Practice Exercises

Part A. Directions: Answer these questions *without* using your calculator.

1. The slope of the curve $y^3 - xy^2 = 4$ at the point where $y = 2$ is

   (A) -2 (B) $\frac{1}{4}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 2

2. The slope of the curve $y^2 - xy - 3x = 1$ at the point $(0, -1)$ is

   (A) -1 (B) -2 (C) $+1$ (D) 2 (E) -3

3. The equation of the tangent to the curve $y = x \sin x$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

   (A) $y = x - \pi$ (B) $y = \frac{\pi}{2}$ (C) $y = \pi - x$

   (D) $y = x + \frac{\pi}{2}$ (E) $y = x$

4. The tangent to the curve of $y = xe^{-x}$ is horizontal when x is equal to

   (A) 0 (B) 1 (C) -1 (D) $\frac{1}{e}$ (E) none of these

5. The minimum value of the slope of the curve $y = x^5 + x^3 - 2x$ is

   (A) 0 (B) 2 (C) 6 (D) -2 (E) none of these

6. The equation of the tangent to the hyperbola $x^2 - y^2 = 12$ at the point $(4, 2)$ on the curve is

   (A) $x - 2y + 6 = 0$ (B) $y = 2x$ (C) $y = 2x - 6$

   (D) $y = \frac{x}{2}$ (E) $x + 2y = 6$

7. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

   (A) $y = 0$ (B) $y = \pm\sqrt{3}$ (C) $y = \frac{1}{2}$

   (D) $y = \pm 3$ (E) none of these

8. The best approximation, in cubic inches, to the increase in volume of a sphere when the radius is increased from 3 to 3.1 in. is

   (A) $\frac{0.04\pi}{3}$ (B) 0.04π (C) 1.2π (D) 3.6π (E) 36π

9. When $x = 3$, the equation $2x^2 - y^3 = 10$ has the solution $y = 2$. When $x = 3.04$, $y \simeq$

   (A) 1.6 (B) 1.96 (C) 2.04 (D) 2.14 (E) 2.4

10. If the side e of a square is increased by 1%, then the area is increased approximately

(A) $0.02e$ (B) $0.02e^2$ (C) $0.01e^2$ (D) 1% (E) $0.01e$

11. The edge of a cube has length 10 in., with a possible error of 1%. The possible error, in cubic inches, in the volume of the cube is

(A) 3 (B) 1 (C) 10 (D) 30 (E) none of these

12. The function $f(x) = x^4 - 4x^2$ has

(A) one relative minimum and two relative maxima
(B) one relative minimum and one relative maximum
(C) two relative maxima and no relative minimum
(D) two relative minima and no relative maximum
(E) two relative minima and one relative maximum

13. The number of inflection points of the curve in Question 12 is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

14. The maximum value of the function $y = -4\sqrt{2-x}$ is

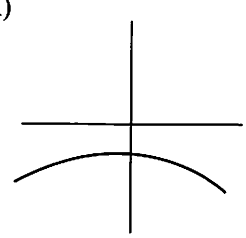
(A) 0 (B) −4 (C) 2 (D) −2 (E) none of these

15. The total number of local maximum and minimum points of the function whose derivative, for all x, is given by $f'(x) = x(x-3)^2(x+1)^4$ is

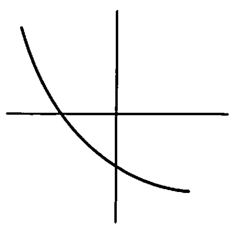
(A) 0 (B) 1 (C) 2 (D) 3 (E) none of these

16. For which curve shown below are both f' and f'' negative?

(A)



(B)



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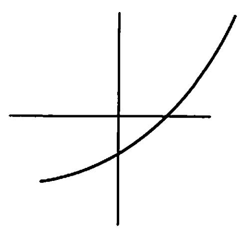

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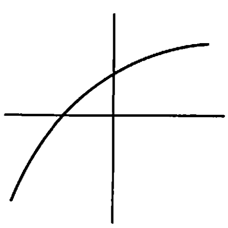

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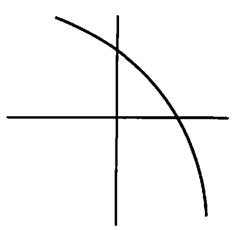


17. For which curve shown in question 16 is f'' positive but f' negative?

In Questions 18–21, the position of a particle moving along a straight line is given by $s = t^3 - 6t^2 + 12t - 8$.

18. The distance s is increasing for

(A) $t < 2$ (B) all t except $t = 2$ (C) $1 < t < 3$
(D) $t < 1$ or $t > 3$ (E) $t > 2$

19. The minimum value of the speed is

(A) 1 (B) 2 (C) 3 (D) 0 (E) none of these

20. The acceleration is positive

(A) when $t > 2$ (B) for all t, $t \neq 2$ (C) when $t < 2$
(D) for $1 < t < 3$ (E) for $1 < t < 2$

21. The speed of the particle is decreasing for

(A) $t > 2$ (B) $t < 3$ (C) all t
(D) $t < 1$ or $t > 2$ (E) none of these

In Questions 22–24, a particle moves along a horizontal line and its position at time t is $s = t^4 - 6t^3 + 12t^2 + 3$.

22. The particle is at rest when t is equal to

(A) 1 or 2 (B) 0 (C) $\frac{9}{4}$ (D) 0, 2, or 3 (E) none of these

23. The velocity, v, is increasing when

(A) $t > 1$ (B) $1 < t < 2$ (C) $t < 2$
(D) $t < 1$ or $t > 2$ (E) $t > 0$

24. The speed of the particle is increasing for

(A) $0 < t < 1$ or $t > 2$ (B) $1 < t < 2$ (C) $t < 2$
(D) $t < 0$ or $t > 2$ (E) $t < 0$

25. The displacement from the origin of a particle moving on a line is given by $s = t^4 - 4t^3$. The maximum displacement during the time interval $-2 \leqq t \leqq 4$ is

(A) 27 (B) 3 (C) $12\sqrt{3} + 3$
(D) 48 (E) none of these

26. If a particle moves along a line according to the law $s = t^5 + 5t^4$, then the number of times it reverses direction is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

BC ONLY

In Questions 27–30, $\mathbf{R} = \left\langle 3\cos\frac{\pi}{3}t,\ 2\sin\frac{\pi}{3}t \right\rangle$ is the (position) vector $\langle x, y\rangle$ from the origin to a moving point $P(x, y)$ at time t.

