A)** is $-1$ **(B)** is infinity **(C)** oscillates between $-1$ and 1
**(D)** is zero **(E)** does not exist

**31.** The function $f(x) = \begin{cases} x^2/x & (x \neq 0) \\ 0 & (x = 0) \end{cases}$

**(A)** is continuous everywhere
**(B)** is continuous except at $x = 0$
**(C)** has a removable discontinuity at $x = 0$
**(D)** has an infinite discontinuity at $x = 0$
**(E)** has $x = 0$ as a vertical asymptote

Questions 32–36 are based on the function $f$ shown in the graph and defined below:

$$f(x)=\begin{cases}1-x & (-1\leqslant x<0)\\ 2x^2-2 & (0\leqslant x\leqslant 1)\\ -x+2 & (1<x<2)\\ 1 & (x=2)\\ 2x-4 & (2<x\leqslant 3)\end{cases}$$




**32.** $\lim_{x\to 2} f(x)$

**(A)** equals 0 **(B)** equals 1 **(C)** equals 2
**(D)** does not exist **(E)** none of these

**33.** The function $f$ is defined on $[-1,3]$

**(A)** if $x\neq 0$ **(B)** if $x\neq 1$ **(C)** if $x\neq 2$
**(D)** if $x\neq 3$ **(E)** at each $x$ in $[-1,3]$

**34.** The function $f$ has a removable discontinuity at

**(A)** $x=0$ **(B)** $x=1$ **(C)** $x=2$ **(D)** $x=3$ **(E)** none of these

**35.** On which of the following intervals is $f$ continuous?

**(A)** $-1\leqslant x\leqslant 0$ **(B)** $0<x<1$ **(C)** $1\leqslant x\leqslant 2$
**(D)** $2\leqslant x\leqslant 3$ **(E)** none of these

**36.** The function $f$ has a jump discontinuity at

**(A)** $x=-1$ **(B)** $x=1$ **(C)** $x=2$
**(D)** $x=3$ **(E)** none of these

**CHALLENGE**

**37.** $\lim_{x\to 0}\sqrt{3+\arctan\frac{1}{x}}$ is

**(A)** $-\infty$ **(B)** $\sqrt{3-\frac{\pi}{2}}$ **(C)** $\sqrt{3+\frac{\pi}{2}}$

**(D)** $\infty$ **(E)** none of these

**38.** Suppose $\lim_{x \to -3^-} f(x) = -1$, $\lim_{x \to -3^+} f(x) = -1$, and $f(-3)$ is not defined. Which of the following statements is (are) true?

I. $\lim_{x \to -3} f(x) = -1$.
II. $f$ is continuous everywhere except at $x = -3$.
III. $f$ has a removable discontinuity at $x = -3$.

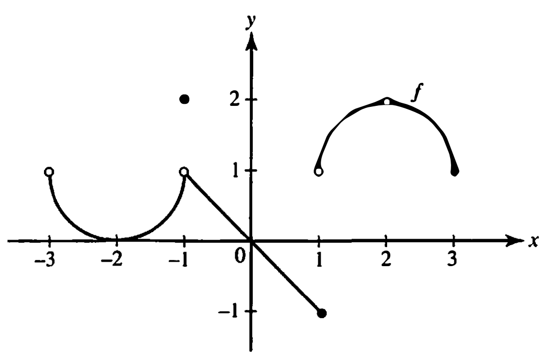
**(A)** None of them **(B)** I only **(C)** III only
**(D)** I and III only **(E)** All of them

**39.** If $y = \dfrac{1}{2 + 10^{\frac{1}{x}}}$, then $\lim_{x \to 0} y$ is

CHALLENGE

**(A)** 0 **(B)** $\dfrac{1}{12}$ **(C)** $\dfrac{1}{2}$ **(D)** $\dfrac{1}{3}$ **(E)** nonexistent

Questions 40–42 are based on the function $f$ shown in the graph.




