Chapter Summary

In this chapter, we have reviewed basic skills for finding indefinite integrals. We've looked at the antiderivative formulas for all of the basic functions and reviewed techniques for finding antiderivatives of other functions.

We've also reviewed the more advanced techniques of integration by partial fractions and integration by parts, both topics only for the BC Calculus course .

Practice Exercises

Directions: Answer these questions *without* using your calculator.

1. $\int(3x^2 - 2x + 3)\,dx =$

(A) $x^3 - x^2 + C$ **(B)** $3x^3 - x^2 + 3x + C$ **(C)** $x^3 - x^2 + 3x + C$

(D) $\frac{1}{2}(3x^2 - 2x + 3)^2 + C$ **(E)** none of these

2. $\int\left(x - \frac{1}{2x}\right)^2 dx =$

(A) $\frac{1}{3}\left(x - \frac{1}{2x}\right)^3 + C$ **(B)** $x^2 - 1 + \frac{1}{4x^2} + C$ **(C)** $\frac{x^3}{3} - 2x - \frac{1}{4x} + C$

(D) $\frac{x^3}{3} - x - \frac{4}{x} + C$ **(E)** none of these

3. $\int\sqrt{4 - 2t}\;dt =$

(A) $-\frac{1}{3}(4 - 2t)^{3/2} + C$ **(B)** $\frac{2}{3}(4 - 2t)^{3/2} + C$ **(C)** $-\frac{1}{6}(4 - 2t)^3 + C$

(D) $+\frac{1}{2}(4 - 2t)^2 + C$ **(E)** $\frac{4}{3}(4 - 2t)^{3/2} + C$

4. $\int(2 - 3x)^5\,dx =$

(A) $\frac{1}{6}(2 - 3x)^6 + C$ **(B)** $-\frac{1}{2}(2 - 3x)^6 + C$ **(C)** $\frac{1}{2}(2 - 3x)^6 + C$

(D) $-\frac{1}{18}(2 - 3x)^6 + C$ **(E)** none of these

5. $\int\frac{1 - 3y}{\sqrt{2y - 3y^2}}\,dy =$

(A) $4\sqrt{2y - 3y^2} + C$ **(B)** $\frac{1}{4}(2y - 3y^2)^2 + C$ **(C)** $\frac{1}{2}\ln\sqrt{2y - 3y^2} + C$

(D) $\frac{1}{4}(2y - 3y^2)^{1/2} + C$ **(E)** $\sqrt{2y - 3y^2} + C$

6. $\int \frac{dx}{3(2x-1)^2}$ =d

(A) $\frac{-3}{2x-1}+C$ (B) $\frac{1}{6-12x}+C$ (C) $+\frac{6}{2x-1}+C$

(D) $\frac{2}{3\sqrt{2x-1}}+C$ (E) $\frac{1}{3}\ln|2x-1|+C$

7. $\int \frac{2\,du}{1+3u} =$

(A) $\frac{2}{3}\ln|1+3u|+C$ (B) $-\frac{1}{3(1+3u)^2}+C$ (C) $2\ln|1+3u|+C$

(D) $\frac{3}{(1+3u)^2}+C$ (E) none of these

8. $\int \frac{t}{\sqrt{2t^2-1}}\,dt =$

(A) $\frac{1}{2}\ln\sqrt{2t^2-1}+C$ (B) $4\ln\sqrt{2t^2-1}+C$ (C) $8\sqrt{2t^2-1}+C$

(D) $-\frac{1}{4(2t^2-1)}+C$ (E) $\frac{1}{2}\sqrt{2t^2-1}+C$

9. $\int \cos 3x\,dx =$

(A) $3\sin 3x+C$ (B) $-\sin 3x+C$ (C) $-\frac{1}{3}\sin 3x+C$

(D) $\frac{1}{3}\sin 3x+C$ (E) $\frac{1}{2}\cos^2 3x+C$

10. $\int \frac{x\,dx}{1+4x^2} =$

(A) $\frac{1}{8}\ln(1+4x^2)+C$ (B) $\frac{1}{8(1+4x^2)^2}+C$ (C) $\frac{1}{4}\sqrt{1+4x^2}+C$

(D) $\frac{1}{2}\ln|1+4x^2|+C$ (E) $\frac{1}{2}\tan^{-1}2x+C$

11. $\int \frac{dx}{1+4x^2} =$

(A) $\tan^{-1}(2x)+C$ (B) $\frac{1}{8}\ln(1+4x^2)+C$ (C) $\frac{1}{8(1+4x^2)^2}+C$

(D) $\frac{1}{2}\tan^{-1}(2x)+C$ (E) $\frac{1}{8x}\ln|1+4x^2|+C$

12. $\int \frac{x}{(1+4x^2)^2}\,dx =$

(A) $\frac{1}{8}\ln(1+4x^2)^2 + C$ (B) $\frac{1}{4}\sqrt{1+4x^2} + C$ (C) $-\frac{1}{8(1+4x^2)} + C$

(D) $-\frac{1}{3(1+4x^2)^3} + C$ (E) $-\frac{1}{(1+4x^2)} + C$

13. $\int \frac{x\,dx}{\sqrt{1+4x^2}} =$

(A) $\frac{1}{8}\sqrt{1+4x^2} + C$ (B) $\frac{\sqrt{1+4x^2}}{4} + C$ (C) $\frac{1}{2}\sin^{-1} 2x + C$

(D) $\frac{1}{2}\tan^{-1} 2x + C$ (E) $\frac{1}{8}\ln\sqrt{1+4x^2} + C$

14. $\int \frac{dy}{\sqrt{4-y^2}} =$

(A) $\frac{1}{2}\sin^{-1}\frac{y}{2} + C$ (B) $-\sqrt{4-y^2} + C$ (C) $\sin^{-1}\frac{y}{2} + C$