27. A single equation in x and y for the path of the point is

(A) $x^2 + y^2 = 13$ (B) $9x^2 + 4y^2 = 36$ (C) $2x^2 + 3y^2 = 13$
(D) $4x^2 + 9y^2 = 1$ (E) $4x^2 + 9y^2 = 36$

28. When $t = 3$, the speed of the particle is

(A) $\frac{2\pi}{3}$ (B) 2 (C) 3 (D) π (E) $\frac{\sqrt{13}}{3}\pi$

29. The magnitude of the acceleration when $t = 3$ is

(A) 2 (B) $\frac{\pi^2}{3}$ (C) 3 (D) $\frac{2\pi^2}{9}$ (E) π

30. At the point where $t = \frac{1}{2}$, the slope of the curve along which the particle moves is

(A) $-\frac{2\sqrt{3}}{9}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{2}{\sqrt{3}}$
(D) $-\frac{2\sqrt{3}}{3}$ (E) none of these

31. A balloon is being filled with helium at the rate of 4 ft^3/min. The rate, in square feet per minute, at which the surface area is increasing when the volume is $\frac{32\pi}{3}$ ft^3 is

(A) 4π (B) 2 (C) 4 (D) 1 (E) 2π

32. A circular conical reservoir, vertex down, has depth 20 ft and radius of the top 10 ft. Water is leaking out so that the surface is falling at the rate of $\frac{1}{2}$ ft/hr. The rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 ft deep is

(A) 4π (B) 8π (C) 16π (D) $\frac{1}{4\pi}$ (E) $\frac{1}{8\pi}$

33. A local minimum value of the function $y = \frac{e^x}{x}$ is

(A) $\frac{1}{e}$ (B) 1 (C) -1 (D) e (E) 0

34. The area of the largest rectangle that can be drawn with one side along the x-axis and two vertices on the curve of $y = e^{-x^2}$ is CHALLENGE

(A) $\sqrt{\frac{2}{e}}$ (B) $\sqrt{2e}$ (C) $\frac{2}{e}$ (D) $\frac{1}{\sqrt{2e}}$ (E) $\frac{2}{e^2}$

CHALLENGE

35. A line is drawn through the point (1, 2) forming a right triangle with the positive x- and y-axes. The slope of the line forming the triangle of least area is

(A) -1 (B) -2 (C) -4 (D) $-\frac{1}{2}$ (E) -3

36. The point(s) on the curve $x^2 - y^2 = 4$ closest to the point (6, 0) is (are)

(A) (2, 0) (B) $(\sqrt{5}, \pm 1)$ (C) $(3, \pm\sqrt{5})$
(D) $(\sqrt{13}, \pm\sqrt{3})$ (E) none of these

37. The sum of the squares of two positive numbers is 200; their minimum product is

(A) 100 (B) $25\sqrt{7}$ (C) 28
(D) $24\sqrt{14}$ (E) none of these

38. The first-quadrant point on the curve $y^2x = 18$ that is closest to the point (2, 0) is

(A) (2, 3) (B) $(6, \sqrt{3})$ (C) $(3, \sqrt{6})$
(D) $(1, 3\sqrt{2})$ (E) none of these

39. If h is a small negative number, then the local linear approximation for $\sqrt[3]{27+h}$ is

(A) $3 + \frac{h}{27}$ (B) $3 - \frac{h}{27}$ (C) $\frac{h}{27}$
(D) $-\frac{h}{27}$ (E) $3 - \frac{h}{9}$

40. If $f(x) = xe^{-x}$, then at $x = 0$

(A) f is increasing (B) f is decreasing (C) f has a relative maximum
(D) f has a relative minimum (E) f' does not exist

41. A function f has a derivative for each x such that $|x| < 2$ and has a local minimum at (2, –5). Which statement below must be true?

(A) $f'(2) = 0$.
(B) f' exists at $x = 2$.
(C) The graph is concave up at $x = 2$.
(D) $f'(x) < 0$ if $x < 2$, $f'(x) > 0$ if $x > 2$.
(E) None of the preceding is necessarily true.

42. The height of a rectangular box is 10 in. Its length increases at the rate of 2 in./sec; its width decreases at the rate of 4 in./sec. When the length is 8 in. and the width is 6 in., the rate, in cubic inches per second, at which the volume of the box is changing is

(A) 200 (B) 80 (C) -80 (D) -200 (E) -20

43. The tangent to the curve $x^3 + x^2y + 4y = 1$ at the point (3, –2) has slope

(A) -3 (B) $-\frac{23}{9}$ (C) $-\frac{27}{13}$ (D) $-\frac{11}{9}$ (E) $-\frac{15}{13}$

44. If $f(x) = ax^4 + bx^2$ and $ab > 0$, then

(A) the curve has no horizontal tangents
(B) the curve is concave up for all x
(C) the curve is concave down for all x
(D) the curve has no inflection point
(E) none of the preceding is necessarily true

45. A function f is continuous and differentiable on the interval [0,4], where f' is positive but f'' is negative. Which table could represent points on f?

(A)

x	0	1	2	3	4
y	10	12	14	16	18

(B)

x	0	1	2	3	4
y	10	12	15	19	24

(C)

x	0	1	2	3	4
y	10	18	24	28	30

(D)

x	0	1	2	3	4
y	30	28	24	18	10

(E)

x	0	1	2	3	4
y	24	19	15	12	10

BC ONLY

46. The equation of the tangent to the curve with parametric equations $x = 2t + 1$, $y = 3 - t^3$ at the point where $t = 1$ is

(A) $2x + 3y = 12$ **(B)** $3x + 2y = 13$ **(C)** $6x + y = 20$
(D) $3x - 2y = 5$ **(E)** none of these

47. Approximately how much less than 4 is $\sqrt[3]{63}$?

(A) $\frac{1}{48}$ **(B)** $\frac{1}{16}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{2}{3}$ **(E)** 1

48. The best linear approximation for $f(x) = \tan x$ near $x = \frac{\pi}{4}$ is

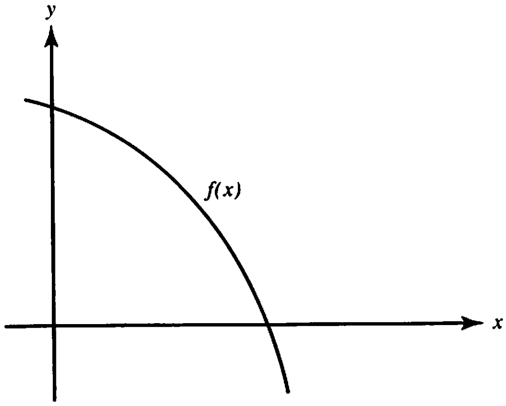
(A) $1 + \frac{1}{2}(x - \frac{\pi}{4})$ **(B)** $1 + (x - \frac{\pi}{4})$ **(C)** $1 + \sqrt{2}\,(x - \frac{\pi}{4})$

(D) $1 + 2(x - \frac{\pi}{4})$ **(E)** $2 + 2(x - \frac{\pi}{4})$

49. When h is near zero, e^{kh}, using the tangent-line approximation, is approximately

(A) k **(B)** kh **(C)** 1 **(D)** $1 + k$ **(E)** $1 + kh$

50. If $f(x) = cx^2 + dx + e$ for the function shown in the graph, then



(A) c, d, and e are all positive
(B) $c > 0, d < 0, e < 0$
(C) $c > 0, d < 0, e > 0$
(D) $c < 0, d > 0, e > 0$
(E) $c < 0, d < 0, e > 0$

Part B. Directions: Some of the following questions require the use of a graphing calculator.

