Practice Exercises

Part A. Directions: Answer these questions *without* using your calculator.

In each of Questions 1–20 a function is given. Choose the alternative that is the derivative, $\frac{dy}{dx}$, of the function.

1. $y = x^5 \tan x$

(A) $5x^4 \tan x$ **(B)** $x^5 \sec^2 x$ **(C)** $5x^4 \sec^2 x$

(D) $5x^4 + \sec^2 x$ **(E)** $5x^4 \tan x + x^5 \sec^2 x$

2. $y = \frac{2-x}{3x+1}$

(A) $-\frac{7}{(3x+1)^2}$ **(B)** $\frac{6x-5}{(3x+1)^2}$ **(C)** $-\frac{9}{(3x+1)^2}$

(D) $\frac{7}{(3x+1)^2}$ **(E)** $\frac{7-6x}{(3x+1)^2}$

3. $y = \sqrt{3-2x}$

(A) $\frac{1}{2\sqrt{3-2x}}$ **(B)** $-\frac{1}{\sqrt{3-2x}}$ **(C)** $-\frac{(3-2x)^{3/2}}{3}$

(D) $-\frac{1}{3-2x}$ **(E)** $\frac{2}{3}(3-2x)^{3/2}$

4. $y = \frac{2}{(5x+1)^3}$

(A) $-\frac{30}{(5x+1)^2}$ **(B)** $-30(5x+1)^{-4}$ **(C)** $\frac{-6}{(5x+1)^4}$

(D) $-\frac{10}{3}(5x+1)^{-4/3}$ **(E)** $\frac{30}{(5x+1)^4}$

5. $y = 3x^{2/3} - 4x^{1/2} - 2$

(A) $2x^{1/3} - 2x^{-1/2}$ (B) $3x^{-1/3} - 2x^{-1/2}$ (C) $\frac{9}{5}x^{5/3} - 8x^{3/2}$

(D) $\frac{2}{x^{1/3}} - \frac{2}{x^{1/2}} - 2$ (E) $2x^{-1/3} - 2x^{-1/2}$

6. $y = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$

(A) $x + \frac{1}{x\sqrt{x}}$ (B) $x^{-1/2} + x^{-3/2}$ (C) $\frac{4x-1}{4x\sqrt{x}}$

(D) $\frac{1}{\sqrt{x}} + \frac{1}{4x\sqrt{x}}$ (E) $\frac{4}{\sqrt{x}} + \frac{1}{x\sqrt{x}}$

7. $y = \sqrt{x^2 + 2x - 1}$

(A) $\frac{x+1}{y}$ (B) $4y(x+1)$ (C) $\frac{1}{2\sqrt{x^2+2x-1}}$

(D) $-\frac{x+1}{(x^2+2x-1)^{3/2}}$ (E) none of these

8. $y = \frac{x^2}{\cos x}$

(A) $\frac{2x}{\sin x}$ (B) $-\frac{2x}{\sin x}$ (C) $\frac{2x\cos x - x^2 \sin x}{\cos^2 x}$

(D) $\frac{2x\cos x + x^2 \sin x}{\cos^2 x}$ (E) $\frac{2x\cos x + x^2 \sin x}{\sin^2 x}$

9. $y = \ln \frac{e^x}{e^x - 1}$

(A) $x - \frac{e^x}{e^x - 1}$ (B) $\frac{1}{e^x - 1}$ (C) $-\frac{1}{e^x - 1}$

(D) 0 (E) $\frac{e^x - 2}{e^x - 1}$

10. $y = \tan^{-1} \frac{x}{2}$

(A) $\frac{4}{4+x^2}$ (B) $\frac{1}{2\sqrt{4-x^2}}$ (C) $\frac{2}{\sqrt{4-x^2}}$

(D) $\frac{1}{2+x^2}$ (E) $\frac{2}{x^2+4}$

11. $y = \ln(\sec x + \tan x)$

(A) $\sec x$ **(B)** $\dfrac{1}{\sec x}$ **(C)** $\tan x + \dfrac{\sec^2 x}{\tan x}$

(D) $\dfrac{1}{\sec x + \tan x}$ **(E)** $-\dfrac{1}{\sec x + \tan x}$

12. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(A) 0 **(B)** 1 **(C)** $\dfrac{2}{(e^x + e^{-x})^2}$

(D) $\dfrac{4}{(e^x + e^{-x})^2}$ **(E)** $\dfrac{1}{e^{2x} + e^{-2x}}$

13. $y = \ln\left(\sqrt{x^2+1}\right)$

(A) $\dfrac{1}{\sqrt{x^2+1}}$ **(B)** $\dfrac{2x}{\sqrt{x^2+1}}$ **(C)** $\dfrac{1}{2\left(x^2+1\right)}$

(D) $\dfrac{x}{x^2+1}$ **(E)** $\dfrac{2x}{x^2+1}$

14. $y = \sin\left(\dfrac{1}{x}\right)$

(A) $\cos\left(\dfrac{1}{x}\right)$ **(B)** $\cos\left(-\dfrac{1}{x^2}\right)$ **(C)** $-\dfrac{1}{x^2}\cos\left(\dfrac{1}{x}\right)$

(D) $-\dfrac{1}{x^2}\sin\left(\dfrac{1}{x}\right) + \dfrac{1}{x}\cos\left(\dfrac{1}{x}\right)$ **(E)** $\cos(\ln x)$

15. $y = \dfrac{1}{2\sin 2x}$

(A) $-\csc 2x \cot 2x$ **(B)** $\dfrac{1}{4\cos 2x}$ **(C)** $-4\csc 2x \cot 2x$

(D) $\dfrac{\cos 2x}{2\sqrt{\sin 2x}}$ **(E)** $-\csc^2 2x$

16. $y = e^{-x}\cos 2x$

(A) $-e^{-x}(\cos 2x + 2\sin 2x)$
(B) $e^{-x}(\sin 2x - \cos 2x)$
(C) $2e^{-x}\sin 2x$
(D) $-e^{-x}(\cos 2x + \sin 2x)$
(E) $-e^{-x}\sin 2x$

17. $y = \sec^2(x)$

(A) $2 \sec x$ (B) $2 \sec x \tan x$ (C) $2 \sec^2 x \tan x$

(D) $\sec^2 x \tan^2 x$ (E) $\tan x$

18. $y = x \ln^3 x$

(A) $\frac{3 \ln^2 x}{x}$ (B) $3 \ln^2 x$ (C) $3x \ln^2 x + \ln^3 x$

(D) $3(\ln x + 1)$ (E) none of these

19. $y = \frac{1+x^2}{1-x^2}$

(A) $-\frac{4x}{(1-x^2)^2}$ (B) $\frac{4x}{(1-x^2)^2}$ (C) $\frac{-4x^3}{(1-x^2)^2}$

(D) $\frac{2x}{1-x^2}$ (E) $\frac{4}{1-x^2}$

20. $y = \sin^{-1} x - \sqrt{1-x^2}$

(A) $\frac{1}{2\sqrt{1-x^2}}$ (B) $\frac{2}{\sqrt{1-x^2}}$ (C) $\frac{1+x}{\sqrt{1-x^2}}$

(D) $\frac{x^2}{\sqrt{1-x^2}}$ (E) $\frac{1}{\sqrt{1+x}}$

In each of Questions 21–24, y is a differentiable function of x. Choose the alternative that is the derivative $\frac{dy}{dx}$.

