Practice Exercises

Part A. Directions: Answer these questions *without* using your calculator.

In Questions 1–10, $a(t)$ denotes the acceleration function, $v(t)$ the velocity function, and $s(t)$ the position or height function at time t. (The acceleration due to gravity is –32 ft/sec^2.)

1. If $a(t) = 4t - 1$ and $v(1) = 3$, then $v(t)$ equals

(A) $2t^2 - t$ **(B)** $2t^2 - t + 1$ **(C)** $2t^2 - t + 2$
(D) $2t^2 + 1$ **(E)** $2t^2 + 2$

2. If $a(t) = 20t^3 - 6t$, $s(-1) = 2$, and $s(1) = 4$, then $v(t)$ equals

(A) $t^5 - t^3$ **(B)** $5t^4 - 3t^2 + 1$ **(C)** $5t^4 - 3t^2 + 3$
(D) $t^5 - t^3 + t + 3$ **(E)** $t^5 - t^3 + 1$

3. Given $a(t)$, $s(-1)$, and $s(1)$ as in Question 2, then $s(0)$ equals

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. A stone is thrown straight up from the top of a building with initial velocity 40 ft/sec and hits the ground 4 sec later. The height of the building, in feet, is

(A) 88 **(B)** 96 **(C)** 112 **(D)** 128 **(E)** 144

5. The maximum height is reached by the stone in Question 4 after

(A) 4/5 sec **(B)** 4 sec **(C)** 5/4 sec **(D)** 5/2 sec **(E)** 2 sec

6. If a car accelerates from 0 to 60 mph in 10 sec, what distance does it travel in those 10 sec? (Assume the acceleration is constant and note that 60 mph = 88 ft/sec.)

(A) 40 ft **(B)** 44 ft **(C)** 88 ft **(D)** 400 ft **(E)** 440 ft

7. A stone is thrown at a target so that its velocity after t sec is $(100 - 20t)$ ft/sec. If the stone hits the target in 1 sec, then the distance from the sling to the target is

(A) 80 ft **(B)** 90 ft **(C)** 100 ft **(D)** 110 ft **(E)** 120 ft

8. What should the initial velocity be if you want a stone to reach a height of 100 ft when you throw it straight up?

(A) 80 ft/sec **(B)** 92 ft/sec **(C)** 96 ft/sec
(D) 112 ft/sec **(E)** none of these

9. If the velocity of a car traveling in a straight line at time t is $v(t)$, then the difference in its odometer readings between times $t = a$ and $t = b$ is

(A) $\int_a^b |v(t)|\,dt$

(B) $\int_a^b v(t)\,dt$

(C) the net displacement of the car's position from $t = a$ to $t = b$
(D) the change in the car's position from $t = a$ to $t = b$
(E) none of these

10. If an object is moving up and down along the y-axis with velocity $v(t)$ and $s'(t) = v(t)$, then it is false that $\int_a^b v(t)\,dt$ gives

(A) $s(b) - s(a)$
(B) the net distance traveled by the object between $t = a$ and $t = b$
(C) the total change in $s(t)$ between $t = a$ and $t = b$
(D) the shift in the object's position from $t = a$ to $t = b$
(E) the total distance covered by the object from $t = a$ to $t = b$

11. Solutions of the differential equation $y\,dy = x\,dx$ are of the form

(A) $x^2 - y^2 = C$ (B) $x^2 + y^2 = C$ (C) $y^2 = Cx^2$
(D) $x^2 - Cy^2 = 0$ (E) $x^2 = C - y^2$

12. Find the domain of the particular solution to the differential equation in Question 11 that passes through point (–2, 1).

(A) $x < 0$ (B) $-2 \le x < 0$ (C) $x < -\sqrt{3}$
(D) $|x| < \sqrt{3}$ (E) $|x| > \sqrt{3}$