51. The point on the curve $y = \sqrt{2x+1}$ at which the normal is parallel to the line $y = -3x + 6$ is

(A) (4, 3) (B) (0, 1) (C) $(1, \sqrt{3})$
(D) (4, –3) (E) $(2, \sqrt{5})$

52. The equation of the tangent to the curve $x^2 = 4y$ at the point on the curve where $x = -2$ is

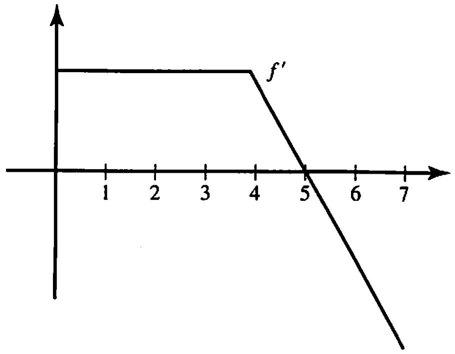
(A) $x + y - 3 = 0$ (B) $y - 1 = 2x(x + 2)$ (C) $x - y + 3 = 0$
(D) $x + y - 1 = 0$ (E) $x + y + 1 = 0$

53. The table shows the velocity at time t of an object moving along a line. Estimate the acceleration (in ft/sec^2) at $t = 6$ sec.

t (sec)	0	4	8	10
vel.	18	16	10	0

(A) –6 (B) –1.8 (C) –1.5 (D) 1.5 (E) 6

Use the graph shown, sketched on [0, 7], for Questions 54–56.



54. From the graph it follows that

(A) f is discontinuous at $x = 4$
(B) f is decreasing for $4 < x < 7$
(C) f is constant for $0 < x < 4$
(D) f has a local maximum at $x = 0$
(E) f has a local minimum at $x = 7$

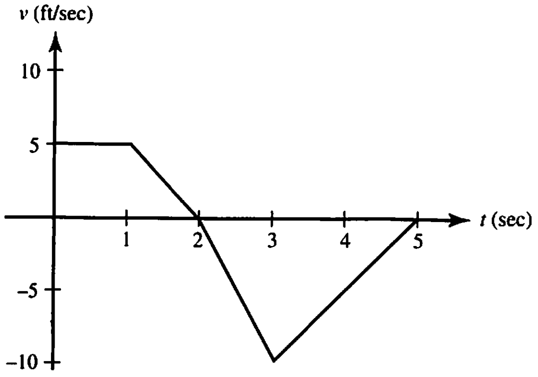
55. Which statement best describes f at $x = 5$?

(A) f has a root. (B) f has a maximum. (C) f has a minimum.
(D) The graph of f has a point of inflection. (E) none of these

56. For which interval is the graph of f concave downward?

(A) (0,4) (B) (4,5) (C) (5,7)
(D) (4,7) (E) none of these

Use the graph shown for Questions 57–63. It shows the velocity of an object moving along a straight line during the time interval $0 \leq t \leq 5$.



57. The object attains its maximum speed when $t =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

58. The speed of the object is increasing during the time interval

(A) (0,1) (B) (1,2) (C) (0,2) (D) (2,3) (E) (3,5)

59. The acceleration of the object is positive during the time interval

(A) (0,1) (B) (1,2) (C) (0,2) (D) (2,3) (E) (3,5)

60. How many times on $0 < t < 5$ is the object's acceleration undefined?

(A) none (B) 1 (C) 2 (D) 3 (E) more than 3

61. During $2 < t < 3$ the object's acceleration (in ft/sec²) is

(A) −10 (B) −5 (C) 0 (D) 5 (E) 10

62. The object is furthest to the right when $t =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

63. The object's average acceleration (in ft/sec²) for the interval $0 \leq t \leq 3$ is

(A) −15 (B) −5 (C) −3 (D) −1 (E) none of these

64. The line $y = 3x + k$ is tangent to the curve $y = x^3$ when k is equal to

(A) 1 or −1 (B) 0 (C) 3 or −3 (D) 4 or −4 (E) 2 or −2

65. The two tangents that can be drawn from the point (3,5) to the parabola $y = x^2$ have slopes

(A) 1 and 5 (B) 0 and 4 (C) 2 and 10

(D) 2 and $-\frac{1}{2}$ (E) 2 and 4

66. The table shows the velocity at various times of an object moving along a line. An estimate of its acceleration (in ft/sec²) at $t = 1$ is

t (sec)	1.0	1.5	2.0	2.5
v (ft/sec)	12.2	13.0	13.4	13.7

(A) 0.8 (B) 1.0 (C) 1.2 (D) 1.4 (E) 1.6

For Questions 67 and 68, $f'(x) = x \sin x - \cos x$ for $0 < x < 4$.

67. f has a local maximum when x is approximately

(A) 0.9 (B) 1.2 (C) 2.3 (D) 3.4 (E) 3.7

68. The graph of f has a point of inflection when x is approximately

(A) 0.9 (B) 1.2 (C) 2.3 (D) 3.4 (E) 3.7

BC ONLY

In Questions 69–72, the motion of a particle in a plane is given by the pair of equations $x = 2t$ and $y = 4t - t^2$.

69. The particle moves along

(A) an ellipse (B) a circle (C) a hyperbola
(D) a line (E) a parabola

70. The speed of the particle at any time t is

(A) $\sqrt{6-2t}$ (B) $2\sqrt{t^2-4t+5}$ (C) $2\sqrt{t^2-2t+5}$

(D) $\sqrt{8}(|t-2|)$ (E) $2(|3-t|)$

71. The minimum speed of the particle is

(A) 2 (B) $2\sqrt{2}$ (C) 0 (D) 1 (E) 4

72. The acceleration of the particle

(A) depends on t
(B) is always directed upward
(C) is constant both in magnitude and in direction
(D) never exceeds 1 in magnitude
(E) is none of these

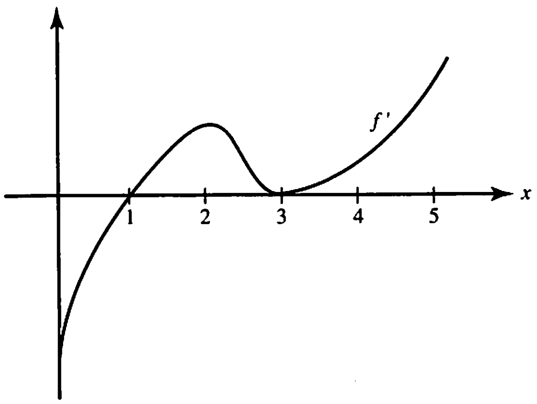
73. If a particle moves along a curve with constant speed, then

(A) the magnitude of its acceleration must equal zero
(B) the direction of acceleration must be constant
(C) the curve along which the particle moves must be a straight line
(D) its velocity and acceleration vectors must be perpendicular
(E) the curve along which the particle moves must be a circle

74. A particle is moving on the curve of $y = 2x - \ln x$ so that $\frac{dx}{dt} = -2$ at all times t. At the point (1,2), $\frac{dy}{dt}$ is

(A) 4 (B) 2 (C) −4 (D) 1 (E) −2

Use the graph of f' on [0,5], shown below, for Questions 75 and 76.