(D) $-\frac{1}{2}\ln\sqrt{4-y^2} + C$ (E) $-\frac{1}{3(4-y^2)^{3/2}} + C$

15. $\int \frac{y\,dy}{\sqrt{4-y^2}} =$

(A) $\frac{1}{2}\sin^{-1}\frac{y}{2} + C$ (B) $-\sqrt{4-y^2} + C$ (C) $\sin^{-1}\frac{y}{2} + C$

(D) $-\frac{1}{2}\ln\sqrt{4-y^2} + C$ (E) $2\sqrt{4-y^2} + C$

16. $\int \frac{2x+1}{2x}\,dx =$

(A) $x + \frac{1}{2}\ln|x| + C$ (B) $1 + \frac{1}{2}x^{-1} + C$ (C) $x + 2\ln|x| + C$

(D) $x + \ln|2x| + C$ (E) $\frac{1}{2}\left(2x - \frac{1}{x^2}\right) + C$

17. $\int \frac{(x-2)^3}{x^2}\,dx =$

(A) $\frac{(x-2)^4}{4x^2} + C$ (B) $\frac{x^2}{2} - 6x + 6\ln|x| - \frac{8}{x} + C$

(C) $\frac{x^2}{2} - 3x + 6\ln|x| + \frac{4}{x} + C$ (D) $-\frac{(x-2)^4}{4x} + C$

(E) none of these

18. $\int\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)^2 dt =$

(A) $t-2+\frac{1}{t}+C$ (B) $\frac{t^3}{3}-2t-\frac{1}{t}+C$ (C) $\frac{t^2}{2}+\ln|t|+C$

(D) $\frac{t^2}{2}-2t+\ln|t|+C$ (E) $\frac{t^2}{2}-t-\frac{1}{t^2}+C$

19. $\int(4x^{1/3}-5x^{3/2}-x^{-1/2})\,dx =$

(A) $3x^{4/3}-2x^{5/2}-2x^{1/2}+C$
(B) $3x^{4/3}-2x^{5/2}+2x^{1/2}+C$
(C) $6x^{2/3}-2x^{5/2}-\frac{1}{2}x^2+C$
(D) $\frac{4}{3}x^{-2/3}-\frac{15}{2}x^{1/2}+\frac{1}{2}x^{-3/2}+C$
(E) none of these

20. $\int\frac{x^3-x-1}{x^2}\,dx =$

(A) $\dfrac{\frac{1}{4}x^4-\frac{1}{2}x^2-x}{\frac{1}{3}x^3}+C$

(B) $1+\frac{1}{x^2}+\frac{2}{x^3}+C$

(C) $\frac{x^2}{2}-\ln|x|-\frac{1}{x}+C$

(D) $\frac{x^2}{2}-\ln|x|+\frac{1}{x}+C$

(E) $\frac{x^2}{2}-\ln|x|+\frac{2}{x^3}+C$

21. $\int\frac{dy}{\sqrt{y}(1-\sqrt{y})} =$

(A) $4\sqrt{1-\sqrt{y}}+C$ (B) $\frac{1}{2}\ln\left|1-\sqrt{y}\right|+C$ (C) $2\ln(1-\sqrt{y})+C$

(D) $2\sqrt{y}-\ln|y|+C$ (E) $-2\ln\left|1-\sqrt{y}\right|+C$

22. $\int\frac{u\,du}{\sqrt{4-9u^2}} =$

(A) $\frac{1}{3}\sin^{-1}\frac{3u}{2}+C$ (B) $-\frac{1}{18}\ln\sqrt{4-9u^2}+C$ (C) $2\sqrt{4-9u^2}+C$

(D) $\frac{1}{6}\sin^{-1}\frac{3}{2}u+C$ (E) $-\frac{1}{9}\sqrt{4-9u^2}+C$

23. $\int \sin\theta\cos\theta\, d\theta =$

(A) $-\dfrac{\sin^2\theta}{2}+C$ (B) $-\dfrac{1}{4}\cos 2\theta + C$ (C) $\dfrac{\cos^2\theta}{2}+C$

(D) $\dfrac{1}{2}\sin 2\theta + C$ (E) $\cos 2\theta + C$

24. $\int \dfrac{\sin\sqrt{x}}{\sqrt{x}}\, dx =$

(A) $-2\cos^{1/2} x + C$ (B) $-\cos\sqrt{x} + C$ (C) $-2\cos\sqrt{x} + C$

(D) $\dfrac{3}{2}\sin^{3/2} x + C$ (E) $\dfrac{1}{2}\cos\sqrt{x} + C$

25. $\int t\cos(2t)^2\, dt =$

(A) $\dfrac{1}{8}\sin(4t^2) + C$ (B) $\dfrac{1}{2}\cos^2(2t) + C$ (C) $-\dfrac{1}{8}\sin(4t^2) + C$

(D) $\dfrac{1}{4}\sin(2t)^2 + C$ (E) none of these

26. $\int \cos^2 2x\, dx =$

(A) $\dfrac{x}{2}+\dfrac{\sin 4x}{8}+C$ (B) $\dfrac{x}{2}-\dfrac{\sin 4x}{8}+C$ (C) $\dfrac{x}{4}+\dfrac{\sin 4x}{4}+C$

(D) $\dfrac{x}{4}+\dfrac{\sin 4x}{16}+C$ (E) $\dfrac{1}{4}(x+\sin 4x)+C$

27. $\int \sin 2\theta\, d\theta =$

(A) $\dfrac{1}{2}\cos 2\theta + C$ (B) $-2\cos 2\theta + C$ (C) $-\sin^2\theta + C$