**40.** For what value(s) of $a$ is it true that $\lim_{x \to a} f(x)$ exists and $f(a)$ exists, but $\lim_{x \to a} f(x) \neq f(a)$? It is possible that $a =$

**(A)** −1 only **(B)** 1 only **(C)** 2 only
**(D)** −1 or 1 only **(E)** −1 or 2 only

**41.** $\lim_{x \to a} f(x)$ does not exist for $a =$

**(A)** −1 only **(B)** 1 only **(C)** 2 only
**(D)** 1 and 2 only **(E)** −1, 1, and 2

**42.** Which statements about limits at $x = 1$ are true?

I. $\lim_{x \to 1^-} f(x)$ exists.
II. $\lim_{x \to 1^+} f(x)$ exists.
III. $\lim_{x \to 1} f(x)$ exists.

**(A)** none of I, II, or III **(B)** I only **(C)** II only
**(D)** I and II only **(E)** I, II, and III

## Answer Key

1. B
2. D
3. C
4. A
5. D
6. B
7. A
8. E
9. C
10. D
11. D
12. B
13. B
14. C
15. B
16. C
17. A
18. B
19. B
20. E
21. A
22. C
23. B
24. B
25. C
26. C
27. D
28. D
29. E
30. E
31. A
32. A
33. E
34. C
35. B
36. B
37. E
38. D
39. E
40. A
41. B
42. D

## Answers Explained

1. **(B)** The limit as $x \to 2$ is $0 \div 8$.

2. **(D)** Use the Rational Function Theorem (page 96). The degrees of $P(x)$ and $Q(x)$ are the same.

3. **(C)** Remove the common factor $x - 3$ from numerator and denominator.

4. **(A)** The fraction equals 1 for all nonzero $x$.

5. **(D)** Note that $\frac{x^3-8}{x^2-4} = \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}$.

6. **(B)** Use the Rational Function Theorem.

7. **(A)** Use the Rational Function Theorem.

8. **(E)** Use the Rational Function Theorem.

9. **(C)** The fraction is equivalent to $\frac{1}{2^{2x}}$; the denominator approaches $\infty$.

10. **(D)** Since $\frac{2^{-x}}{2^x} = 2^{-2x}$, therefore, as $x \to -\infty$, the fraction $\to +\infty$.

11. **(D)** $\lim_{x\to 0} \frac{\sin 5x}{x} = \lim_{x\to 0} \frac{\sin 5x}{x} \cdot \frac{5}{5} = 5 \lim_{x\to 0} \frac{\sin 5x}{5x} = 5$

12. **(B)** $\lim_{x\to 0} \frac{\sin 2x}{3x} = \frac{1}{3} \lim_{x\to 0} \frac{\sin 2x}{x} \cdot \frac{2}{2} = \frac{2}{3} \lim_{x\to 0} \frac{\sin 2x}{2x} = \frac{2}{3}$

13. **(B)** Because the graph of $y = \tan x$ has vertical asymptotes at $x = \pm\frac{\pi}{2}$, the graph of the inverse function $y = \arctan x$ has horizontal asymptotes at $y = \pm\frac{\pi}{2}$.

14. **(C)** Since $\frac{x^2-9}{3x-9} = \frac{(x-3)(x+3)}{3(x-3)} = \frac{x+3}{3}$ (provided $x \neq 3$), $y$ can be defined to be equal to 2 at $x = 3$, removing the discontinuity at that point.

15. **(B)** Note that $\frac{\sin x}{x^2+3x} = \frac{\sin x}{x(x+3)} = \frac{\sin x}{x} \cdot \frac{1}{x+3} \to 1 \cdot \frac{1}{3}$.

16. (C) As $x \to 0$, $\frac{1}{x}$ takes on varying finite values as it increases. Since the sine function repeats, $\sin \frac{1}{x}$ oscillates, taking on, infinitely many times, each value between $-1$ and $1$. The calculator graph of $Y_1 = \sin(1/X)$ exhibits this oscillating discontinuity at $x = 0$.

17. (A) Note that, since $y = \frac{2x^2+4}{(2-x)(1+4x)}$, both $x = 2$ and $x = -\frac{1}{4}$ are vertical asymptotes. Also, $y = -\frac{1}{2}$ is a horizontal asymptote.

18. (B) $\frac{2x^2+1}{(2-x)(2+x)} = \frac{2x^2+1}{4-x^2}$. Use the Rational Function Theorem (page 96).

19. (B) Since $|x| = x$ if $x > 0$ but equals $-x$ if $x < 0$, $\lim_{x\to0^+} \frac{|x|}{x} = \lim_{x\to0^+} \frac{x}{x} = 1$ while $\lim_{x\to0^-} \frac{|x|}{x} = \lim_{x\to0^-} \frac{-x}{x} = -1$.