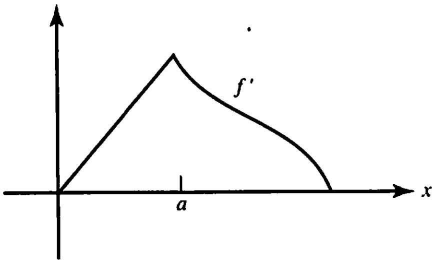
75. f has a local minimum at $x =$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 5

76. The graph of f has a point of inflection at $x =$

(A) 1 only **(B)** 2 only **(C)** 3 only
(D) 2 and 3 only **(E)** none of these

77. It follows from the graph of f', shown at the right, that



(A) f is not continuous at $x = a$
(B) f is continuous but not differentiable at $x = a$
(C) f has a relative maximum at $x = a$
(D) The graph of f has a point of inflection at $x = a$
(E) none of these

78. A vertical circular cylinder has radius r ft and height h ft. If the height and radius both increase at the constant rate of 2 ft/sec, then the rate, in square feet per second, at which the lateral surface area increases is

(A) $4\pi r$ **(B)** $2\pi(r + h)$ **(C)** $4\pi(r + h)$ **(D)** $4\pi rh$ **(E)** $4\pi h$

79. A tangent drawn to the parabola $y = 4 - x^2$ at the point (1, 3) forms a right triangle with the coordinate axes. The area of the triangle is

(A) $\frac{5}{4}$ **(B)** $\frac{5}{2}$ **(C)** $\frac{25}{2}$ **(D)** 1 **(E)** $\frac{25}{4}$

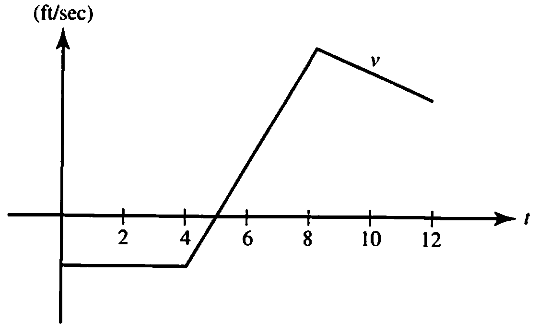
80. Two cars are traveling along perpendicular roads, car A at 40 mph, car B at 60 mph. At noon, when car A reaches the intersection, car B is 90 mi away, and moving toward it. At 1 P.M. the rate, in miles per hour, at which the distance between the cars is changing is

(A) –40 **(B)** 68 **(C)** 4 **(D)** –4 **(E)** 40

81. For Question 80, if t is the number of hours of travel after noon, then the cars are closest together when t is

(A) 0 (B) $\frac{27}{26}$ (C) $\frac{9}{5}$ (D) $\frac{3}{2}$ (E) $\frac{14}{13}$

The graph for Questions 82 and 83 shows the velocity of an object moving along a straight line during the time interval $0 \leqslant t \leqslant 12$.



82. For what t does this object attain its maximum acceleration?

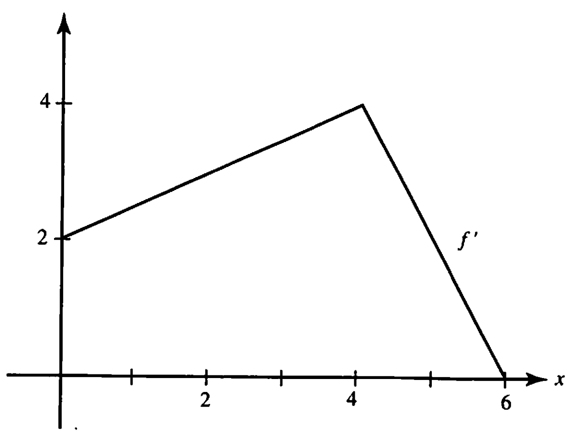
(A) $0 < t < 4$ (B) $4 < t < 8$ (C) $t = 5$ (D) $t = 8$ (E) $t = 12$

83. The object reverses direction at $t =$

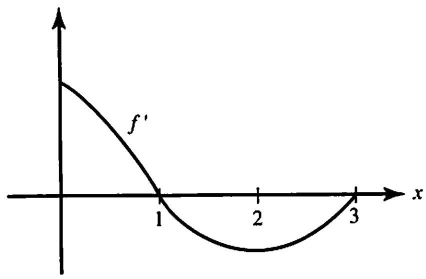
(A) 4 only (B) 5 only (C) 8 only
(D) 5 and 8 (E) none of these

84. The graph of f' is shown below. If we know that $f(2) = 10$, then the local linearization of f at $x = 2$ is $f(x) \simeq$

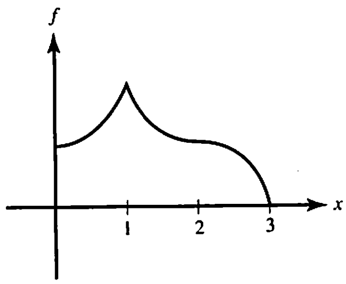
(A) $\frac{x}{2} + 2$ (B) $\frac{x}{2} + 9$ (C) $3x - 3$
(D) $3x + 4$ (E) $10x - 17$



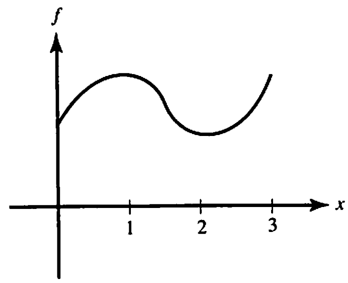
85. Given f' as graphed, which could be the graph of f?