21. $x^3 - y^3 = 1$

(A) x (B) $3x^2$ (C) $\sqrt[3]{3x^2}$ (D) $\frac{x^2}{y^2}$ (E) $\frac{3x^2-1}{y^2}$

22. $x + \cos(x + y) = 0$

(A) $\csc(x + y) - 1$ (B) $\csc(x + y)$ (C) $\frac{x}{\sin(x+y)}$

(D) $\frac{1}{\sqrt{1-x^2}}$ (E) $\frac{1-\sin x}{\sin y}$

23. $\sin x - \cos y - 2 = 0$

(A) $-\cot x$ (B) $-\cot y$ (C) $\frac{\cos x}{\sin y}$

(D) $-\csc y \cos x$ (E) $\frac{2-\cos x}{\sin y}$

24. $3x^2 - 2xy + 5y^2 = 1$

(A) $\frac{3x+y}{x-5y}$ (B) $\frac{y-3x}{5y-x}$ (C) $3x + 5y$

(D) $\frac{3x+4y}{x}$ (E) none of these

BC ONLY

25. If $x = t^2 + 1$ and $y = 2t^3$, then $\frac{dy}{dx} =$

(A) $3t$ (B) $6t^2$ (C) $\frac{6t^2}{t^2+1}$ (D) $\frac{6t^2}{(t^2+1)^2}$ (E) $\frac{2t^4+6t^2}{(t^2+1)^2}$

26. If $f(x) = x^4 - 4x^3 + 4x^2 - 1$, then the set of values of x for which the derivative equals zero is

(A) $\{1, 2\}$ (B) $\{0, -1, -2\}$ (C) $\{-1, +2\}$
(D) $\{0\}$ (E) $\{0, 1, 2\}$

27. If $f(x) = 16\sqrt{x}$, then $f''(4)$ is equal to

(A) -32 (B) -16 (C) -4 (D) -2 (E) $-\frac{1}{2}$

28. If $f(x) = \ln x^3$, then $f''(3)$ is

(A) $-\frac{1}{3}$ (B) -1 (C) -3 (D) 1 (E) none of these

29. If a point moves on the curve $x^2 + y^2 = 25$, then, at $(0, 5)$, $\frac{d^2y}{dx^2}$ is

(A) 0 (B) $\frac{1}{5}$ (C) -5 (D) $-\frac{1}{5}$ (E) nonexistent

BC ONLY

30. If $x = t^2 - 1$ and $y = t^4 - 2t^3$, then, when $t = 1$, $\frac{d^2y}{dx^2}$ is

(A) 1 (B) -1 (C) 0 (D) 3 (E) $\frac{1}{2}$

31. If $f(x) = 5^x$ and $5^{1.002} \simeq 5.016$, which is closest to $f'(1)$?

(A) 0.016 (B) 1.0 (C) 5.0 (D) 8.0 (E) 32.0

32. If $y = e^x(x - 1)$, then $y''(0)$ equals

(A) -2 (B) -1 (C) 0 (D) 1 (E) none of these

BC ONLY

33. If $x = e^\theta \cos\theta$ and $y = e^\theta \sin\theta$, then, when $\theta = \frac{\pi}{2}$, $\frac{dy}{dx}$ is

(A) 1 (B) 0 (C) $e^{\pi/2}$ (D) nonexistent (E) -1

BC ONLY

34. If $x = \cos t$ and $y = \cos 2t$, then $\frac{d^2y}{dx^2}$ $(\sin t \neq 0)$ is

(A) $4 \cos t$ (B) 4 (C) $\frac{4y}{x}$ (D) -4 (E) $-4 \cot t$

35. $\lim_{h\to 0} \frac{(1+h)^6 - 1}{h}$ is

(A) 0 (B) 1 (C) 6 (D) ∞ (E) nonexistent

36. $\lim_{h\to 0} \frac{\sqrt[3]{8+h} - 2}{h}$ is

(A) 0 (B) $\frac{1}{12}$ (C) 1 (D) 192 (E) ∞

37. $\lim_{h\to 0} \frac{\ln(e+h) - 1}{h}$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) nonexistent

38. $\lim_{x\to 0} \frac{\cos x - 1}{x}$ is

(A) -1 (B) 0 (C) 1 (D) ∞ (E) none of these

39. If $f(x) = \begin{cases} \frac{4x^2 - 4}{x - 1}, & x \neq 1 \\ 4, & x = 1 \end{cases}$, which of these statements are true?

I. $\lim_{x\to 1} f(x)$ exists.

II. f is continuous at $x = 1$.

III. f is differentiable at $x = 1$.

(A) none (B) I only (C) I and II only
(D) I and III only (E) I, II, and III

40. If $g(x) = \begin{cases} x^2, & x \leq 3 \\ 6x - 9, & x > 3 \end{cases}$, which of these statements are true?

I. $\lim_{x\to 3} g(x)$ exists.

II. g is continuous at $x = 3$.

III. g is differentiable at $x = 3$.

(A) I only (B) II only (C) III only
(D) I and II only (E) I, II, and III

41. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the Mean Value Theorem because

(A) $f(0)$ is not defined (B) $f(x)$ is not continuous on $[-8, 8]$
(C) $f'(-1)$ does not exist (D) $f(x)$ is not defined for $x < 0$
(E) $f'(0)$ does not exist

42. If $f(x) = 2x^3 - 6x$, at what point on the interval $0 \leq x \leq \sqrt{3}$, if any, is the tangent to the curve parallel to the secant line on that interval?

(A) 1 (B) −1 (C) $\sqrt{2}$ (D) 0 (E) nowhere

43. If h is the inverse function of f and if $f(x) = \frac{1}{x}$, then $h'(3) =$

(A) −9 (B) $-\frac{1}{9}$ (C) $\frac{1}{9}$ (D) 3 (E) 9

BC ONLY

44. $\lim_{x\to\infty} \frac{e^x}{x^{50}}$ equals

(A) 0 (B) 1 (C) $\frac{1}{50!}$ (D) ∞ (E) none of these

45. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

(A) $\sec(xy)$ (B) $\frac{\sec(xy)}{x}$ (C) $\frac{\sec(xy) - y}{x}$

(D) $\frac{1 + \sec(xy)}{x}$ (E) $\sec(xy) - 1$

46. $\lim_{x\to 0} \frac{\sin 2x}{x}$ is

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) 0 (E) ∞

47. $\lim_{x\to 0} \frac{\sin 3x}{\sin 4x}$ is

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{3}{4}$ (D) 0 (E) nonexistent

48. $\lim_{x\to 0} \frac{1 - \cos x}{x}$ is

(A) nonexistent (B) 1 (C) 2 (D) ∞ (E) none of these

49. $\lim_{x\to 0} \frac{\tan \pi x}{x}$ is

(A) $\frac{1}{\pi}$ (B) 0 (C) 1 (D) π (E) ∞

50. $\lim_{x\to\infty} x^2 \sin \frac{1}{x}$

(A) is 1 (B) is 0 (C) is ∞

(D) oscillates between −1 and 1 (E) is none of these

51. The graph in the xy-plane represented by $x = 3 + 2\sin t$ and $y = 2\cos t - 1$, for $-\pi \leqslant t \leqslant \pi$, is

BC ONLY

(A) a semicircle **(B)** a circle **(C)** an ellipse
(D) half of an ellipse **(E)** a hyperbola

52. $\lim\limits_{x\to 0} \dfrac{\sec x - \cos x}{x^2}$ equals

(A) 0 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** none of these

BC ONLY

In each of Questions 53–56 a pair of equations that represent a curve parametrically is given. Choose the alternative that is the derivative $\dfrac{dy}{dx}$.