(D) $\cos^2\theta + C$ (E) $-\dfrac{1}{2}\cos 2\theta + C$

BC ONLY

28. $\int x\cos x\, dx =$

(A) $x\sin x + C$ (B) $x\sin x + \cos x + C$ (C) $x\sin x - \cos x + C$

(D) $\cos x - x\sin x + C$ (E) $\dfrac{x^2}{2}\sin x + C$

29. $\int \dfrac{du}{\cos^2 3u} =$

(A) $-\dfrac{\sec 3u}{3} + C$ (B) $\tan 3u + C$ (C) $u + \dfrac{\sec 3u}{3} + C$

(D) $\dfrac{1}{3}\tan 3u + C$ (E) $\dfrac{1}{3\cos 3u} + C$

30. $\int \frac{\cos x\, dx}{\sqrt{1+\sin x}} =$

(A) $-\frac{1}{2}(1+\sin x)^{1/2}+C$

(B) $\ln\sqrt{1+\sin x}+C$

(C) $2\sqrt{1+\sin x}+C$

(D) $\ln|1+\sin x|+C$

(E) $\frac{2}{3(1+\sin x)^{3/2}}+C$

31. $\int \frac{\cos(\theta-1)\, d\theta}{\sin^2(\theta-1)} =$

(A) $2\ln\sin|\theta-1|+C$ (B) $-\csc(\theta-1)+C$ (C) $-\frac{1}{3}\sin^{-3}(\theta-1)+C$

(D) $-\cot(\theta-1)+C$ (E) $\csc(\theta-1)+C$

32. $\int \sec\frac{t}{2}\, dt =$

(A) $\ln\left|\sec\frac{t}{2}+\tan\frac{t}{2}\right|+C$ (B) $2\tan^2\frac{t}{2}+C$ (C) $2\ln\cos\frac{t}{2}+C$

(D) $\ln|\sec t+\tan t|+C$ (E) $2\ln\left|\sec\frac{t}{2}+\tan\frac{t}{2}\right|+C$

33. $\int \frac{\sin 2x\, dx}{\sqrt{1+\cos^2 x}} =$

(A) $-2\sqrt{1+\cos^2 x}+C$ (B) $\frac{1}{2}\ln(1+\cos^2 x)+C$

(C) $\sqrt{1+\cos^2 x}+C$ (D) $-\ln\sqrt{1+\cos^2 x}+C$

(E) $2\ln|\sin x|+C$

34. $\int \sec^{3/2} x\tan x\, dx =$

(A) $\frac{2}{5}\sec^{5/2} x+C$ (B) $-\frac{2}{3}\cos^{-3/2} x+C$ (C) $\sec^{3/2} x+C$

(D) $\frac{2}{3}\sec^{3/2} x+C$ (E) none of these

35. $\int \tan\theta\, d\theta =$

(A) $-\ln|\sec\theta|+C$ (B) $\sec^2\theta+C$ (C) $\ln|\sin\theta|+C$

(D) $\sec\theta+C$ (E) $-\ln|\cos\theta|+C$

36. $\int \frac{dx}{\sin^2 2x} =$

(A) $\frac{1}{2}\csc 2x \cot 2x + C$ **(B)** $-\frac{2}{\sin 2x} + C$ **(C)** $-\frac{1}{2}\cot 2x + C$
(D) $-\cot x + C$ **(E)** $-\csc 2x + C$

37. $\int \frac{\tan^{-1} y}{1 + y^2}\, dy =$

(A) $\sec^{-1} y + C$ **(B)** $(\tan^{-1} y)^2 + C$ **(C)** $\ln(1 + y^2) + C$
(D) $\ln(\tan^{-1} y) + C$ **(E)** none of these

38. $\int \sin 2\theta \cos \theta \, d\theta =$

(A) $-\frac{2}{3}\cos^3\theta + C$ **(B)** $\frac{2}{3}\cos^3\theta + C$ **(C)** $\sin^2\theta\cos\theta + C$
(D) $\cos^3\theta + C$ **(E)** none of these

39. $\int \frac{\sin 2t}{1 - \cos 2t}\, dt =$

(A) $\frac{2}{(1-\cos 2t)^2} + C$ **(B)** $-\ln|1-\cos 2t| + C$ **(C)** $\ln\sqrt{|1-\cos 2t|} + C$
(D) $\sqrt{1-\cos 2t} + C$ **(E)** $2\ln|1-\cos 2t| + C$

40. $\int \cot 2u \, du =$

(A) $\ln|\sin u| + C$ **(B)** $\frac{1}{2}\ln|\sin 2u| + C$ **(C)** $-\frac{1}{2}\csc^2 2u + C$
(D) $-\sec 2u + C$ **(E)** $2\ln|\sin 2u| + C$

41. $\int \frac{e^x}{e^x - 1}\, dx =$

(A) $x + \ln|e^x - 1| + C$ **(B)** $x - e^x + C$ **(C)** $x - \frac{1}{(e^x-1)^2} + C$
(D) $1 + \frac{1}{e^x - 1} + C$ **(E)** $\ln|e^x - 1| + C$

BC ONLY

42. $\int \frac{x-1}{x(x-2)}\, dx =$

(A) $\frac{1}{2}\ln|x| + \ln|x-2| + C$ **(B)** $\frac{1}{2}\ln\left|\frac{x-2}{x}\right| + C$

(C) $\ln|x-2| + \ln|x| + C$ **(D)** $\frac{1}{2}\ln|x(x-2)| + C$

(E) none of these

43. $\int xe^{x^2}\,dx =$

(A) $\frac{1}{2}e^{x^2}+C$ **(B)** $e^{x^2}(2x^2+1)+C$ **(C)** $2e^{x^2}+C$

(D) $e^{x^2}+C$ **(E)** $\frac{1}{2}e^{x^2+1}+C$

44. $\int \cos\theta\, e^{\sin\theta}\,d\theta =$

(A) $e^{\sin\theta+1}+C$ **(B)** $e^{\sin\theta}+C$ **(C)** $-e^{\sin\theta}+C$

(D) $e^{\cos\theta}+C$ **(E)** $e^{\sin\theta}(\cos\theta-\sin\theta)+C$

45. $\int e^{2\theta}\sin e^{2\theta}\,d\theta =$

(A) $\cos e^{2\theta}+C$ **(B)** $2e^{4\theta}(\cos e^{2\theta}+\sin e^{2\theta})+C$ **(C)** $-\frac{1}{2}\cos e^{2\theta}+C$