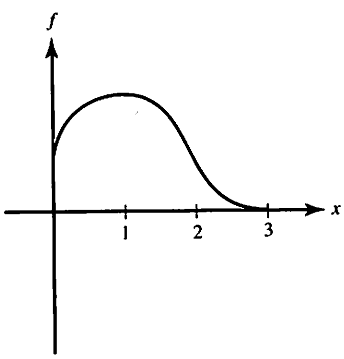
(A)



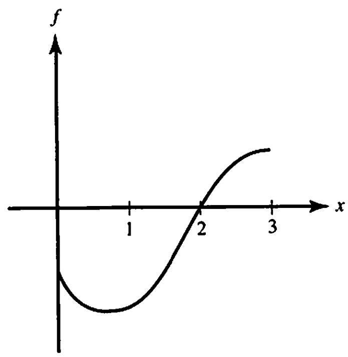
(B)



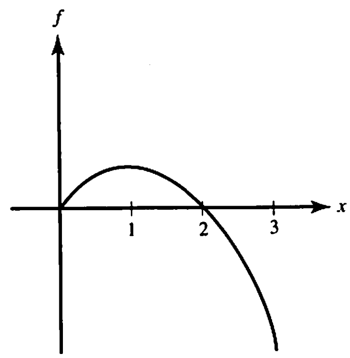
(C)



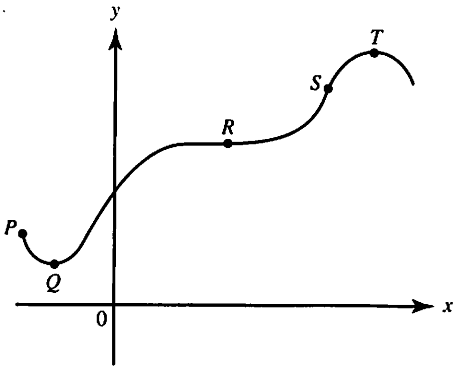
(D)



(E)



Use the following graph for Questions 86–88.



86. At which labeled point do both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ equal zero?

(A) P (B) Q (C) R (D) S (E) T

87. At which labeled point is $\frac{dy}{dx}$ positive and $\frac{d^2y}{dx^2}$ equal to zero?

(A) P (B) Q (C) R (D) S (E) T

88. At which labeled point is $\frac{dy}{dx}$ equal to zero and $\frac{d^2y}{dx^2}$ negative?

(A) P (B) Q (C) R (D) S (E) T

89. If $f(6) = 30$ and $f'(x) = \frac{x^2}{x+3}$, estimate $f(6.02)$ using the line tangent to f at $x = 6$.

(A) 29.92 (B) 30.02 (C) 30.08
(D) 34.00 (E) none of these

90. The local linear approximation for $f(x) = \sqrt{x^2 + 16}$ near $x = -3$ is

(A) $5 - \frac{3}{5}(x-3)$ (B) $5 + \frac{3}{5}(x-3)$ (C) $5 - \frac{3}{5}(x+3)$

(D) $3 - \frac{5}{3}(x-3)$ (E) $3 + \frac{3}{5}(x+3)$

Answer Key

1. D	21. E	41. E	61. A	81. B
2. A	22. B	42. D	62. C	82. B
3. E	23. D	43. E	63. B	83. B
4. B	24. A	44. D	64. E	84. D
5. D	25. D	45. C	65. C	85. C
6. C	26. C	46. B	66. E	86. C
7. D	27. E	47. A	67. D	87. D
8. D	28. A	48. D	68. C	88. E
9. C	29. B	49. E	69. E	89. C
10. B	30. D	50. E	70. B	90. C
11. D	31. C	51. A	71. A	
12. E	32. B	52. E	72. C	
13. C	33. D	53. C	73. D	
14. A	34. A	54. E	74. E	
15. B	35. B	55. B	75. B	
16. E	36. C	56. D	76. D	
17. B	37. E	57. D	77. D	
18. B	38. C	58. D	78. C	
19. D	39. A	59. E	79. E	
20. A	40. A	60. D	80. D	

Answers Explained

1. **(D)** Substituting $y = 2$ yields $x = 1$. We find y' implicitly.

$$3y^2y' - (2xyy' + y^2) = 0; \quad (3y^2 - 2xy)y' - y^2 = 0.$$

Replace x by 1 and y by 2; solve for y'.

2. **(A)** $2yy' - (xy' + y) - 3 = 0$. Replace x by 0 and y by -1; solve for y'.

3. **(E)** Find the slope of the curve at $x = \frac{\pi}{2}$: $y' = x\cos x + \sin x$. At $x = \frac{\pi}{2}$, $y' = \frac{\pi}{2} \cdot 0 + 1$. The equation is $y - \frac{\pi}{2} = 1\left(x - \frac{\pi}{2}\right)$.

4. **(B)** Since $y' = e^{-x}(1 - x)$ and $e^{-x} > 0$ for all x, $y' = 0$ when $x = 1$.

5. **(D)** The slope $y' = 5x^4 + 3x^2 - 2$. Let $g = y'$. Since $g'(x) = 20x^3 + 6x = 2x(10x^2 + 3)$, $g'(x) = 0$ only if $x = 0$. Since $g''(x) = 60x^2 + 6$, g'' is always positive, assuring that $x = 0$ yields the minimum slope. Find y' when $x = 0$.

6. **(C)** Since $2x - 2yy' = 0$, $y' = \frac{x}{y}$. At (4, 2), $y' = 2$. The equation of the tangent at (4, 2) is $y - 2 = 2(x - 4)$.

7. **(D)** Since $y' = \frac{y}{2y - x}$, the tangent is vertical for $x = 2y$. Substitute in the given equation and solve for y.

8. **(D)** Since $V = \frac{4}{3}\pi r^3$, therefore, $dV = 4\pi r^2 dr$. The approximate increase in volume is $dV \approx 4\pi(3^2)(0.1)$ in^3.

9. (C) Differentiating implicitly yields $4x - 3y^2y' = 0$. So $y' = \frac{4x}{3y^2}$. The linear approximation for the true value of y when x changes from 3 to 3.04 is

$$y_{\text{at } x=3} + y'_{\text{at point }(3,2)} \cdot (3.04 - 3).$$

Since it is given that, when $x = 3$, $y = 2$, the approximate value of y is

$$2 + \frac{4x}{3y^2}_{\text{at }(3,2)} \cdot (0.04)$$

or

$$2 + \frac{12}{12} \cdot (0.04) = 2.04.$$

10. (B) We want to approximate the change in area of the square when a side of length e increases by $0.01e$. The answer is

$$A'(e)(0.01e) \text{ or } 2e(0.01e).$$

11. (D) Since $V = e^3$, $V' = 3e^2$. Therefore at $e = 10$, the slope of the tangent line is 300. The change in volume is approximately $300(\pm 0.1) = 30$ in.^{3}

12. (E) $f'(x) = 4x^3 - 8x = 4x(x^2 - 2)$. $f' = 0$ if $x = 0$ or $\pm\sqrt{2}$. $f''(x) = 12x^2 - 8$; f'' is positive if $x = \pm\sqrt{2}$, negative if $x = 0$.

13. (C) Since $f''(x) = 4(3x^2 - 2)$, it equals 0 if $x = \pm\sqrt{\frac{2}{3}}$. Since f'' changes sign from positive to negative at $x = -\sqrt{\frac{2}{3}}$ and from negative to positive at $x = +\sqrt{\frac{2}{3}}$ both locate inflection points.