53. $x = t - \sin t$ and $y = 1 - \cos t$

(A) $\dfrac{\sin t}{1-\cos t}$ **(B)** $\dfrac{1-\cos t}{\sin t}$ **(C)** $\dfrac{\sin t}{\cos t - 1}$

(D) $\dfrac{1-x}{y}$ **(E)** $\dfrac{1-\cos t}{t - \sin t}$

BC ONLY

54. $x = \cos^3\theta$ and $y = \sin^3\theta$

(A) $\tan^3\theta$ **(B)** $-\cot\theta$ **(C)** $\cot\theta$ **(D)** $-\tan\theta$ **(E)** $-\tan^2\theta$

55. $x = 1 - e^{-t}$ and $y = t + e^{-t}$

(A) $\dfrac{e^{-t}}{1-e^{-t}}$ **(B)** $e^{-t} - 1$ **(C)** $e^t + 1$ **(D)** $e^t - e^{-2t}$ **(E)** $e^t - 1$

56. $x = \dfrac{1}{1-t}$ and $y = 1 - \ln(1-t)$ $(t < 1)$

(A) $\dfrac{1}{1-t}$ **(B)** $t - 1$ **(C)** $\dfrac{1}{x}$ **(D)** $\dfrac{(1-t)^2}{t}$ **(E)** $1 + \ln x$

Part B. Directions: Some of the following questions require the use of a graphing calculator.

In Questions 57–64, differentiable functions f and g have the values shown in the table.

x	f	f'	g	g'
0	2	1	5	–4
1	3	2	3	–3
2	5	3	1	–2
3	10	4	0	–1

57. If $A = f + 2g$, then $A'(3) =$

(A) -2 (B) 2 (C) 7 (D) 8 (E) 10

58. If $B = f \cdot g$, then $B'(2) =$

(A) -20 (B) -7 (C) -6 (D) -1 (E) 13

59. If $D = \frac{1}{g}$, then $D'(1) =$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$ (C) $-\frac{1}{9}$ (D) $\frac{1}{9}$ (E) $\frac{1}{3}$

60. If $H(x) = \sqrt{f(x)}$, then $H'(3) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2\sqrt{10}}$ (C) 2 (D) $\frac{2}{\sqrt{10}}$ (E) $4\sqrt{10}$

61. If $K(x) = \left(\frac{f}{g}\right)(x)$, then $K'(0) =$

(A) $\frac{-13}{25}$ (B) $-\frac{1}{4}$ (C) $\frac{13}{25}$ (D) $\frac{13}{16}$ (E) $\frac{22}{25}$

62. If $M(x) = f(g(x))$, then $M'(1) =$

(A) -12 (B) -6 (C) 4 (D) 6 (E) 12

63. If $P(x) = f(x^3)$, then $P'(1) =$

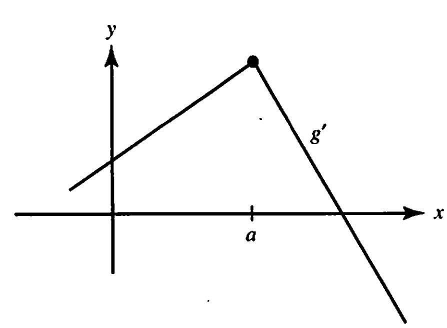
(A) 2 (B) 6 (C) 8 (D) 12 (E) 54

64. If $S(x) = f^{-1}(x)$, then $S'(3) =$

(A) -2 (B) $-\frac{1}{25}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 2

65. The graph of g' is shown here. Which of the following statements is (are) true of g at $x = a$?

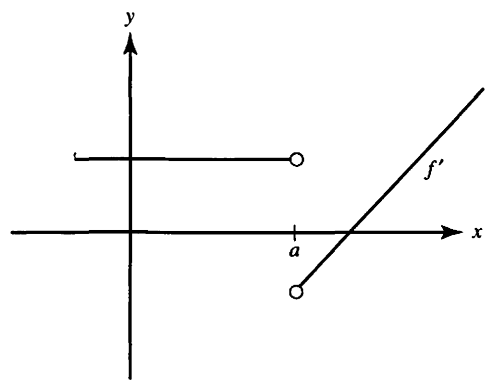
I. g is continuous.
II. g is differentiable.
III. g is increasing.



(A) I only (B) III only (C) I and III only
(D) II and III only (E) I, II, and III

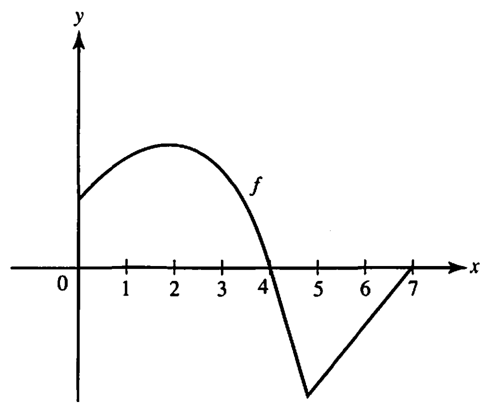
66. A function f has the derivative shown. Which of the following statements must be false?

(A) f is continuous at $x = a$.
(B) $f(a) = 0$.
(C) f has a vertical asymptote at $x = a$.
(D) f has a jump discontinuity at $x = a$.
(E) f has a removable discontinuity at $x = a$.



67. The function f whose graph is shown has $f' = 0$ at $x =$

(A) 2 only
(B) 2 and 5
(C) 4 and 7
(D) 2, 4, and 7
(E) 2, 4, 5, and 7



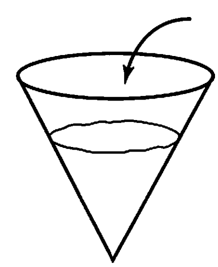
68. A differentiable function f has the values shown. Estimate $f'(1.5)$.

x	1.0	1.2	1.4	1.6
$f(x)$	8	10	14	22

(A) 8 (B) 12 (C) 18 (D) 40 (E) 80

69. Water is poured into a conical reservoir at a constant rate. If $h(t)$ is the rate of change of the depth of the water, then h is

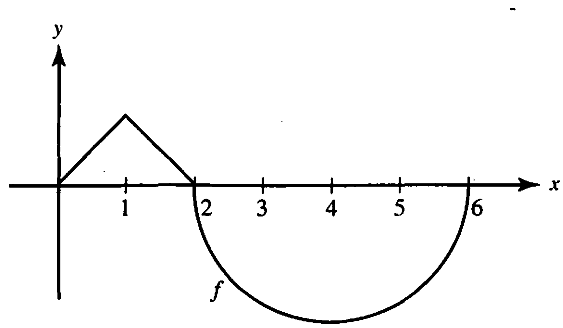
(A) constant
(B) linear and increasing
(C) linear and decreasing
(D) nonlinear and increasing
(E) nonlinear and decreasing



Use the figure to answer Questions 70–72. The graph of f consists of two line segments and a semicircle.

70. $f'(x) = 0$ for $x =$

(A) 1 only
(B) 2 only
(C) 4 only
(D) 1 and 4
(E) 2 and 6



71. $f'(x)$ does not exist for $x =$

(A) 1 only **(B)** 2 only **(C)** 1 and 2
(D) 2 and 6 **(E)** 1, 2, and 6

72. $f'(5) =$

(A) $\frac{1}{2}$ **(B)** $\frac{1}{\sqrt{3}}$ **(C)** 1 **(D)** 2 **(E)** $\sqrt{3}$

73. At how many points on the interval $[-5,5]$ is a tangent to $y = x + \cos x$ parallel to the secant line?

(A) none **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 3

74. From the values of f shown, estimate $f'(2)$.

x	1.92	1.94	1.96	1.98	2.00
$f(x)$	6.00	5.00	4.40	4.10	4.00

(A) -0.10 **(B)** -0.20 **(C)** -5 **(D)** -10 **(E)** -25

75. Using the values shown in the table for Question 74, estimate $(f^{-1})'(4)$.

(A) -0.2 **(B)** -0.1 **(C)** -5 **(D)** -10 **(E)** -25

76. The "left half" of the parabola defined by $y = x^2 - 8x + 10$ for $x \leq 4$ is a one-to-one function; therefore its inverse is also a function. Call that inverse g. Find $g'(3)$.