(D) $-2\cos e^{2\theta}+C$ **(E)** none of these

46. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$

(A) $2\sqrt{x}(e^{\sqrt{x}}-1)+C$ **(B)** $2e^{\sqrt{x}}+C$ **(C)** $\frac{e^{\sqrt{x}}}{2}\left(\frac{1}{x}+\frac{1}{x\sqrt{x}}\right)+C$

(D) $\frac{1}{2}e^{\sqrt{x}}+C$ **(E)** none of these

BC ONLY

47. $\int xe^{-x}\,dx =$

(A) $e^{-x}(1-x)+C$ **(B)** $\frac{e^{1-x}}{1-x}+C$ **(C)** $-e^{-x}(x+1)+C$

(D) $-\frac{x^2}{2}e^{-x}+C$ **(E)** $e^{-x}(x+1)+C$

48. $\int x^2e^x\,dx =$

(A) $e^x(x^2+2x)+C$ **(B)** $e^x(x^2-2x-2)+C$ **(C)** $e^x(x^2-2x+2)+C$

(D) $e^x(x-1)^2+C$ **(E)** $e^x(x+1)^2+C$

49. $\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx =$

(A) $x-\ln\left|e^x-e^{-x}\right|+C$ **(B)** $x+2\ln\left|e^x-e^{-x}\right|+C$

(C) $-\frac{1}{2}(e^x-e^{-x})^{-2}+C$ **(D)** $\ln\left|e^x-e^{-x}\right|+C$

(E) $\ln(e^x+e^{-x})+C$

50. $\int \frac{e^x}{1+e^{2x}}\,dx =$

(A) $\tan^{-1} e^x + C$ (B) $\frac{1}{2}\ln(1+e^{2x}) + C$ (C) $\ln(1+e^{2x}) + C$

(D) $\frac{1}{2}\tan^{-1} e^x + C$ (E) $2\tan^{-1} e^x + C$

51. $\int \frac{\ln v\,dv}{v} =$

(A) $\ln|\ln v| + C$ (B) $\ln\frac{v^2}{2} + C$ (C) $\frac{1}{2}(\ln v)^2 + C$

(D) $2\ln v + C$ (E) $\frac{1}{2}\ln v^2 + C$

52. $\int \frac{\ln\sqrt{x}}{x}\,dx =$

(A) $\frac{\ln^2\sqrt{x}}{\sqrt{x}} + C$ (B) $\ln^2 x + C$ (C) $\frac{1}{2}\ln|\ln x| + C$

(D) $\frac{(\ln\sqrt{x})^2}{2} + C$ (E) $\frac{1}{4}\ln^2 x + C$

BC ONLY

53. $\int x^3 \ln x\,dx =$

(A) $x^2(3\ln x + 1) + C$ (B) $\frac{x^4}{16}(4\ln x - 1) + C$ (C) $\frac{x^4}{4}(\ln x - 1) + C$

(D) $3x^2\left(\ln x - \frac{1}{2}\right) + C$ (E) none of these

54. $\int \ln\eta\,d\eta =$

(A) $\frac{1}{2}\ln^2\eta + C$ (B) $\eta(\ln\eta - 1) + C$ (C) $\frac{1}{2}\ln\eta^2 + C$

(D) $\ln\eta(\eta - 1) + C$ (E) $\eta\ln\eta + \eta + C$

55. $\int \ln x^3\,dx =$

(A) $\frac{3}{2}\ln^2 x + C$ (B) $3x(\ln x - 1) + C$ (C) $3\ln x(x-1) + C$

(D) $\frac{3x\ln^2 x}{2} + C$ (E) none of these

BC ONLY

56. $\int \frac{\ln y}{y^2}\, dy =$

(A) $\frac{1}{y}(1 - \ln y) + C$ **(B)** $\frac{1}{2y}\ln^2 y + C$ **(C)** $-\frac{1}{3y^3}(4\ln y + 1) + C$

(D) $-\frac{1}{y}(\ln y + 1) + C$ **(E)** $\frac{\ln y}{y} - \frac{1}{y} + C$

57. $\int \frac{dv}{v \ln v} =$

(A) $\frac{1}{\ln v^2} + C$ **(B)** $-\frac{1}{\ln^2 v} + C$ **(C)** $-\ln|\ln v| + C$

(D) $\ln\frac{1}{v} + C$ **(E)** $\ln|\ln v| + C$

58. $\int \frac{y-1}{y+1}\, dy =$

(A) $y - 2\ln|y+1| + C$ **(B)** $1 - \frac{2}{y+1} + C$ **(C)** $\ln\frac{|y|}{(y+1)^2} + C$

(D) $1 - 2\ln|y+1| + C$ **(E)** $\ln\left|\frac{e^y}{y+1}\right| + C$

59. $\int \frac{dx}{x^2 + 2x + 2} =$

(A) $\ln(x^2 + 2x + 2) + C$ **(B)** $\ln|x+1| + C$ **(C)** $\arctan(x+1) + C$

(D) $\frac{1}{\frac{1}{3}x^3 + x^2 + 2x} + C$ **(E)** $-\frac{1}{x} + \frac{1}{2}\ln|x| + \frac{x}{2} + C$

60. $\int \sqrt{x}(\sqrt{x} - 1)\, dx =$

(A) $2(x^{3/2} - x) + C$ **(B)** $\frac{x^2}{2} - x + C$ **(C)** $\frac{1}{2}(\sqrt{x} - 1)^2 + C$

(D) $\frac{1}{2}x^2 - \frac{2}{3}x^{3/2} + C$ **(E)** $x - 2\sqrt{x} + C$

BC ONLY

61. $\int e^\theta \cos\theta\, d\theta =$

(A) $e^\theta(\cos\theta - \sin\theta) + C$
(B) $e^\theta \sin\theta + C$
(C) $\frac{1}{2}e^\theta(\sin\theta + \cos\theta) + C$
(D) $2e^\theta(\sin\theta + \cos\theta) + C$
(E) $\frac{1}{2}e^\theta(\sin\theta - \cos\theta) + C$