14. (A) The domain of y is $\{x \mid x \leqq 2\}$. Note that y is negative for each x in the domain except 2, where $y = 0$.

15. (B) $f'(x)$ changes sign (from negative to positive) as x passes through zero only.

16. (E) The graph must be decreasing and concave downward.

17. (B) The graph must be concave upward but decreasing.

18. (B) The distance is increasing when v is positive. Since $v = \frac{ds}{dt} = 3(t-2)^2$, $v > 0$ for all $t \neq 2$.

19. (D) The speed $= |v|$. From Question 18, $|v| = v$. The least value of v is 0.

20. (A) The acceleration $a = \frac{dv}{dt}$. From Question 18, $a = 6(t - 2)$.

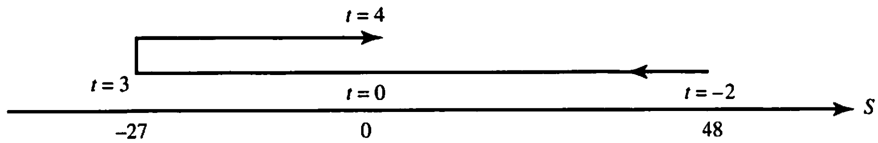
21. (E) The speed is decreasing when v and a have opposite signs. The answer is $t < 2$, since for all such t the velocity is positive while the acceleration is negative. For $t > 2$, both v and a are positive.

22. (B) The particle is at rest when $v = 0$; $v = 2t(2t^2 - 9t + 12) = 0$ only if $t = 0$. Note that the discriminant of the quadratic factor $(b^2 - 4ac)$ is negative.

23. (D) Since $a = 12(t - 1)(t - 2)$, we check the signs of a in the intervals $t < 1$, $1 < t < 2$, and $t > 2$. We choose those where $a > 0$.

24. (A) From Questions 22 and 23 we see that $v > 0$ if $t > 0$ and that $a > 0$ if $t < 1$ or $t > 2$. So both v and a are positive if $0 < t < 1$ or $t > 2$. There are no values of t for which both v and a are negative.

25. (D) See the figure, which shows the motion of the particle during the time interval $-2 \leqslant t \leqslant 4$. The particle is at rest when $t = 0$ or 3, but reverses direction only at 3. The endpoints need to be checked here, of course. Indeed, the maximum displacement occurs at one of those, namely, when $t = -2$.



26. (C) Since $v = 5t^3(t + 4)$, $v = 0$ when $t = -4$ or 0. Note that v does change sign at each of these times.

27. (E) Since $x = 3 \cos \frac{\pi}{3} t$ and $y = 2 \sin \frac{\pi}{3} t$, we note that $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$.

28. (A) Note that $\mathbf{v} = \left\langle -\pi \sin \frac{\pi}{3} t, \frac{2\pi}{3} \cos \frac{\pi}{3} t \right\rangle$. At $t = 3$,

$$|\mathbf{v}| = \sqrt{(-\pi \cdot 0)^2 + \left(\frac{2\pi}{3} \cdot -1\right)^2}.$$

29. (B) $\mathbf{a} = \left\langle -\frac{\pi^2}{3} \cos \frac{\pi}{3} t, \frac{2\pi^2}{9} \sin \frac{\pi}{3} t \right\rangle$. At $t = 3$,

$$|\mathbf{a}| = \sqrt{\left(\frac{-\pi^2}{3} \cdot -1\right)^2 + \left(\frac{-2\pi^2}{9} \cdot 0\right)^2}.$$

30. (D) The slope of the curve is the slope of $\mathbf{v}$, namely, $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. At $t = \frac{1}{2}$, the slope is equal to

$$\frac{\frac{2\pi}{3} \cdot \cos \frac{\pi}{6}}{-\pi \cdot \sin \frac{\pi}{6}} = -\frac{2}{3} \cot \frac{\pi}{6}.$$

31. (C) Since $V = \frac{4}{3} \pi r^3$, $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. Since $\frac{dV}{dt} = 4$, $\frac{dr}{dt} = \frac{1}{\pi r^2}$. When $V = \frac{32\pi}{3}$, $r = 2$ and $\frac{dr}{dt} = \frac{1}{4\pi}$.

$$S = 4\pi r^2; \qquad \frac{dS}{dt} = 8\pi r \frac{dr}{dt};$$

when $r = 2$, $\frac{dS}{dt} = 8\pi(2)\left(\frac{1}{4\pi}\right) = 4$.

32. (B) See Figure N4–22 on page 189. Replace the printed measurements of the radius and height by 10 and 20, respectively. We are given here that $r = \frac{h}{2}$ and that $\frac{dh}{dt} = -\frac{1}{2}$. Since $V = \frac{1}{3}\pi r^2 h$, we have $V = \frac{\pi}{3}\frac{h^3}{4}$, so

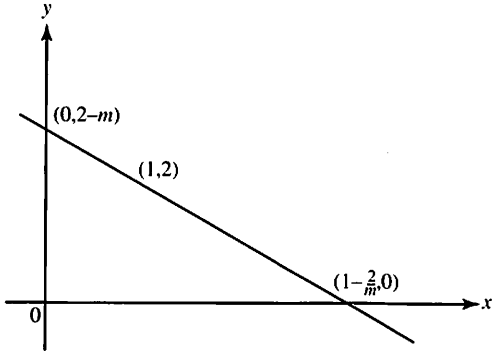
$$\frac{dV}{dt} = \frac{\pi h^2}{4}\frac{dh}{dt} = \frac{-\pi h^2}{8}.$$

Replace h by 8.

33. (D) $y' = \frac{e^x(x-1)}{x^2}(x \neq 0)$. Since $y' = 0$ if $x = 1$ and changes from negative to positive as x increases through 1, $x = 1$ yields a minimum. Evaluate y at $x = 1$.

34. (A) The domain of y is $-\infty < x < \infty$. The graph of y, which is nonnegative, is symmetric to the y-axis. The inscribed rectangle has area $A = 2xe^{-x^2}$. Thus $A' = \frac{2(1-2x^2)}{e^{x^2}}$, which is 0 when the positive value of x is $\frac{\sqrt{2}}{2}$. This value of x yields maximum area. Evaluate A.

35. (B) See the figure. If we let m be the slope of the line, then its equation is $y - 2 = m(x - 1)$ with intercepts as indicated in the figure.



The area A of the triangle is given by

$$A = \frac{1}{2}(2-m)\left(1-\frac{2}{m}\right) = \frac{1}{2}\left(4-\frac{4}{m}-m\right).$$

Then $\frac{dA}{dm} = \frac{1}{2}\left(\frac{4}{m^2}-1\right)$ and equals 0 when $m = \pm 2$; m must be negative.

36. (C) Let $q = (x-6)^2 + y^2$ be the quantity to be minimized. Then

$$q = (x-6)^2 + (x^2 - 4);$$

$q' = 0$ when $x = 3$. Note that it suffices to minimize the square of the distance.