(A) $-\frac{1}{2}$ **(B)** $-\frac{1}{6}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{11}{2}$

77. The table below shows some points on a function f that is both continuous and differentiable on the closed interval $[2,10]$.

x	2	4	6	8	10
$f(x)$	30	25	20	25	30

Which must be true?

(A) $f(x) > 0$ for $2 < x < 10$
(B) $f'(6) = 0$
(C) $f'(8) > 0$
(D) The maximum value of f on the interval $[2,10]$ is 30.
(E) For some value of x on the interval $[2,10]$ $f'(x) = 0$.

78. If f is differentiable and difference quotients overestimate the slope of f at $x = a$ for all $h > 0$, which must be true?

(A) $f'(a) > 0$ **(B)** $f'(a) < 0$ **(C)** $f''(a) > 0$
(D) $f''(a) < 0$ **(E)** none of these

79. If $f(u) = \sin u$ and $u = g(x) = x^2 - 9$, then $(f \circ g)'(3)$ equals

(A) 0 **(B)** 1 **(C)** 6 **(D)** 9 **(E)** none of these

80. If $f(x) = \frac{x}{(x-1)^2}$, then the set of x's for which $f'(x)$ exists is

(A) all reals
(B) all reals except $x = 1$ and $x = -1$
(C) all reals except $x = -1$
(D) all reals except $x = \frac{1}{3}$ and $x = -1$
(E) all reals except $x = 1$

81. If $y = \sqrt{x^2+1}$, then the derivative of y^2 with respect to x^2 is

BC ONLY

(A) 1 (B) $\frac{x^2+1}{2x}$ (C) $\frac{x}{2(x^2+1)}$ (D) $\frac{2}{x}$ (E) $\frac{x^2}{x^2+1}$

82. If $y = x^2 + x$, then the derivative of y with respect to $\frac{1}{1-x}$ is

BC ONLY

(A) $(2x+1)(x-1)^2$ (B) $\frac{2x+1}{(1-x)^2}$ (C) $2x+1$

(D) $\frac{3-x}{(1-x)^3}$ (E) none of these

83. If $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sqrt{x}$, then the derivative of $f(g(x))$ is

(A) $\frac{-\sqrt{x}}{(x^2+1)^2}$ (B) $-(x+1)^{-2}$ (C) $\frac{-2x}{(x^2+1)^2}$

(D) $\frac{1}{(x+1)^2}$ (E) $\frac{1}{2\sqrt{x}(x+1)}$

84. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

(A) $f(x)$ must be identically zero
(B) $f'(x)$ may be different from zero for all x on $[a, b]$
(C) there exists at least one number c, $a < c < b$, such that $f'(c) = 0$
(D) $f'(x)$ must exist for every x on (a, b)
(E) none of the preceding is true

85. Suppose $y = f(x) = 2x^3 - 3x$. If $h(x)$ is the inverse function of f, then $h'(-1) =$

(A) -1 (B) $\frac{1}{5}$ (C) $\frac{1}{3}$ (D) 1 (E) 3

86. Suppose $f(1) = 2$, $f'(1) = 3$, and $f'(2) = 4$. Then $(f^{-1})'(2)$

(A) equals $-\frac{1}{3}$ (B) equals $-\frac{1}{4}$ (C) equals $\frac{1}{4}$

(D) equals $\frac{1}{3}$ (E) cannot be determined

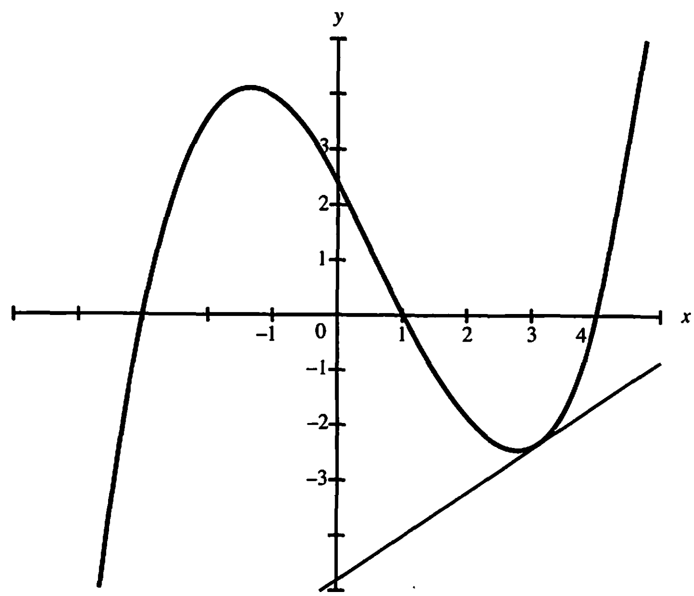
87. If $f(x) = x^3 - 3x^2 + 8x + 5$ and $g(x) = f^{-1}(x)$, then $g'(5) =$

(A) 8 (B) $\frac{1}{8}$ (C) 1 (D) $\frac{1}{53}$ (E) 5

88. Suppose $\lim_{x \to 0} \frac{g(x) - g(0)}{x} = 1$. It follows necessarily that

(A) g is not defined at $x = 0$
(B) g is not continuous at $x = 0$
(C) the limit of $g(x)$ as x approaches 0 equals 1
(D) $g'(0) = 1$
(E) $g'(1) = 0$

Use this graph of $y = f(x)$ for Questions 89 and 90.



89. $f'(3)$ is most closely approximated by

(A) 0.3 (B) 0.8 (C) 1.5 (D) 1.8 (E) 2

90. The rate of change of $f(x)$ is least at $x \simeq$

(A) –3 (B) –1.3 (C) 0 (D) 0.7 (E) 2.7

Use the following definition of the *symmetric difference quotient for* $f'(x_0)$ for Questions 91–93: For small values of h,

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}.$$

91. For $f(x) = 5^x$, what is the estimate of $f'(2)$ obtained by using the symmetric difference quotient with $h = 0.03$?

(A) 25.029 (B) 40.236 (C) 40.252 (D) 41.223 (E) 80.503

92. To how many places is the symmetric difference quotient accurate when it is used to approximate $f'(0)$ for $f(x) = 4^x$ and $h = 0.08$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4

93. To how many places is $f'(x_0)$ accurate when it is used to approximate $f'(0)$ for $f(x) = 4^x$ and $h = 0.001$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4

94. The value of $f'(0)$ obtained using the symmetric difference quotient with $f(x) = |x|$ and $h = 0.001$ is

(A) −1 (B) 0 (C) ±1 (D) 1 (E) indeterminate

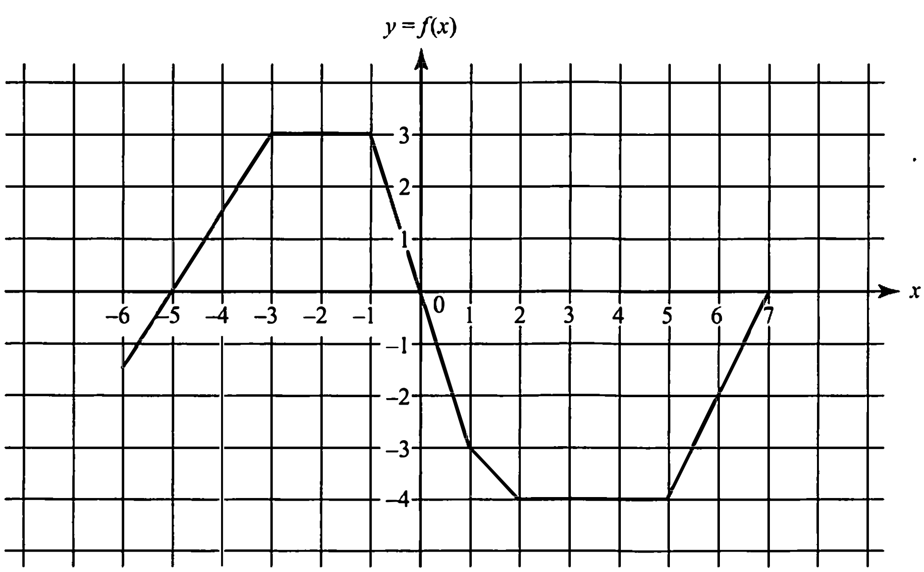
95. If $\frac{d}{dx} f(x) = g(x)$ and $h(x) = \sin x$, then $\frac{d}{dx} f(h(x))$ equals