13. If $\frac{dy}{dx} = \frac{y}{2\sqrt{x}}$ and $y = 1$ when $x = 4$, then

(A) $y^2 = 4\sqrt{x} - 7$ (B) $\ln y = 4\sqrt{x} - 8$ (C) $\ln y = \sqrt{x} - 2$

(D) $y = e^{\sqrt{x}}$ (E) $y = e^{\sqrt{x}-2}$

14. If $\frac{dy}{dx} = e^y$ and $y = 0$ when $x = 1$, then

(A) $y = \ln|x|$ (B) $y = \ln|2 - x|$ (C) $e^{-y} = 2 - x$

(D) $y = -\ln|x|$ (E) $e^{-y} = x - 2$

15. If $\frac{dy}{dx} = \frac{x}{\sqrt{9 + x^2}}$ and $y = 5$ when $x = 4$, then y equals

(A) $\sqrt{9 + x^2} - 5$ (B) $\sqrt{9 + x^2}$ (C) $2\sqrt{9 + x^2} - 5$

(D) $\frac{\sqrt{9 + x^2} + 5}{2}$ (E) none of these

16. The general solution of the differential equation $x\,dy = y\,dx$ is a family of

(A) circles (B) hyperbolas (C) parallel lines
(D) parabolas (E) lines passing through the origin

17. The general solution of the differential equation $\frac{dy}{dx} = y$ is a family of

(A) parabolas (B) straight lines (C) hyperbolas
(D) ellipses (E) none of these

18. A function $f(x)$ that satisfies the equations $f(x)f'(x) = x$ and $f(0) = 1$ is

(A) $f(x) = \sqrt{x^2+1}$ (B) $f(x) = \sqrt{1-x^2}$ (C) $f(x) = x$
(D) $f(x) = e^x$ (E) none of these

19. The curve that passes through the point (1, 1) and whose slope at any point (x, y) is equal to $\frac{3y}{x}$ has the equation

(A) $3x - 2 = y$ (B) $y^3 = x$ (C) $y = x^3$
(D) $3y^2 = x^2 + 2$ (E) $3y^2 - 2x = 1$

20. What is the domain of the particular solution in Question 19?

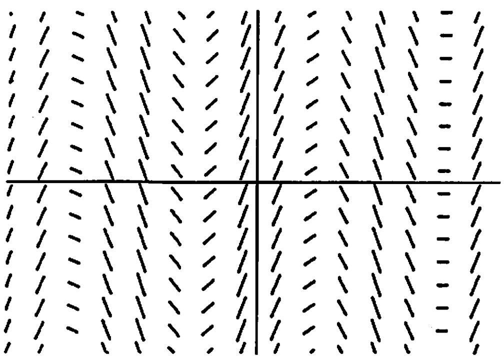
(A) all real numbers (B) $|x| \le 1$ (C) $x \ne 0$
(D) $x < 0$ (E) $x > 0$

21. If $\frac{dy}{dx} = \frac{k}{x}$, k a constant, and if $y = 2$ when $x = 1$ and $y = 4$ when $x = e$, then, when $x = 2$, y equals

(A) 2 (B) 4 (C) ln 8 (D) ln 2 + 2 (E) ln 4 + 2

22. The slope field shown at the right is for the differential equation

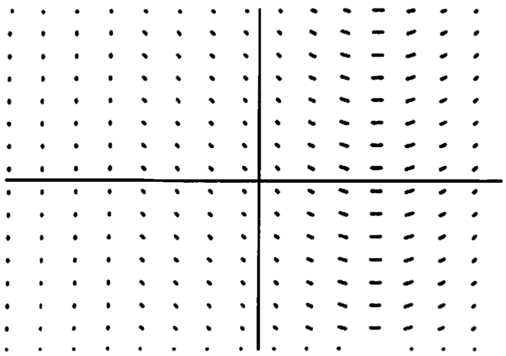
(A) $y' = x + 1$
(B) $y' = \sin x$
(C) $y' = -\sin x$
(D) $y' = \cos x$
(E) $y' = -\cos x$



$[-2\pi, 2\pi] \times [-1.5, 1.5]$

23. The slope field at the right is for the differential equation

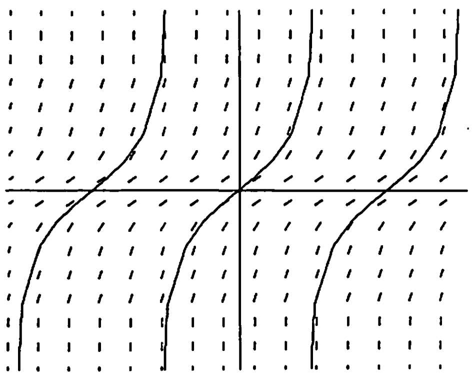
(A) $y' = 2x$
(B) $y' = 2x - 4$
(C) $y' = 4 - 2x$
(D) $y' = y$
(E) $y' = x + y$