62. $\int \frac{(1-\ln t)^2}{t}\,dt =$

(A) $\frac{1}{3}(1-\ln t)^3 + C$ (B) $\ln t - 2\ln^2 t + \ln^3 t + C$ (C) $-2(1-\ln t) + C$

(D) $\ln t - \ln^2 t + \frac{\ln t^3}{3} + C$ (E) $-\frac{(1-\ln t)^3}{3} + C$

BC ONLY

63. $\int u \sec^2 u\,du =$

(A) $u\tan u + \ln|\cos u| + C$ (B) $\frac{u^2}{2}\tan u + C$ (C) $\frac{1}{2}\sec u\,\tan u + C$

(D) $u\tan u - \ln|\sin u| + C$ (E) $u\sec u - \ln|\sec u + \tan u| + C$

64. $\int \frac{2x+1}{4+x^2}\,dx =$ **CHALLENGE**

(A) $\ln(x^2+4) + C$ (B) $\ln(x^2+4) + \tan^{-1}\frac{x}{2} + C$ (C) $\frac{1}{2}\tan^{-1}\frac{x}{2} + C$

(D) $\ln(x^2+4) + \frac{1}{2}\tan^{-1}\frac{x}{2} + C$ (E) none of these

65. $\int \frac{1-x}{\sqrt{1-x^2}}\,dx =$ **CHALLENGE**

(A) $\sqrt{1-x^2} + C$ (B) $\sin^{-1} x + C$

(C) $\frac{1}{2}\ln\sqrt{1-x^2} + C$ (D) $\sin^{-1} x + \sqrt{1-x^2} + C$

(E) $\sin^{-1} x + \frac{1}{2}\ln\sqrt{1-x^2} + C$

66. $\int \frac{2x-1}{\sqrt{4x-4x^2}}\,dx =$ **CHALLENGE**

(A) $4\ln\sqrt{4x-4x^2} + C$ (B) $\sin^{-1}(1-2x) + C$

(C) $\frac{1}{2}\sqrt{4x-4x^2} + C$ (D) $-\frac{1}{4}\ln(4x-4x^2) + C$

(E) $-\frac{1}{2}\sqrt{4x-4x^2} + C$

67. $\int \frac{e^{2x}}{1+e^x}\,dx =$ **CHALLENGE**

(A) $\tan^{-1} e^x + C$ (B) $e^x - \ln(1+e^x) + C$ (C) $e^x - x + \ln|1+e^x| + C$

(D) $e^x + \frac{1}{(e^x+1)^2} + C$ (E) none of these

68. $\int \frac{\cos\theta}{1+\sin^2\theta}\,d\theta =$

(A) $\sec\theta\tan\theta + C$ (B) $\sin\theta - \csc\theta + C$ (C) $\ln(1+\sin^2\theta) + C$

(D) $\tan^{-1}(\sin\theta) + C$ (E) $-\frac{1}{(1+\sin^2\theta)^2} + C$

BC ONLY

69. $\int \arctan x\,dx =$

(A) $\arctan x + C$

(B) $x\arctan x - \ln(1+x^2) + C$

(C) $x\arctan x + \ln(1+x^2) + C$

(D) $x\arctan x + \frac{1}{2}\ln(1+x^2) + C$

(E) $x\arctan x - \frac{1}{2}\ln(1+x^2) + C$

70. $\int \frac{dx}{1-e^x} =$

CHALLENGE

(A) $-\ln|1-e^x| + C$ (B) $x - \ln|1-e^x| + C$ (C) $\frac{1}{(1-e^x)^2} + C$

(D) $e^{-x}\ln|1+e^x| + C$ (E) none of these

71. $\int \frac{(2-y)^2}{4\sqrt{y}}\,dy =$

(A) $\frac{1}{6}(2-y)^3\sqrt{y} + C$

(B) $2\sqrt{y} - \frac{2}{3}y^{3/2} + \frac{8}{5}y^{5/2} + C$

(C) $\ln|y| - y + 2y^2 + C$

(D) $2y^{1/2} - \frac{2}{3}y^{3/2} + \frac{1}{10}y^{5/2} + C$

(E) none of these

72. $\int e^{2\ln u}\,du =$

(A) $\frac{1}{3}e^{u^3} + C$ (B) $e^{u^3/3} + C$ (C) $\frac{1}{3}u^3 + C$

(D) $\frac{2}{u}e^{2\ln u} + C$ (E) $e^{1+2\ln u} + C$

73. $\int \frac{dy}{y(1+\ln y^2)} =$

(A) $\frac{1}{2}\ln|1+\ln y^2| + C$ (B) $-\frac{1}{(1+\ln y^2)^2} + C$

(C) $\ln|y| + \frac{1}{2}\ln|\ln y| + C$ (D) $\tan^{-1}(\ln|y|) + C$ (E) none of these

74. $\int(\tan\theta - 1)^2\, d\theta =$

CHALLENGE

(A) $\sec\theta + \theta + 2\ln|\cos\theta| + C$ (B) $\tan\theta + 2\ln|\cos\theta| + C$
(C) $\tan\theta - 2\sec^2\theta + C$ (D) $\sec\theta + \theta - \tan^2\theta + C$
(E) $\tan\theta - 2\ln|\cos\theta| + C$

75. $\int \frac{d\theta}{1+\sin\theta} =$

CHALLENGE

(A) $\sec\theta - \tan\theta + C$ (B) $\ln(1+\sin\theta) + C$
(C) $\ln|\sec\theta + \tan\theta| + C$ (D) $\theta + \ln|\csc\theta - \cot\theta| + C$
(E) none of these

76. A particle starting at rest at $t = 0$ moves along a line so that its acceleration at time t is $12t$ ft/sec². How much distance does the particle cover during the first 3 sec?