(A) $g(\sin x)$ (B) $\cos x \cdot g(x)$ (C) $g'(x)$
(D) $\cos x \cdot g(\sin x)$ (E) $\sin x \cdot g(\sin x)$

96. Let $f(x) = 3^x - x^3$. The tangent to the curve is parallel to the secant through (0,1) and (3,0) for $x =$

(A) 0.984 only (B) 1.244 only (C) 2.727 only
(D) 0.984 and 2.804 only (E) 1.244 and 2.727 only

Questions 97–101 are based on the following graph of $f(x)$, sketched on $-6 \leq x \leq 7$. Assume the horizontal and vertical grid lines are equally spaced at unit intervals.



97. On the interval $1 < x < 2$, $f(x)$ equals

(A) $-x - 2$ (B) $-x - 3$ (C) $-x - 4$ (D) $-x + 2$ (E) $x - 2$

98. Over which of the following intervals does $f'(x)$ equal zero?

I. (−6,−3) II. (−3,−1) III. (2,5)

(A) I only (B) II only (C) I and II only
(D) I and III only (E) II and III only

99. How many points of discontinuity does $f'(x)$ have on the interval $-6 < x < 7$?

(A) none (B) 2 (C) 3 (D) 4 (E) 5

100. For $-6 < x < -3$, $f'(x)$ equals

(A) $-\frac{3}{2}$ (B) -1 (C) 1 (D) $\frac{3}{2}$ (E) 2

101. Which of the following statements about the graph of $f'(x)$ is false?

(A) It consists of six horizontal segments.
(B) It has four jump discontinuities.
(C) $f'(x)$ is discontinuous at each x in the set $\{-3,-1,1,2,5\}$.
(D) $f'(x)$ ranges from -3 to 2.
(E) On the interval $-1 < x < 1$, $f'(x) = -3$.

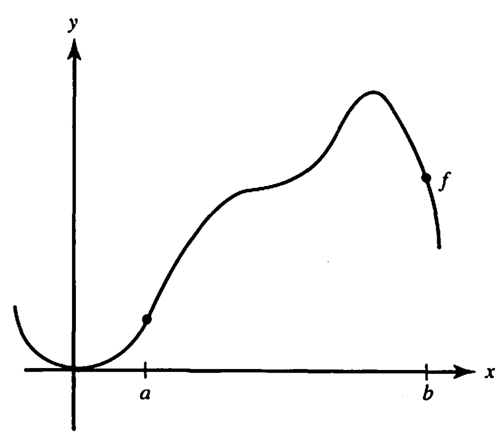
102. The table gives the values of a function f that is differentiable on the interval $[0,1]$:

x	0.10	0.20	0.30	0.40	0.50	0.60
$f(x)$	0.171	0.288	0.357	0.384	0.375	0.336

According to this table, the best approximation of $f'(0.10)$ is

(A) 0.12 (B) 1.08 (C) 1.17 (D) 1.77 (E) 2.88

103. At how many points on the interval $[a,b]$ does the function graphed satisfy the Mean Value Theorem?



(A) none (B) 1 (C) 2 (D) 3 (E) 4

Answer Key

1. E
2. A
3. B
4. B
5. E
6. D
7. A
8. D
9. C
10. E
11. A
12. D
13. D
14. C
15. A
16. A
17. C
18. E
19. B
20. C
21. D
22. A
23. D
24. B
25. A
26. E
27. E
28. A
29. D
30. E
31. D
32. D
33. E
34. B
35. C
36. B
37. B
38. B
39. B
40. E
41. E
42. A
43. B
44. D
45. C
46. B
47. C
48. E
49. D
50. C
51. B
52. C
53. A
54. D
55. E
56. C
57. B
58. B
59. E
60. D
61. C
62. A
63. B
64. D
65. E
66. C
67. A
68. D
69. E
70. C
71. E
72. B
73. D
74. C
75. A
76. B
77. E
78. C
79. C
80. E
81. A
82. A
83. B
84. B
85. C
86. D
87. B
88. D
89. B
90. D
91. C
92. B
93. E
94. B
95. D
96. E
97. A
98. E
99. E
100. D
101. B
102. C
103. D

Answers Explained

Many of the explanations provided include intermediate steps that would normally be reached on the way to a final algebraically simplified result. You may not need to reach the final answer.

NOTE: the formulas or rules cited in parentheses in the explanations are given on pages 113 and 114.

1. (E) By the Product Rule, (5),

$$y' = x^5(\tan x)' + (x^5)'(\tan x).$$

2. (A) By the Quotient Rule, (6),

$$y' = \frac{(3x+1)(-1)-(2-x)(3)}{(3x+1)^2} = -\frac{7}{(3x+1)^2}.$$

3. (B) Since $y = (3-2x)^{1/2}$, by the Power Rule, (3),

$$y' = \frac{1}{2}(3-3x)^{-1/2}\cdot(-2) = -\frac{1}{\sqrt{3-2x}}.$$

4. (B) Since $y = 2(5x+1)^{-3}, y' = -6(5x+1)^{-4}(5).$

5. (E) $y' = 3\left(\frac{2}{3}\right)x^{-1/3} - 4\left(\frac{1}{2}\right)x^{-1/2}$

6. (D) Rewrite: $y = 2x^{1/2} - \frac{1}{2}x^{1/2}$, so $y' = x^{-1/2} + \frac{1}{4}x^{-3/2}$.

7. (A) Rewrite: $y = (x^2 + 2x - 1)^{1/2}$; then $y = \frac{1}{2}(x^2 + 2x - 1)^{-1/2}(2x + 2)$.

8. (D) Use the Quotient Rule:

$$y' = \frac{2x\cos x - x^2(-\sin x)}{\cos^2 x}.$$

9. (C) Since

$$\begin{aligned} y &= \ln e^x - \ln(e^x - 1) \\ &= x - \ln(e^x - 1), \end{aligned}$$

then

$$y' = 1 - \frac{e^x}{e^x - 1} = \frac{e^x - 1 - e^x}{e^x - 1} = -\frac{1}{e^x - 1}.$$

10. (E) Use formula (18): $y' = \dfrac{\frac{1}{2}}{1 + \frac{x^2}{4}}$.

11. (A) Use formulas (13), (11), and (9):

$$y' = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}.$$

12. (D) By the Quotient Rule,

$$\begin{aligned} y' &= \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2} \\ &= \frac{(e^{2x} + 2 + e^{-2x}) - (e^{2x} - 2 + e^{-2x})}{(e^x + e^{-x})^2} = \frac{4}{(e^x + e^{-x})^2}. \end{aligned}$$

13. (D) Since $y = \frac{1}{2}\ln(x^2 + 1)$,

$$y' = \frac{1}{2} \cdot \frac{2x}{x^2 + 1}.$$

14. (C) $y' = \sin'\left(\frac{1}{x}\right) \cdot \left(\frac{1}{x}\right)' = \cos\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right)$.

15. (A) Since $y = \frac{1}{2}\csc 2x$, $y' = \frac{1}{2}(-\csc 2x \cot 2x \cdot 2)$.