$[-4, 4] \times [-12, 12]$

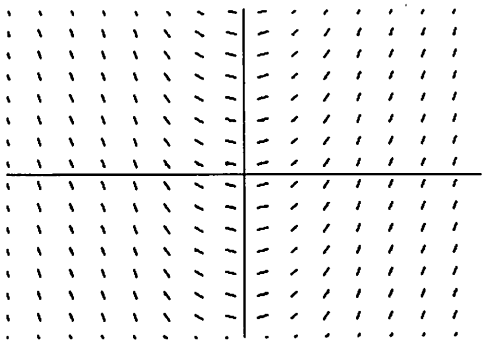
24. A solution curve has been superimposed on the slope field shown at the right. The solution is for the differential equation and initial condition

(A) $y' = \tan x;\ y(0) = 0$
(B) $y' = \cot x,\ y(\pi/4) = 1$
(C) $y' = 1 + x^2;\ y(0) = 0$
(D) $y' = \dfrac{1}{1+x^2};\ y\left(\dfrac{\pi}{4}\right) = 1$
(E) $y' = 1 + y^2;\ y(0) = 0$

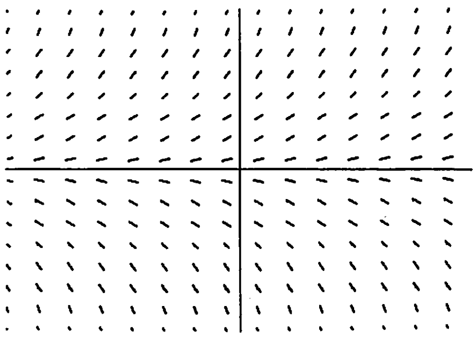


$[-4, 4] \times [-4, 4]$

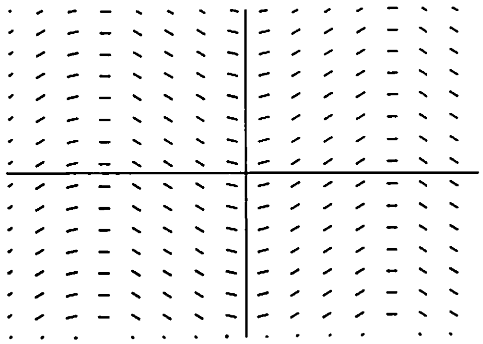
The slope fields below are for Questions 25–30.



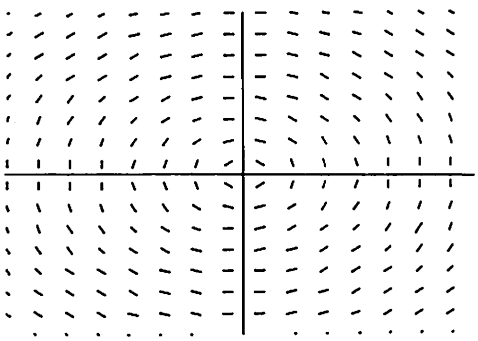
I $[-3, 3] \times [-3, 3]$



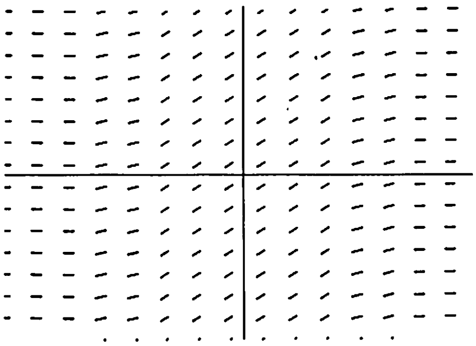
II $[-3, 3] \times [-3, 3]$



III $[-5, 5] \times [-5, 5]$



IV $[-3, 3] \times [-3, 3]$



V $[-2, 2] \times [-2, 2]$

25. Which slope field is for the differential equation $y' = y$?

(A) I (B) II (C) III (D) IV (E) V

26. Which slope field is for the differential equation $y' = -\frac{x}{y}$?

(A) I (B) II (C) III (D) IV (E) V

27. Which slope field is for the differential equation $y' = \sin x$?

(A) I (B) II (C) III (D) IV (E) V

28. Which slope field is for the differential equation $y' = 2x$?

(A) I (B) II (C) III (D) IV (E) V

29. Which slope field is for the differential equation $y' = e^{-x^2}$?

(A) I (B) II (C) III (D) IV (E) V

30. A particular solution curve of a differential equation whose slope field is shown above in II passes through the point (0,–1). The equation is