37. (E) Minimize, if possible, xy, where $x^2 + y^2 = 200$ $(x, y > 0)$. The derivative of the product is $\frac{2(100-x^2)}{\sqrt{200-x^2}}$, which equals 0 for $x = 10$. The derivative is positive to the left of that point and negative to the right, showing that $x = 10$ yields a maximum product. No minimum exists.

38. (C) Minimize $q = (x-2)^2 + \frac{18}{x}$. Since

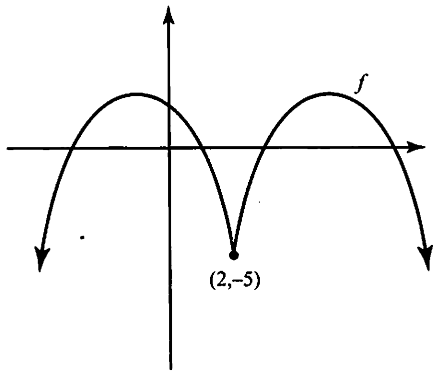
$$q' = 2(x-2) - \frac{18}{x^2} = \frac{2(x^3 - 2x^2 - 9)}{x^2},$$

$q' = 0$ if $x = 3$. Since q' is negative to the left of $x = 3$ and positive to the right, the minimum value of q occurs at $x = 3$.

39. (A) The best approximation for $\sqrt[3]{27+h}$ when h is small is the local linear (or tangent line) approximation. If we let $f(h) = \sqrt[3]{27+h}$, then $f'(h) = \frac{1}{3(27+h)^{2/3}}$ and $f'(0) = \frac{1}{3 \cdot 9}$. The approximation for $f(h)$ is $f(0) + f'(0) \cdot h$, which equals $3 + \frac{1}{27}h$.

40. (A) Since $f'(x) = e^{-x}(1-x)$, $f'(0) > 0$.

41. (E) The graph shown serves as a counterexample for A–D.



42. (D) Since $V = 10\ell w$, $V' = 10\left(\ell\frac{dw}{dt} + w\frac{d\ell}{dt}\right) = 10(8 \cdot -4 + 6 \cdot 2)$.

43. (E) We differentiate implicitly: $3x^2 + x^2y' + 2xy + 4y' = 0$. Then $y' = -\frac{3x^2 + 2xy}{x^2 + 4}$. At $(3,-2)$, $y' = -\frac{27-12}{9+4} = -\frac{15}{13}$.

44. (D) Since $ab > 0$, a and b have the same sign; therefore $f''(x) = 12ax^2 + 2b$ never equals 0. The curve has one horizontal tangent at $x = 0$.

45. (C) Since the first derivative is positive, the function must be increasing. However, the negative second derivative indicates that the rate of increase is slowing down, as seen in table C.

46. (B) Since $\frac{dy}{dx} = -\frac{3t^2}{2}$, therefore, at $t = 1$, $\frac{dy}{dx} = -\frac{3}{2}$. Also, $x = 3$ and $y = 2$.

47. **(A)** Let $f(x) = x^{1/3}$, and find the slope of the tangent line at (64, 4). Since $f'(x) = \frac{1}{3}x^{-2/3}$, $f'(64) = \frac{1}{48}$. If we move one unit to the left of 64, the tangent line will drop approximately $\frac{1}{48}$ unit.

48. **(D)** $\tan x \simeq \tan\left(\frac{\pi}{4}\right) + \sec^2\left(\frac{\pi}{4}\right)\left(x-\frac{\pi}{4}\right) = 1 + 2\left(x-\frac{\pi}{4}\right)$

49. **(E)** $e^{kh} \simeq e^{k\cdot 0} + ke^{k\cdot 0}(h-0) = 1 + kh$

50. **(E)** Since the curve has a positive y-intercept, $e > 0$. Note that $f'(x) = 2cx + d$ and $f''(x) = 2c$. Since the curve is concave down, $f''(x) < 0$, implying that $c < 0$. Since the curve is decreasing at $x = 0$, $f'(0)$ must be negative, implying, since $f'(0) = d$, that $d < 0$. Therefore $c < 0$, $d < 0$, and $e > 0$.

51. **(A)** Since the slope of the tangent to the curve is $y' = \frac{1}{\sqrt{2x+1}}$, the slope of the normal is $-\sqrt{2x+1}$. So $-\sqrt{2x+1} = -3$ and $2x + 1 = 9$.

52. **(E)** The slope $y' = \frac{2x}{4}$; at the given point $y' = -\frac{4}{4} = -1$ and $y = 1$. The equation is therefore

$$y - 1 = -1(x + 2) \qquad \text{or} \qquad x + y + 1 = 0.$$

53. **(C)** $a \simeq \frac{\Delta v}{\Delta t} = \frac{v(8) - v(4)}{8-4} = \frac{10-16}{4}$ ft/sec².

54. **(E)** Since $f' < 0$ on $5 \leqslant x < 7$, the function decreases as it approaches the right endpoint.

55. **(B)** For $x < 5$, $f' > 0$, so f is increasing; for $x > 5$, f is decreasing.

56. **(D)** The graph of f being concave downward implies that $f'' < 0$, which implies that f' is decreasing.

57. **(D)** Speed is the magnitude of velocity; at $t = 3$, speed = 10 ft/sec.

58. **(D)** Speed increases from 0 at $t = 2$ to 10 at $t = 3$; it is constant or decreasing elsewhere.

59. **(E)** Acceleration is positive when the *velocity* increases.

60. **(D)** Acceleration is undefined when velocity is not differentiable. Here that occurs at $t = 1, 2, 3$.

61. **(A)** Acceleration is the derivative of velocity. Since the velocity is linear, its derivative is its slope.

62. **(C)** Positive velocity implies motion to the right ($t < 2$); negative velocity ($t > 2$) implies motion to the left.

63. (B) The average rate of change of velocity is $\frac{v(3)-v(0)}{3-0}=\frac{-10-5}{-3}$ ft/sec².

64. (E) The slope of $y=x^3$ is $3x^2$. It is equal to 3 when $x=\pm 1$. At $x=1$, the equation of the tangent is

$$y-1=3(x-1) \quad \text{or} \quad y=3x+2.$$

At $x=-1$, the equation is

$$y+1=3(x+1) \quad \text{or} \quad y=3x+2.$$

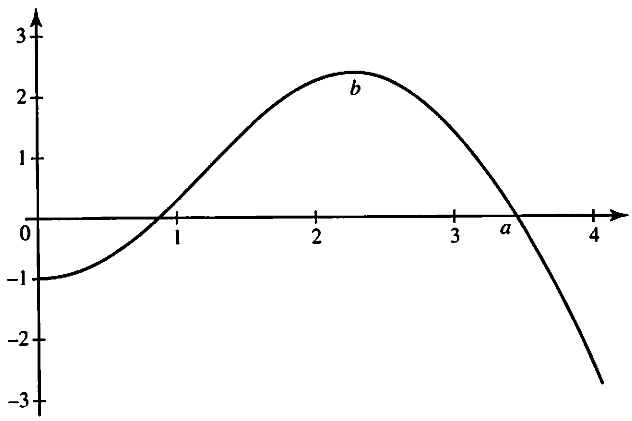
65. (C) Let the tangent to the parabola from (3, 5) meet the curve at (x_1, y_1). Its equation is $y-5=2x_1(x-3)$. Since the point (x_1, y_1) is on both the tangent and the parabola, we solve simultaneously:

$$y_1-5=2x_1(x_1-3) \quad \text{and} \quad y_1=x_1^2$$

The points of tangency are (5, 25) and (1, 1). The slopes, which equal $2x_1$, are 10 and 2.