16. (A) $y' = e^{-x}(-2\sin 2x) + \cos 2x(-e^{-x})$.

17. (C) $y' = (2\sec x)(\sec x \tan x)$.

18. (E) $y' = \dfrac{x(3\ln^2 x)}{x} + \ln^3 x$. The correct answer is $3\ln^2 x + \ln^3 x$.

19. (B) $y' = \dfrac{(1 - x^2)(2x) - (1 + x^2)(-2x)}{(1 - x^2)^2}$.

20. (C) $y' = \dfrac{1}{\sqrt{1 - x^2}} - \dfrac{1 \cdot (-2x)}{2\sqrt{1 - x^2}}$.

21. **(D)** Let y' be $\frac{dy}{dx}$; then $3x^2 - 3y^2y' = 0$; $y' = \frac{-3x^2}{-3y^2}$.

22. **(A)** $1 - \sin(x + y)(1 + y') = 0$; $\frac{1-\sin(x+y)}{\sin(x+y)} = y'$.

23. **(D)** $\cos x + \sin y \cdot y' = 0$; $y' = -\frac{\cos x}{\sin y}$.

24. **(B)** $6x - 2(xy' + y) + 10yy' = 0$; $y'(10y - 2x) = 2y - 6x$.

25. **(A)** $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t^2}{2t}$.

26. **(E)** $f'(x) = 4x^3 - 12x^2 + 8x = 4x(x - 1)(x - 2)$.

27. **(E)** $f'(x) = 8x^{-1/2}$; $f''(x) = -4x^{-3/2} = -\frac{4}{x^{3/2}}$; $f''(4) = -\frac{4}{8}$.

28. **(A)** $f(x) = 3\ln x$; $f'(x) = \frac{3}{x}$; $f''(x) = \frac{-3}{x^2}$. Replace x by 3.

29. **(D)** $2x + 2yy' = 0$; $y' = -\frac{x}{y}$; $y'' = -\frac{y-xy'}{y^2}$. At (0,5), $y'' = -\frac{5-0}{25}$.

30. **(E)** $\frac{dy}{dx} = \frac{4t^3 - 6t^2}{2t} = 2t^2 - 3t\ (t \neq 0)$; $\frac{d^2y}{dx^2} = \frac{4t-3}{2t}$. Replace t by 1.

31. **(D)** $f'(1) \simeq \frac{5^{1.002} - 5^1}{0.002} = \frac{5.016 - 5}{0.002}$.

32. **(D)** $y' = e^x \cdot 1 + e^x(x - 1) = xe^x$;
$y'' = xe^x + e^x$ and $y''(0) = 0 \cdot 1 + 1 = 1$.

33. **(E)** When simplified, $\frac{dy}{dx} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$.

34. **(B)** Since (if $\sin t \neq 0$)

$$\frac{dy}{dt} = -2\sin 2t = -4\sin t\cos t \text{ and } \frac{dx}{dt} = -\sin t,$$

then $\frac{dy}{dx} = 4\cos t$. Thus:

$$\frac{d^2y}{dx^2} = -\frac{4\sin t}{-\sin t}.$$

BC ONLY

NOTE: Since each of the limits in Questions 35–39 yields an indeterminate form of the type $\frac{0}{0}$, we can apply L'Hôpital's Rule in each case, getting identical answers.

35. **(C)** The given limit is the derivative of $f(x) = x^6$ at $x = 1$.

36. **(B)** The given limit is the definition for $f'(8)$, where $f(x) = \sqrt[3]{x}$;

$$f'(x) = \frac{1}{3x^{2/3}}.$$

37. (B) The given limit is $f'(e)$, where $f(x) = \ln x$.

38. (B) The given limit is the derivative of $f(x) = \cos x$ at $x = 0$; $f'(x) = -\sin x$.

39. (B) $\lim\limits_{x\to 1}\frac{4x^2-4}{x-1} = \lim\limits_{x\to 1}\frac{4(x+1)(x-1)}{x-1} = \lim\limits_{x\to 1} 4(x+1) = 8$, but $f(1) = 4$.

Thus f is discontinuous at $x = 1$, so it cannot be differentiable.

40. (E) $\lim\limits_{x\to 3^-} x^2 = \lim\limits_{x\to 3^+}(6x-9) = 9$, so the limit exists. Because $g(3) = 9$, g is continuous at $x = 3$. Since $g'(x) = \begin{cases} 2x, & x<3 \\ 6, & x>3 \end{cases}$, so $g'(3) = 6$.

41. (E) Since $f'(x) = \frac{2}{3x^{1/3}}$, $f'(0)$ is not defined; $f'(x)$ must be defined on $(-8,8)$.

42. (A) Note that $f(0) = f(\sqrt{3}) = 0$ and that $f'(x)$ exists on the given interval. By the MVT, there is a number, c, in the interval such that $f'(c) = 0$. If $c = 1$, then $6c^2 - 6 = 0$. (-1 is not in the interval.)

43. (B) Since the inverse, h, of $f(x) = \frac{1}{x}$ is $h(x) = \frac{1}{x}$, then $h'(x) = -\frac{1}{x^2}$. Replace x by 3.

44. (D) After 50(!) applications of L'Hôpital's Rule we get $\lim\limits_{x\to\infty}\frac{e^x}{50!}$, which "equals" ∞. A perfunctory examination of the limit, however, shows immediately that the answer is ∞. In fact, $\lim\limits_{x\to\infty}\frac{e^x}{x^n}$ for any positive integer n, no matter how large, is ∞.

45. (C) $\cos(xy)(xy'+y) = 1;\ x\cos(xy)y' = 1 - y\cos(xy);$

$$y' = \frac{1 - y\cos(xy)}{x\cos(xy)}.$$

NOTE: In Questions 46–50 the limits are all indeterminate forms of the type $\frac{0}{0}$. We have therefore applied L'Hôpital's Rule in each one. The indeterminacy can also be resolved by introducing $\frac{\sin a}{a}$, which approaches 1 as a approaches 0. The latter technique is presented in square brackets.

46. (B) $\lim\limits_{x\to 0}\frac{\sin 2x}{x} = \lim\limits_{x\to 0}\frac{2\cos 2x}{1} = \frac{2\cdot 1}{1} = 2.$

[Using $\sin 2x = 2\sin x\cos x$ yields $\lim\limits_{x\to 0} 2\left(\frac{\sin x}{x}\right)\cos x = 2\cdot 1\cdot 1 = 2$.]

47. (C) $\lim\limits_{x\to 0}\frac{\sin 3x}{\sin 4x} = \lim\limits_{x\to 0}\frac{3\cos 3x}{4\cos 4x} = \frac{3\cdot 1}{4\cdot 1} = \frac{3}{4}.$

[We rewrite $\frac{\sin 3x}{\sin 4x}$ as $\frac{\sin 3x}{3x}\cdot\frac{4x}{\sin 4x}\cdot\frac{3}{4}$. As $x \to 0$, so do $3x$ and $4x$; the fraction approaches $1\cdot 1\cdot \frac{3}{4}$.]

48. (E) $\lim_{x\to 0} \frac{1-\cos x}{x} = \lim_{x\to 0} \frac{\sin x}{1} = 0.$

[We can replace $1 - \cos x$ by $2 \sin^2 \frac{x}{2}$, getting

$$\lim_{x\to 0} \frac{2\sin^2 \frac{x}{2}}{x} = \lim_{x\to 0} \frac{\sin^2 \frac{x}{2}}{\frac{x}{2}} = \lim_{x\to 0} \sin\frac{x}{2} \left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right) = 0 \bullet 1.]$$

49. (D) $\lim_{x\to 0} \frac{\tan \pi x}{x} = \lim_{x\to 0} \frac{(\sec^2 \pi x)\cdot \pi}{1} = 1 \cdot \pi = \pi.$

[$\frac{\tan \pi x}{x} = \frac{\sin \pi x}{x \cos \pi x} = \pi \cdot \frac{\sin \pi x}{\pi x} \cdot \frac{1}{\cos x}$; as x (or πx) approaches 0, the original fraction approaches $\pi \cdot 1 \cdot \frac{1}{1} = \pi$.]