(A) $y = -e^x$ (B) $y = -e^{-x}$ (C) $y = x^2 - 1$ (D) $y = -\cos x$

(E) $y = -\sqrt{1 - x^2}$

BC ONLY

31. If you use Euler's method with $\Delta x = 0.1$ for the d.e. $y' = x$, with initial value $y(1) = 5$, then, when $x = 1.2$, y is approximately

(A) 5.10 (B) 5.20 (C) 5.21 (D) 6.05 (E) 7.10

32. The error in using Euler's method in Question 31 is

(A) 0.005 (B) 0.010 (C) 0.050 (D) 0.500 (E) 0.720

33. Which differential equation has the slope field shown?

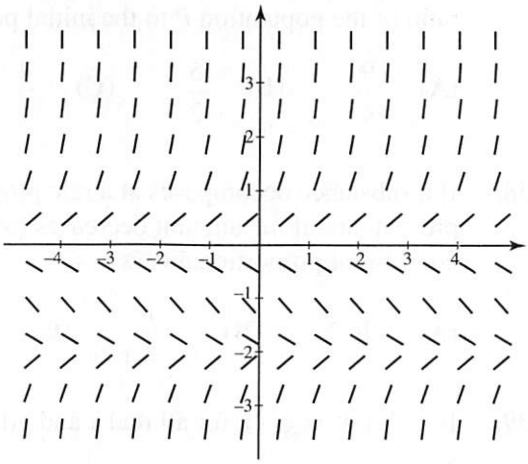
(A) $y' = y(y + 2)$

(B) $y' = x(y + 2)$

(C) $y' = xy + 2$

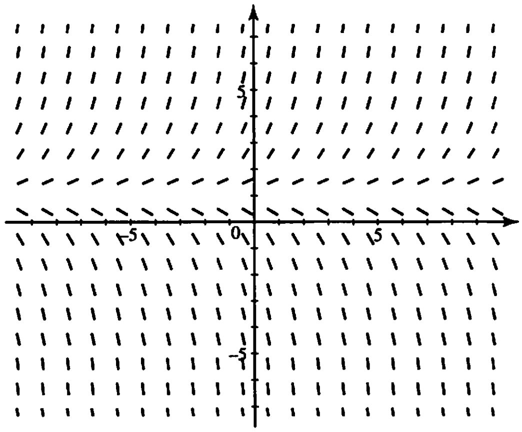
(D) $y' = \dfrac{x}{y + 2}$

(E) $y' = \dfrac{y}{y + 2}$



34. Which function is a possible solution of the slope field shown?

(A) $y = 1 - \frac{1}{x}$
(B) $y = 1 - \ln x$
(C) $y = 1 + \ln x$
(D) $y = 1 + e^x$
(E) $y = 1 + \tan x$



Part B. Directions: Some of the following questions require the use of a graphing calculator.

35. If $\frac{ds}{dt} = \sin^2\left(\frac{\pi}{2}s\right)$ and if $s = 1$ when $t = 0$, then, when $s = \frac{3}{2}$, t is equal to

(A) $\frac{1}{2}$ (B) $\frac{\pi}{2}$ (C) 1 (D) $\frac{2}{\pi}$ (E) $-\frac{2}{\pi}$

36. If radium decomposes at a rate proportional to the amount present, then the amount R left after t yr, if R_0 is present initially and c is the negative constant of proportionality, is given by

(A) $R = R_0ct$ (B) $R = R_0e^{ct}$ (C) $R = R_0 + \frac{1}{2}ct^2$
(D) $R = e^{R_0ct}$ (E) $R = e^{R_0+ct}$

37. The population of a city increases continuously at a rate proportional, at any time, to the population at that time. The population doubles in 50 yr. After 75 yr the ratio of the population P to the initial population P_0 is

(A) $\frac{9}{4}$ (B) $\frac{5}{2}$ (C) $\frac{4}{1}$ (D) $\frac{2\sqrt{2}}{1}$ (E) none of these

38. If a substance decomposes at a rate proportional to the amount of the substance present, and if the amount decreases from 40 g to 10 g in 2 hr, then the constant of proportionality is