20. (E) Note that $x \sin \frac{1}{x}$ can be rewritten as $\frac{\sin\frac{1}{x}}{\frac{1}{x}}$ and that, as $x \to \infty$, $\frac{1}{x} \to 0$.

21. (A) As $x \to \pi$, $(\pi - x) \to 0$.

22. (C) Since $f(x) = x + 1$ if $x \neq 1$, $\lim_{x\to1} f(x)$ exists (and is equal to 2).

23. (B) $f(x) = \frac{x(x-1)}{2x} = \frac{x-1}{2}$, for all $x \neq 0$. For $f$ to be continuous at $x = 0$, $\lim_{x\to0} f(x)$ must equal $f(0)$. $\lim_{x\to0} f(x) = -\frac{1}{2}$.

24. (B) Only $x = 1$ and $x = 2$ need be checked. Since $f(x) = \frac{3x}{x-2}$ for $x \neq 1, 2$, and $\lim_{x\to1} f(x) = -3 = f(1)$, $f$ is continuous at $x = 1$. Since $\lim_{x\to2} f(x)$ does not exist, $f$ is not continuous at $x = 2$.

25. (C) As $x \to \pm\infty$, $y = f(x) \to 0$, so the $x$-axis is a horizontal asymptote. Also, as $x \to \pm1$, $y \to \infty$, so $x = \pm1$ are vertical asymptotes.

26. (C) As $x \to \infty$, $y \to \frac{1}{2}$; the denominator (but not the numerator) of $y$ equals 0 at $x = 0$ and at $x = 1$.

27. (D) The function is defined at 0 to be 1, which is also $\lim_{x\to0} \frac{x^2+x}{x} = \lim_{x\to0}(x+1)$.

28. (D) See Figure N2–1 on page 88.

29. (E) Note, from Figure N2–1, that $\lim_{x\to-2^-} [x] = -3$ but $\lim_{x\to-2^+} [x] = -2$.

30. (E) As $x \to \infty$, the function $\sin x$ oscillates between $-1$ and $1$; hence the limit does not exist.

31. (A) Note that $\frac{x^2}{x} = x$ if $x \neq 0$ and that $\lim_{x\to0} f = 0$.

32. (A) $\lim_{x\to2^-} f(x) = \lim_{x\to2^+} f(x) = 0$.

33. (E) Verify that $f$ is defined at $x = 0, 1, 2$, and 3 (as well as at all other points in [−1,3]).

34. (C) Note that $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 0$. However, $f(2) = 1$. Redefining $f(2)$ as 0 removes the discontinuity.

35. (B) The function is not continuous at $x = 0$, 1, or 2.

36. (B) $\lim_{x\to 1^-} f(x) = 0 \neq \lim_{x\to 1^+} f(x) = 1$.

37. (E) As $x \to 0^-$, $\arctan \frac{1}{x} \to -\frac{\pi}{2}$, so $y \to \sqrt{3 - \frac{\pi}{2}}$. As $x \to 0^+$, $y \to \sqrt{3 + \frac{\pi}{2}}$. The graph has a jump discontinuity at $x = 0$. (Verify with a calculator.)

38. (D) No information is given about the domain of $f$ except in the neighborhood of $x = -3$.

39. (E) As $x \to 0^+$, $10^{\frac{1}{x}} \to \infty$ and therefore $y \to 0$. As $x \to 0^-$, $\frac{1}{x} \to -\infty$, so $10^{\frac{1}{x}} \to 0$ and therefore $y \to \frac{1}{2}$. Because the two one-sided limits are not equal, the limit does not exist. (Verify with a calculator.)

40. (A) $\lim_{x\to -1} f(x) = 1$, but $f(-1) = 2$. The limit does not exist at $a = 1$ and $f(2)$ does not exist.

41. (B) $\lim_{x\to -1} f(x) = 1$ and $\lim_{x\to 2} f(x) = 2$.

42. (D) $\lim_{x\to 1^-} f(x) = -1$ and $\lim_{x\to 1^+} f(x) = 1$, but since these two limits are not the same, $\lim_{x\to 1} f(x)$ does not exist.