(A) 16 ft (B) 32 ft (C) 48 ft (D) 54 ft (E) 108 ft

77. The equation of the curve whose slope at point (x, y) is $x^2 - 2$ and which contains the point $(1, -3)$ is

(A) $y = \frac{1}{3}x^3 - 2x$ (B) $y = 2x - 1$ (C) $y = \frac{1}{3}x^3 - \frac{10}{3}$

(D) $y = \frac{1}{3}x^3 - 2x - \frac{4}{3}$ (E) $3y = x^3 - 10$

78. A particle moves along a line with acceleration $2 + 6t$ at time t. When $t = 0$, its velocity equals 3 and it is at position $s = 2$. When $t = 1$, it is at position $s =$

(A) 2 (B) 5 (C) 6 (D) 7 (E) 8

79. Find the acceleration (in ft/sec²) needed to bring a particle moving with a velocity of 75 ft/sec to a stop in 5 sec.

(A) −3 (B) −6 (C) −15 (D) −25 (E) −30

80. $\int \frac{x^2}{x^2-1}\, dx =$

BC ONLY

(A) $x + \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$ (B) $\ln|x^2 - 1| + C$ (C) $x + \tan^{-1}x + C$

(D) $x + \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| + C$ (E) $1 + \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| + C$

CHALLENGE

Answer Key

1. C	17. E	33. A	49. D	65. D
2. E	18. D	34. D	50. A	66. E
3. A	19. A	35. E	51. C	67. B
4. D	20. D	36. C	52. E	68. D
5. E	21. E	37. E	53. B	69. E
6. B	22. E	38. A	54. B	70. B
7. A	23. B	39. C	55. B	71. D
8. E	24. C	40. B	56. D	72. C
9. D	25. A	41. E	57. E	73. A
10. A	26. A	42. D	58. A	74. B
11. D	27. E	43. A	59. C	75. E
12. C	28. B	44. B	60. D	76. D
13. B	29. D	45. C	61. C	77. D
14. C	30. C	46. B	62. E	78. D
15. B	31. B	47. C	63. A	79. C
16. A	32. E	48. C	64. D	80. A

Answers Explained

All the references in parentheses below are to the basic integration formulas on pages 215 and 216. In general, if u is a function of x, then $du = u'(x)\,dx$.

1. **(C)** Use, first, formula (2), then (3), replacing u by x.

2. **(E)** Hint: Expand. $\int (x^2 - 1 + \frac{1}{4x^2})\,dx = \frac{x^3}{3} - x - \frac{1}{4x} + C$.

3. **(A)** By formula (3), with $u = 4 - 2t$ and $n = \frac{1}{2}$,

$$\int \sqrt{4-2t}\,dx = -\frac{1}{2}\int \sqrt{4-2t}\cdot(-2dt) = -\frac{1}{2}\frac{(4-2t)^{3/2}}{3/2} + C.$$

4. **(D)** Rewrite: $-\frac{1}{3}\int (2-3x)^5(-3\,dx)$

5. **(E)** Rewrite:

$$\int (2y-3y^2)^{-1/2}(1-3y)\,dy = \frac{1}{2}\int (2y-3y^2)^{-1/2}(2-6y)\,dy.$$

Use (3).

6. **(B)** Rewrite:

$$\frac{1}{3}\int (2x-1)^{-2}dx = \frac{1}{3}\cdot\frac{1}{2}\int (2x-1)^{-2}\cdot 2dx.$$

Using (3) yields $-\frac{1}{6(2x-1)} + C$.

7. (A) This is equivalent to $\frac{1}{3} \cdot 2\int \frac{3du}{1+3u}$. Use (4).

8. (E) Rewrite as $\frac{1}{4}\int (2t^2 - 1)^{-1/2} \cdot 4t\,dt$. Use (3).

9. (D) Use (5) with $u = 3x$; $du = 3\,dx$: $\frac{1}{3}\int \cos(3x)(3\,dx)$

10. (A) Use (4). If $u = 1 + 4x^2$, $du = 8x\,dx$: $\frac{1}{8}\int \frac{8x\,dx}{1+4x^2}$

11. (D) Use (18). Let $u = 2x$; then $du = 2\,dx$: $\frac{1}{2}\int \frac{2\,dx}{1+(2x)^2}$

12. (C) Rewrite as $\frac{1}{8}\int (1 + 4x^3)^{-2} \cdot (8x\,dx)$. Use (3) with $n = -2$.

13. (B) Rewrite as $\frac{1}{8}\int (1 + 4x^2)^{-1/2} \cdot (8x\,dx)$. Use (3) with $n = -\frac{1}{2}$.
Note carefully the differences in the integrands in Questions 10–13.

14. (C) Use (17); rewrite as $\int \frac{\frac{1}{2}\,dy}{\sqrt{1-\left(\frac{y}{2}\right)^2}}$.

15. (B) Rewrite as $-\frac{1}{2}\int (4 - y^2)^{-1/2} \cdot (-2y\,dy)$. Use (3).
Compare the integrands in Questions 14 and 15, noting the difference.

16. (A) Divide to obtain $\int \left(1 + \frac{1}{2} \cdot \frac{1}{x}\right)dx$. Use (2), (3), and (4). Remember that

$\int k\,dx = kx + C$ whenever $k \neq 0$.

17. (E) $\int \frac{(x-2)^3}{x^2}\,dx = \int \left(x - 6 + \frac{12}{x} - \frac{8}{x^2}\right)dx = \frac{x^2}{2} - 6x + 12\ln|x| + \frac{8}{x} + C.$
(Note the Binomial Theorem on page 673 with $n = 3$ to expand $(x-2)^3$.)

18. (D) The integral is equivalent to $\int \left(t - 2 + \frac{1}{t}\right) dt$. Integrate term by term.

19. (A) Integrate term by term.

20. (D) Division yields

$$\int \left(x - \frac{1}{x} - \frac{1}{x^2}\right) dx = \int x\,dx - \int \frac{1}{x}\,dx - \int \frac{1}{x^2}\,dx.$$

21. (E) Use formula (4) with $u = 1 - \sqrt{y} = 1 - y^{1/2}$. Then $du = -\frac{1}{2\sqrt{y}}\,dy$. Note that the integral can be written as $-2\int \frac{1}{(1-\sqrt{y})}\left(-\frac{1}{2\sqrt{y}}\right) dy$.