(A) $-\ln 2$ (B) $-\frac{1}{2}$ (C) $-\frac{1}{4}$ (D) $\ln\frac{1}{4}$ (E) $\ln\frac{1}{8}$

39. If $(g'(x))^2 = g(x)$ for all real x and $g(0) = 0$, $g(4) = 4$, then $g(1)$ equals

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 4

40. The solution curve of $y' = y$ that passes through point (2, 3) is

(A) $y = e^x + 3$ (B) $y = \sqrt{2x+5}$ (C) $y = 0.406e^x$

(D) $y = e^x - (e^2 + 3)$ (E) $y = e^x/(0.406)$

41. At any point of intersection of a solution curve of the d.e. $y' = x + y$ and the line $x + y = 0$, the function y at that point

(A) is equal to 0 (B) is a local maximum (C) is a local minimum

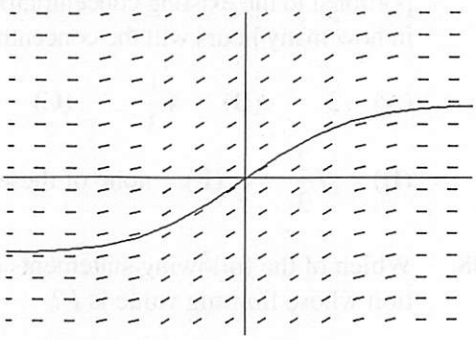
(D) has a point of inflection (E) has a discontinuity

42. The slope field for $F'(x) = e^{-x^2}$ is shown at the right with the particular solution $F(0) = 0$ superimposed. With a graphing calculator, $\lim_{x\to\infty} F(x)$ to three decimal places is

(A) 0.886 (B) 0.987

(C) 1.000 (D) 1.414

(E) ∞



$[-2, 2] \times [-2, 2]$

BC ONLY

43. The graph displays logistic growth for a frog population F. Which differential equation could be the appropriate model?

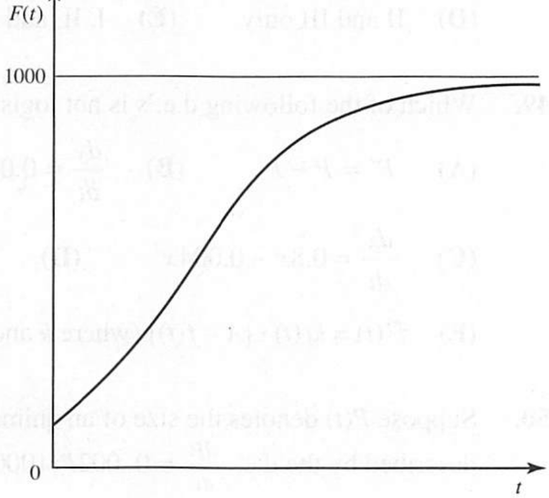
(A) $\dfrac{dF}{dt} = 1.5F - 0.003F^2$

(B) $\dfrac{dF}{dt} = 1.5F^2 - 0.003F$

(C) $\dfrac{dF}{dt} = 3F - 0.003F^2$

(D) $\dfrac{dF}{dt} = 3F^2 - 0.003F$

(E) $\dfrac{dF}{dt} = 0.003F^2 - 3F$



44. The table shows selected values of the derivative for a differentiable function f.

x	2	3	4	5	6	7
$f'(x)$	2.0	2.5	1.0	–0.5	–1.5	0.5

Given that $f(3) = 100$, use Euler's method with a step size of 2 to estimate $f(7)$.

(A) 101.5 (B) 102.5 (C) 103 (D) 104 (E) 104.5

45. A cup of coffee at temperature 180°F is placed on a table in a room at 68°F. The d.e. for its temperature at time t is $\frac{dy}{dt} = -0.11(y - 68)$; $y(0) = 180$. After 10 min the temperature (in °F) of the coffee is

(A) 96 (B) 100 (C) 105 (D) 110 (E) 115

46. Approximately how long does it take the temperature of the coffee in Question 45 to drop to 75°F?

(A) 10 min (B) 15 min (C) 18 min (D) 20 min (E) 25 min

47. The concentration of a medication injected into the bloodstream drops at a rate proportional to the existing concentration. If the factor of proportionality is 30% per hour, in how many hours will the concentration be one-tenth of the initial concentration?