50. (C) The limit is easiest to obtain here if we rewrite:

$$\lim_{x\to\infty} x^2 \sin \frac{1}{x} = \lim_{x\to\infty} x \frac{\sin(1/x)}{(1/x)} = \infty \bullet 1 = \infty.$$

51. (B) Since $x - 3 = 2 \sin t$ and $y + 1 = 2 \cos t$,

$$(x-3)^2 + (y+1)^2 = 4.$$

This is the equation of a circle with center at (3,–1) and radius 2. In the domain given, $-\pi \leqslant t \leqslant \pi$, the entire circle is traced by a particle moving counterclockwise, starting from and returning to (3, –3).

52. (C) Use L'Hôpital's Rule; then

$$\lim_{x\to 0} \frac{\sec x - \cos x}{x^2} = \lim_{x\to 0} \frac{\sec x \tan x + \sin x}{2x}$$
$$= \lim_{x\to 0} \frac{\sec^3 x + \sec x \tan^2 x + \cos x}{2} = \frac{1+1\cdot 0+1}{2} = 1.$$

53. (A) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin t}{1-\cos t}.$

54. (D) $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3\sin^2\theta\cos\theta}{-3\cos^2\theta\sin\theta}.$

55. (E) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1-e^{-t}}{e^{-t}} = e^t - 1.$

56. (C) Since $\frac{dy}{dt} = \frac{1}{1-t}$ and $\frac{dx}{dt} = \frac{1}{(1-t)^2}$, then

$$\frac{dy}{dx} = 1 - t = \frac{1}{x}.$$

57. (B) $(f + 2g)'(3) = f'(3) + 2g'(3) = 4 + 2(-1)$

58. **(B)** $(f \cdot g)'(2) = f(2) \cdot g'(2) + g(2) \cdot f'(2) = 5(-2) + 1(3)$

59. **(E)** $\left(\frac{1}{g}\right)'(1) = -1 \cdot \frac{1}{[g(1)]^2} \cdot g'(1) = -1 \cdot \frac{1}{3^2}(-3).$

60. **(D)** $(\sqrt{f})'(3) = \frac{1}{2}[f(3)]^{-1/2} \cdot f'(3) = \frac{1}{2}(10^{-1/2}) \cdot 4.$

61. **(C)** $\left(\frac{f}{g}\right)'(0) = \frac{g(0) \cdot f'(0) - f(0) \cdot g'(0)}{[g(0)]^2} = \frac{5(1) - 2(-4)}{5^2}.$

62. **(A)** $M'(1) = f'(g(1)) \cdot g'(1) = f'(3)g'(1) = 4(-3).$

63. **(B)** $[f(x^3)]' = f'(x^3) \cdot 3x^2$, so $P'(1) = f'(1^3) \cdot 3 \cdot 1^2 = 2 \cdot 3.$

64. **(D)** $f(S(x)) = x$ implies that $f'(S(x)) \cdot S'(x) = 1$, so

$$S'(3) = \frac{1}{f'(S(3))} = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(1)}.$$

65. **(E)** Since $g'(a)$ exists, g is differentiable and thus continuous; $g'(a) > 0$.

66. **(C)** Near a vertical asymptote the slopes must approach $\pm\infty$.

67. **(A)** There is only one horizontal tangent.

68. **(D)** Use the symmetric difference quotient; then

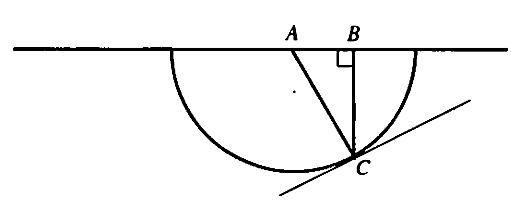
$$f'(1.5) \approx \frac{f(1.6) - f(1.4)}{1.6 - 1.4} = \frac{8}{0.2}.$$

69. **(E)** Since the water level rises more slowly as the cone fills, the rate of depth change is decreasing, as in (C) and (E). However, at every instant the portion of the cone containing water is similar to the entire cone; the volume is proportional to the cube of the depth of the water. The rate of change of depth (the derivative) is therefore not linear, as in (C).

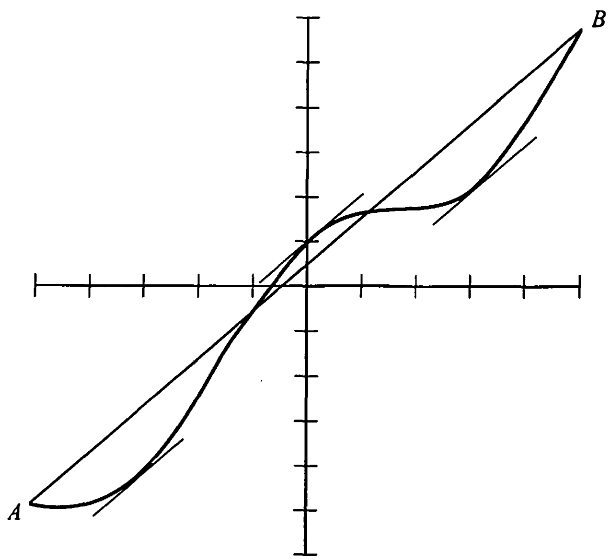
70. **(C)** The only horizontal tangent is at $x = 4$. Note that $f'(1)$ does not exist.

71. **(E)** The graph has corners at $x = 1$ and $x = 2$; the tangent line is vertical at $x = 6$.

72. **(B)** Consider triangle ABC: $AB = 1$; radius $AC = 2$; thus, $BC = \sqrt{3}$ and AC has $m = -\sqrt{3}$. The tangent line is perpendicular to the radius.



73. **(D)** The graph of $y = x + \cos x$ is shown in window $[-5,5] \times [-6,6]$. The average rate of change is represented by the slope of secant segment $\overline{AB}$. There appear to be 3 points at which tangent lines are parallel to $\overline{AB}$.



74. **(C)** $f'(2) \approx \dfrac{f(2)-f(1.98)}{2-1.98} = \dfrac{4.00-4.10}{0.02}$

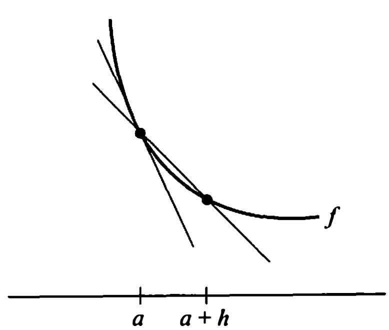
75. **(A)** Since an estimate of the answer for Question 74 is $f'(2) \approx -5$, then $\left(f^{-1}\right)'(4) = \dfrac{1}{f'(2)} \approx \dfrac{1}{-5} = -0.2$.

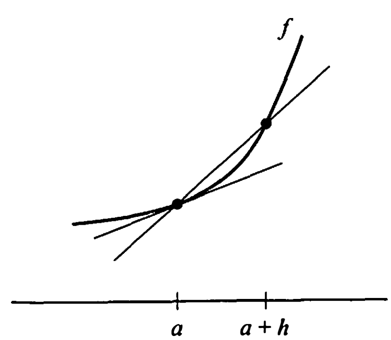
76. **(B)** When $x = 3$ on g^{-1}, $y = 3$ on the original half-parabola. $3 = x^2 - 8x + 10$ at $x = 1$ (and at $x = 7$, but that value is not in the given domain).