22. (E) Rewrite as $-\frac{1}{18}\int (4 - 9u^2)^{-1/2}(-18u\,du)$ and use formula (3).

23. **(B)** The integral is equal to $\frac{1}{2}\int \sin 2\theta \, d\theta$. Use formula (6) with $u = 2\theta$; $du = 2\,d\theta$.

24. **(C)** Use formula (6) with $u = \sqrt{x}$; $du = \frac{1}{2\sqrt{x}}\,dx$; $2\int \sin(\sqrt{x})\left(\frac{1}{2\sqrt{x}}\,dx\right)$

25. **(A)** Use formula (5) with $u = 4t^2$; $du = 8t\,dt$; $\frac{1}{8}\int \cos(4t^2)(8t\,dt)$

26. **(A)** Using the Half-Angle Formula (23) on page 675 with $\alpha = 2x$ yields $\int\left(\frac{1}{2} + \frac{1}{2}\cos 4x\right)dx$.

27. **(E)** Use formula (6): $\frac{1}{2}\int \sin 2\theta\,(2\,d\theta)$. (See #23)

28. **(B)** Integrate by parts (page 224). Let $u = x$ and $dv = \cos x\,dx$. Then $du = dx$ and $v = \sin x$. The given integral equals $x\sin x - \int \sin x\,dx$.

29. **(D)** Replace $\frac{1}{\cos^2 3u}$ by $\sec^2 3u$; then use formula (9): $\frac{1}{3}\int \sec^2 3u\,(3\,du)$

30. **(C)** Rewrite using $u = 1 + \sin x$ and $du = \cos x\,dx$ as $\int (1 + \sin x)^{-1/2}(\cos x\,dx)$.

Use formula (3).

31. **(B)** The integral is equivalent to $\int \csc(\theta - 1)\cot(\theta - 1)\,d\theta$. Use formula (12).

32. **(E)** Use formula (13) with $u = \frac{t}{2}$; $du = \frac{1}{2}dt$: $2\int \sec\frac{t}{2}\left(\frac{1}{2}dt\right)$

33. **(A)** Replace $\sin 2x$ by $2\sin x\cos x$; then the integral is equivalent to

$$-\int \frac{-2\sin x\cos x}{\sqrt{1+\cos^2 x}}\,dx = -\int u^{-1/2}\,du$$

where $u = 1 + \cos^2 x$ and $du = -2\sin x\cos x\,dx$. Use formula (3).

34. **(D)** Rewriting in terms of sines and cosines yields

$$\int \frac{\sin x}{\cos^{5/2} x}\,dx = -\int \cos^{-5/2} x(-\sin x)\,dx = -\left(-\frac{2}{3}\right)\cos^{-3/2} x + C.$$

35. **(E)** Use formula (7).

36. **(C)** Replace $\frac{1}{\sin^2 2x}$ by $\csc^2 2x$ and use formula (10): $\frac{1}{2}\int \csc^2 2x\,(2\,dx)$

37. **(E)** Let $u = \tan^{-1} y$; then integrate $\int u\,du$. The correct answer is $\frac{1}{2}(\tan^{-1} y)^2 + C$.

38. **(A)** Replacing $\sin 2\theta$ by $2\sin\theta\cos\theta$ yields

$$\int 2\sin\theta\cos^2\theta\,d\theta = -2\int(\cos\theta)^2(-\sin\theta\,d\theta) = -\frac{2}{3}\cos^3\theta + C.$$

39. **(C)** $\frac{1}{2}\int \frac{2\sin 2t\,dt}{1-\cos 2t} = \frac{1}{2}\ln|1-\cos 2t| + C$.

40. **(B)** Rewrite as $\frac{1}{2}\int \cot 2u\,(2du)$ and use formula (8).

41. **(E)** Use formula (4) with $u = e^x - 1$; $du = e^x\,dx$.

42. **(D)** Use partial fractions; find A and B such that

$$\frac{x-1}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}.$$

Then $x - 1 = A(x-2) + Bx$.
Set $x = 0$: $-1 = -2A$ and $A = \frac{1}{2}$.
Set $x = 2$: $1 = 2B$ and $B = \frac{1}{2}$.
So the given integral equals

$$\int \left(\frac{1}{2x} + \frac{1}{2(x-2)}\right)dx = \frac{1}{2}\ln|x| + \frac{1}{2}\ln|x-2| + C$$
$$= \frac{1}{2}\ln|x(x-2)| + C.$$

43. **(A)** Use formula (15) with $u = x^2$; $du = 2x\,dx$; $\frac{1}{2}\int e^{x^2}(2x\,dx)$.

44. **(B)** Use formula (15) with $u = \sin\theta$; $du = \cos\theta\,d\theta$.

45. **(C)** Use formula (6) with $u = e^{2\theta}$; $du = 2e^{2\theta}\,d\theta$; $\frac{1}{2}\int \sin e^{2\theta}\,(2e^{2\theta}\,d\theta)$.

46. **(B)** Use formula (15) with $u = \sqrt{x} = x^{1/2}$; $du = \frac{1}{2\sqrt{x}}\,dx$.

47. **(C)** Use the Parts Formula. Let $u = x$, $dv = e^{-x}dx$; $du = dx$, $v = -e^{-x}$. Then,

$$-xe^{-x} + \int e^{-x}\,dx = -xe^{-x} - e^{-x} + C.$$

48. **(C)** See Example 44, page 225.

49. **(D)** The integral is of the form $\int \frac{du}{u}$; use (4).

50. **(A)** The integral has the form $\int \frac{du}{1+u^2}$. Use formula (18), with $u = e^x$, $du = e^x\,dx$.

51. **(C)** Let $u = \ln v$; then $du = \frac{dv}{v}$. Use formula (3) for $\int \ln v(\frac{1}{v}\,dv)$.