66. (E) $a\approx\frac{\Delta v}{\Delta t}=\frac{v(1.5)-v(1.0)}{0.5}=\frac{13.2-12.2}{0.5}$ ft/sec².

67. (D) The graph of $f'(x)=x\sin x-\cos x$ is drawn here in the window $[0,4]\times[-3,3]$:



A local maximum exists at $x=0$, where f' changes from positive to negative; use your calculator to approximate a.

68. (C) f'' changes sign when f' changes from increasing to decreasing (or vice versa). Again, use your calculator to approximate the x-coordinate at b.

69. (E) Eliminating t yields the equation $y=-\frac{1}{4}x^2+2x$.

70. (B) $|\mathbf{v}|=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}=\sqrt{2^2+(4-2t)^2}$.

71. (A) Since $|\mathbf{v}|=2\sqrt{t^2-4t+5}$, $\frac{d|\mathbf{v}|}{dt}=\frac{2t-4}{\sqrt{t^2-4t+5}}=0$ if $t=2$. We note that, as t increases through 2, the sign of $|\mathbf{v}|'$ changes from negative to positive, assuring a minimum of $|\mathbf{v}|$ at $t=2$. Evaluate $|\mathbf{v}|$ at $t=2$.

72. **(C)** The direction of **a** is $\tan^{-1}\dfrac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}}$. Since $\dfrac{d^2x}{dt^2}=0$ and $\dfrac{d^2y}{dt^2}=-2$, the acceleration is always directed downward. Its magnitude, $\sqrt{0^2+(-2)^2}$, is 2 for all t.

73. **(D)** Using the notations v_x, v_y, a_x, and a_y, we are given that $|\mathbf{v}|=\sqrt{v_x^2+v_y^2}=k$, where k is a constant. Then

$$\frac{d|\mathbf{v}|}{dt}=\frac{v_xa_x+v_ya_y}{|\mathbf{v}|}=0 \quad\text{or}\quad \frac{v_x}{v_y}=-\frac{a_y}{a_x}.$$

74. **(E)** $\dfrac{dy}{dt}=\left(2-\dfrac{1}{x}\right)\dfrac{dx}{dt}=\left(2-\dfrac{1}{x}\right)(-2)$.

75. **(B)** A local minimum exists where f changes from decreasing ($f'<0$) to increasing ($f'>0$). Note that f has local maxima at both endpoints, $x=0$ and $x=5$.

76. **(D)** See Answer 68.

77. **(D)** At $x=a$, f' changes from increasing ($f''>0$) to decreasing ($f''<0$). Thus f changes from concave upward to concave downward, and therefore has a point of inflection at $x=a$. Note that f is differentiable at a (because $f'(a)$ exists) and therefore continuous at a.

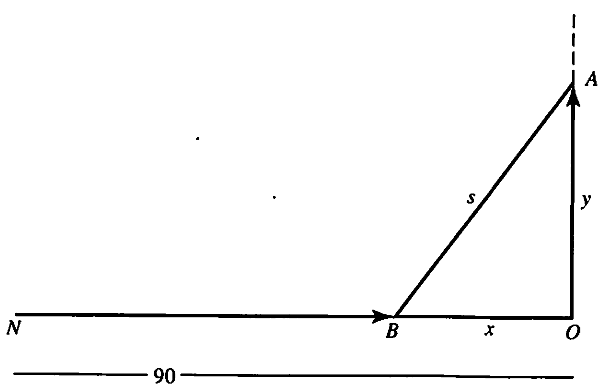
78. **(C)** We know that $\dfrac{dh}{dt}=\dfrac{dr}{dt}=2$. Since $S=2\pi rh$,

$$\frac{dS}{dt}=2\pi\left(r\frac{dh}{dt}+h\frac{dr}{dt}\right).$$

79. **(E)** The equation of the tangent is $y=-2x+5$. Its intercepts are $\frac{5}{2}$ and 5.

80. **(D)** See the figure. At noon, car A is at O, car B at N; the cars are shown t hours after noon. We know that $\dfrac{dx}{dt}=-60$ and that $\dfrac{dy}{dt}=40$. Using $s^2=x^2+y^2$, we get

$$\frac{ds}{dt}=\frac{x\frac{dx}{dt}+y\frac{dy}{dt}}{s}=\frac{-60x+40y}{s}.$$



At 1 P.M., $x=30$, $y=40$, and $s=50$.

81. (B) $\frac{ds}{dt}$ (from Question 80) is zero when $y = \frac{3}{2}x$. Note that $x = 90 - 60t$ and $y = 40t$.

82. (B) Maximum acceleration occurs when the derivative (slope) of velocity is greatest.

83. (B) The object changes direction only when velocity changes sign. Velocity changes sign from negative to positive at $t = 5$.

84. (D) From the graph, $f'(2) = 3$, and we are told the line passes through (2,10). We therefore have $f(x) \simeq 10 + 3(x - 2) = 3x + 4$.

85. (C) At $x = 1$ and 3, $f'(x) = 0$; therefore f has horizontal tangents.
For $x < 1, f' > 0$; therefore f is increasing.
For $x > 1, f' < 0$, so f is decreasing.
For $x < 2, f'$ is decreasing, so $f'' < 0$ and the graph of f is concave downward.
For $x > 2, f'$ is increasing, so $f'' > 0$ and the graph of f is concave upward.

86. (C) Note that $\frac{dy}{dx} = 0$ at Q, R, and T. At Q, $\frac{d^2y}{dx^2} > 0$; at T, $\frac{d^2y}{dx^2} < 0$.

87. (D) Only at S does the graph both rise and change concavity.

88. (E) Only at T is the tangent horizontal and the curve concave down.

89. (C) Since $f'(6) = 4$, the equation of the tangent at (6, 30) is $y - 30 = 4(x - 6)$. Therefore $f(x) \simeq 4x + 6$ and $f(6.02) \simeq 30.08$.

90. (C) $\sqrt{x^2 + 16} \simeq \sqrt{9 + 16} + \frac{(-3)}{\sqrt{(-3)^2 + 16}}(x + 3) = 5 - \frac{3}{5}(x + 3)$