$g'(3) = \dfrac{1}{y'(1)} = \left.\dfrac{1}{2x-8}\right|_{x=1} = -\dfrac{1}{6}$.

77. **(E)** f satisfies Rolle's Theorem on $[2,10]$.

78. **(C)** The diagrams show secant lines (whose slope is the difference quotient) with greater slopes than the tangent line. In both cases, f is concave upward.





79. (C) $(f \circ g)'$ at $x = 3$ equals $f'(g(3)) \cdot g'(3)$ equals $\cos u$ (at $u = 0$) times $2x$ (at $x = 3$) $= 1 \cdot 6 = 6$.

80. (E) Here $f'(x)$ equals $\frac{-x-1}{(x-1)^2}$.

81. (A) $\frac{dy^2}{dx^2} = \frac{\frac{dy^2}{dx}}{\frac{dx^2}{dx}}$. Since $y^2 = x^2 + 1$, $\frac{dy^2}{dx^2} = \frac{2x}{2x}$.

82. (A) $\frac{dy}{d\left(\frac{1}{1-x}\right)} = \frac{\frac{dy}{dx}}{\frac{d\left(\frac{1}{1-x}\right)}{dx}} = \frac{2x+1}{\frac{1}{(1-x)^2}}$.

83. (B) Note that $f(g(x)) = \frac{1}{x+1}$.

84. (B) Sketch the graph of $f(x) = 1 - |x|$; note that $f(-1) = f(1) = 0$ and that f is continuous on $[-1,1]$. Only (B) holds.

85. (C) Since $f'(x) = 6x^2 - 3$, therefore $h'(x) = \frac{1}{6x^2-3}$; also, $f(x)$, or $2x^3 - 3x$, equals -1, by observation, for $x = 1$. So $h'(-1)$ or $\frac{1}{6x^2-3}$ (when $x = 1$) equals $\frac{1}{6-3} = \frac{1}{3}$.

86. (D) $(f^{-1})'(2) = \frac{1}{f'(1)} = \frac{1}{3}$.

87. (B) Since $f(0) = 5$, $g'(5) = \frac{1}{f'(0)} = \frac{1}{3x^2 - 6x + 8}\Big|_{x=0} = \frac{1}{8}$.

88. (D) The given limit is the derivative of $g(x)$ at $x = 0$.

89. (B) The tangent line appears to contain (3,–2.6) and (4,–1.8).

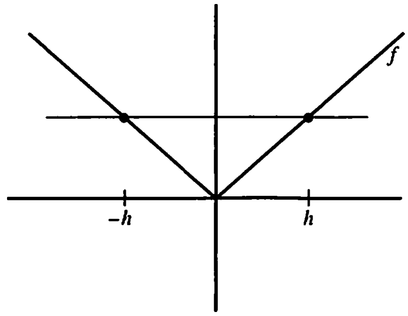
90. (D) $f'(x)$ is least at the point of inflection of the curve, at about 0.7.

91. (C) $\frac{5^{2.03} - 5^{1.97}}{2.03 - 1.97} \approx 40.25158$

92. (B) By calculator, $f'(0) = 1.386294805$ and $\frac{4^{0.08} - 4^{-0.08}}{0.16} = 1.3891\ldots$.

93. (E) Now $\frac{4^{0.001} - 4^{-0.001}}{0.002} = 1.386294805$.

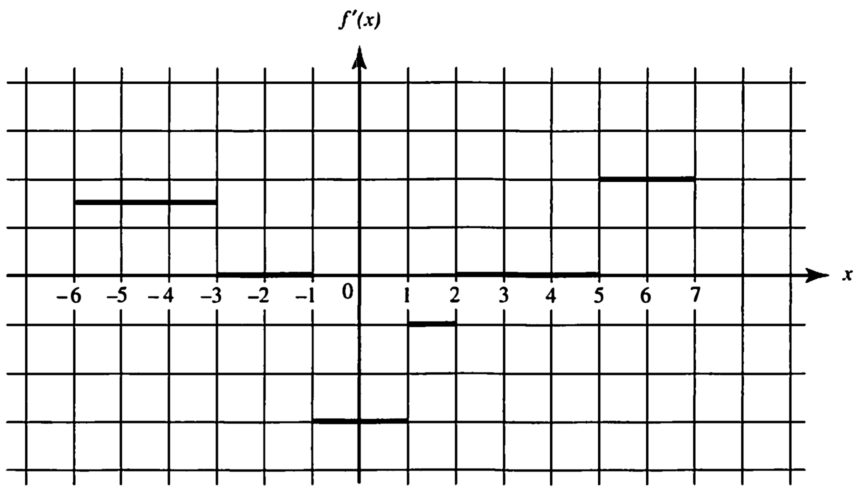
94. (B) Note that any line determined by two points equidistant from the origin will necessarily be horizontal.



95. (D) Note that $\frac{d}{dx}f(h(x)) = f'(h(x)) \cdot h'(x) = g(h(x)) \cdot h'(x) = g(\sin x) \cdot \cos x$.

96. (E) Since $f(x) = 3^x - x^3$, then $f'(x) = 3^x \ln 3 - 3x^2$. Furthermore, f is continuous on [0,3] and f' is differentiable on (0,3), so the MVT applies. We therefore seek c such that $f'(c) = \frac{f(3)-f(0)}{3} = -\frac{1}{3}$. Solving $3^x \ln x - 3x^2 = -\frac{1}{3}$ with a calculator, we find that c may be either 1.244 or 2.727. These values are the x-coordinates of points on the graph of $f(x)$ at which the tangents are parallel to the secant through points (0,1) and (3,0) on the curve.

97. (A) The line segment passes through (1,–3) and (2,–4).



Use the graph of $f'(x)$, shown above, for Questions 98–101.

98. (E) $f'(x) = 0$ when the slope of $f(x)$ is 0; that is, when the graph of f is a horizontal segment.

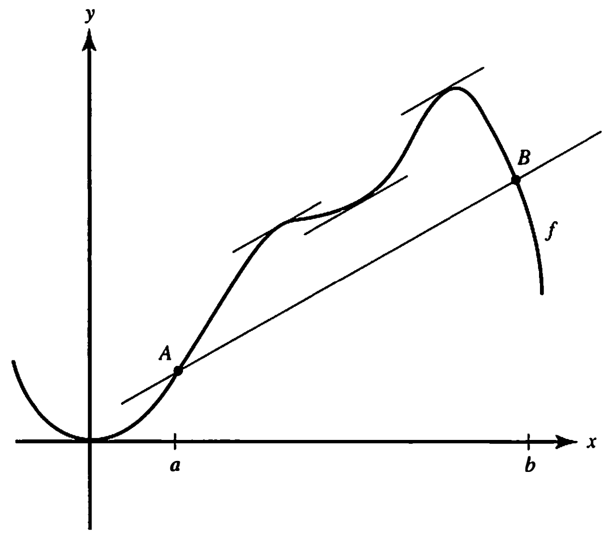
99. (E) The graph of $f'(x)$ jumps at each corner of the graph of $f(x)$, namely, at x equal to –3, –1, 1, 2, and 5.

100. (D) On the interval (–6,–3), $f(x) = \frac{3}{2}(x + 5)$.

101. (B) Verify that all choices but (B) are true. The graph of $f'(x)$ has five (not four) jump discontinuities.

102. (C) The best approximation to $f'(0.10)$ is $\frac{f(0.20)-f(0.10)}{0.20-0.10}$.

103. (D)



The average rate of change is represented by the slope of secant segment $\overline{AB}$. There appear to be 3 points at which the tangent lines are parallel to $\overline{AB}$.