52. **(E)** Hint: $\ln\sqrt{x} = \frac{1}{2}\ln x$; the integral is $\frac{1}{2}\int(\ln x)(\frac{1}{x}\,dx)$

53. **(B)** Use parts, letting $u = \ln x$ and $dv = x^3\,dx$. Then $du = \frac{1}{x}dx$ and $v = \frac{x^4}{4}$. The integral equals $\frac{x^4}{4}\ln x - \frac{1}{4}\int x^3\,dx$.

54. **(B)** Use parts, letting $u = \ln\eta$ and $dv = dx$. Then $du = \frac{1}{\eta}d\eta$ and $v = \eta$. The integral equals $\eta\ln\eta - \int d\eta$.

55. **(B)** Rewrite $\ln x^3$ as $3 \ln x$, and use the method of Answer 54.

56. **(D)** Use parts, letting $u = \ln y$ and $dv = y^{-2}\,dy$. Then $du = \frac{1}{y}\,dy$ and $v = -\frac{1}{y}$. The Parts Formula yields $\frac{-\ln y}{y} + \int \frac{1}{y^2}\,dy$.

57. **(E)** The integral has the form $\int \frac{du}{u}$, where $u = \ln v$: $\int \frac{\left(\frac{1}{v}dv\right)}{\ln v}$

58. **(A)** By long division, the integrand is equivalent to $1 - \frac{2}{y+1}$.

59. **(C)** $\int \frac{dx}{(x+1)^2+1} = \int \frac{dx}{1+(x+1)^2}$; use formula (18) with $u = x + 1$.

60. **(D)** Multiply to get $\int (x - \sqrt{x}\,)dx$.

61. **(C)** See Example 45, page 225. Replace x by θ.

62. **(E)** The integral equals $-\int (1 - \ln t)^2 \left(-\frac{1}{t}\,dt\right)$; it is equivalent to $-\int u^2\,du$, where $u = 1 - \ln t$.

63. **(A)** Replace u by x in the given integral to avoid confusion in applying the Parts Formula. To integrate $\int x \sec^2 x\,dx$, let the variable u in the Parts Formula be x, and let dv be $\sec^2 x\,dx$. Then $du = dx$ and $v = \tan x$, so

$$\int x \sec^2 x\,dx = x \tan x - \int \tan x\,dx$$
$$= x \tan x + \ln|\cos x| + C.$$

64. **(D)** The integral is equivalent to $\int \frac{2x}{4+x^2}\,dx + \int \frac{1}{4+x^2}\,dx$. Use formula (4) on the first integral and (18) on the second.

65. **(D)** The integral is equivalent to $\int \frac{1}{\sqrt{1-x^2}}\,dx - \int \frac{x}{\sqrt{1-x^2}}\,dx$.
Use formula (17) on the first integral. Rewrite the second integral as
$-\frac{1}{2} \int \left(1-x^2\right)^{-\frac{1}{2}} (-2x)\,dx$, and use (3).

66. **(E)** Rewrite: $-\frac{1}{4} \int (4x - 4x^2)^{-\frac{1}{2}} (4 - 8x)\,dx$.

67. **(B)** Hint: Divide, getting $\int \left[e^x - \frac{e^x}{1+e^x}\right] dx$.

68. **(D)** Letting $u = \sin\theta$ yields the integral $\int \frac{du}{1+u^2}$. Use formula (18).

69. **(E)** Use integration by parts, letting $u = \arctan x$ and $dv = dx$. Then

$$du = \frac{dx}{1+x^2} \quad \text{and} \quad v = x.$$

The Parts Formula yields

$$x \arctan x - \int \frac{x\,dx}{1+x^2} \qquad \text{or} \qquad x \arctan x - \frac{1}{2} \ln(1 + x^2) + C.$$

70. **(B)** Hint: Note that

$$\frac{1}{1-e^x} = \frac{1-e^x+e^x}{1-e^x} = 1 + \frac{e^x}{1-e^x}.$$

Or multiply the integrand by $\frac{e^{-x}}{e^{-x}}$, recognizing that the correct answer is equivalent to $-\ln\left|e^{-x}-1\right|$.

71. **(D)** Hint: Expand the numerator and divide. Then integrate term by term.

72. **(C)** Hint: Observe that $e^{2\ln u} = u^2$.

73. **(A)** Let $u = 1 + \ln y^2 = 1 + 2\ln|y|$; integrate $\frac{1}{2}\int \frac{\frac{2}{y}dy}{1+2\ln|y|}$.

74. **(B)** Hint: Expand and note that

$$\int(\tan^2\theta - 2\tan\theta + 1)\,d\theta = \int\sec^2\theta\,d\theta - 2\int\tan\theta\,d\theta.$$

Use formulas (9) and (7).

75. **(E)** Multiply by $\frac{1-\sin\theta}{1-\sin\theta}$. The correct answer is $\tan\theta - \sec\theta + C$.

76. **(D)** Note the initial conditions: when $t = 0$, $v = 0$ and $s = 0$. Integrate twice: $v = 6t^2$ and $s = 2t^3$. Let $t = 3$.

77. **(D)** Since $y' = x^2 - 2$, $y = \frac{1}{3}x^3 - 2x + C$. Replacing x by 1 and y by -3 yields $C = -\frac{4}{3}$.

78. **(D)** When $t = 0$, $v = 3$ and $s = 2$, so

$$v = 2t + 3t^2 + 3 \qquad \text{and} \qquad s = t^2 + t^3 + 3t + 2.$$

Let $t = 1$.

79. **(C)** Let $\frac{dv}{dt} = a$; then

$$v = at + C. \qquad (*)$$

Since $v = 75$ when $t = 0$, therefore $C = 75$. Then (*) becomes

$$v = at + 75$$

so

$$0 = at + 75 \qquad \text{and} \qquad a = -15.$$

80. **(A)** Divide to obtain $\int\left(1 + \frac{1}{x^2-1}\right)dx$. Use partial fractions to get

$$\frac{1}{x^2-1} = \frac{1}{2(x-1)} - \frac{1}{2(x+1)}.$$