(A) 3 (B) $4\frac{1}{3}$ (C) $6\frac{2}{3}$

(D) $7\frac{2}{3}$ (E) none of these

BC ONLY

48. Which of the following statements characterize(s) the logistic growth of a population whose limiting value is L?

I. The rate of growth increases at first.

II. The growth rate attains a maximum when the population equals $\frac{L}{2}$.

III. The growth rate approaches 0 as the population approaches L.

(A) I only (B) II only (C) I and II only
(D) II and III only (E) I, II, and III

49. Which of the following d.e.'s is not logistic?

(A) $P' = P - P^2$ (B) $\frac{dy}{dt} = 0.01y(100 - y)$

(C) $\frac{dx}{dt} = 0.8x - 0.004x^2$ (D) $\frac{dR}{dt} = 0.16(350 - R)$

(E) $f'(t) = kf(t) \cdot [A - f(t)]$ (where k and A are constants)

50. Suppose $P(t)$ denotes the size of an animal population at time t and its growth is described by the d.e. $\frac{dP}{dt} = 0.002P(1000 - P)$. The population is growing fastest

(A) initially (B) when $P = 500$ (C) when $P = 1000$
(D) when $\frac{dP}{dt} = 0$ (E) when $\frac{d^2P}{dt^2} > 0$

51. According to Newton's law of cooling, the temperature of an object decreases at a rate proportional to the difference between its temperature and that of the surrounding air. Suppose a corpse at a temperature of 32°C arrives at a mortuary where the temperature is kept at 10°C. Then the differential equation satisfied by the temperature T of the corpse t hr later is

(A) $\frac{dT}{dt} = -k(T-10)$ **(B)** $\frac{dT}{dt} = k(T-32)$ **(C)** $\frac{dT}{dt} = 32e^{-kt}$

(D) $\frac{dT}{dt} = -kT(T-10)$ **(E)** $\frac{dT}{dt} = kT(T-32)$.

52. If the corpse in Question 51 cools to 27°C in 1 hr, then its temperature (in °C) is given by the equation

(A) $T = 22e^{0.205t}$ **(B)** $T = 10e^{1.163t}$ **(C)** $T = 10 + 22e^{-0.258t}$
(D) $T = 32e^{-0.169t}$ **(E)** $T = 32 - 10e^{-0.093t}$

Answer Key

1. C	**12.** C	**23.** B	**34.** D	**45.** C
2. B	**13.** E	**24.** E	**35.** D	**46.** E
3. D	**14.** C	**25.** B	**36.** B	**47.** D
4. B	**15.** B	**26.** D	**37.** D	**48.** E
5. C	**16.** E	**27.** C	**38.** A	**49.** D
6. E	**17.** E	**28.** A	**39.** A	**50.** B
7. B	**18.** A	**29.** E	**40.** C	**51.** A
8. A	**19.** C	**30.** A	**41.** C	**52.** C
9. A	**20.** E	**31.** C	**42.** A	
10. E	**21.** E	**32.** B	**43.** C	
11. A	**22.** D	**33.** A	**44.** D	

Answers Explained

1. (C) $v(t) = 2t^2 - t + C$; $v(1) = 3$; so $C = 2$.

2. (B) If $a(t) = 20t^3 - 6t$, then

$$v(t) = 5t^4 - 3t^2 + C_1,$$
$$s(t) = t^5 - t^3 + C_1t + C_2,$$

Since

$$s(-1) = -1 + 1 - C_1 + C_2 = 2$$

and

$$s(1) = 1 - 1 + C_1 + C_2 = 4,$$

therefore

$$2C_2 = 6,\ C_2 = 3,$$
$$C_1 = 1.$$

So

$$v(t) = 5t^4 - 3t^2 + 1.$$

3. (D) From Answer 2, $s(t) = t^5 - t^3 + t + 3$, so $s(0) = C_2 = 3$.

4. (B) Since $a(t) = -32$, $v(t) = -32t + 40$, and the height of the stone $s(t) = -16t^2 + 40t + C$. When the stone hits the ground, 4 sec later, $s(t) = 0$, so

$$0 = -16(16) + 40(4) + C,$$
$$C = 96 \text{ ft}.$$

5. (C) From Answer 4

$$s(t) = -16t^2 + 40t + 96.$$

Then

$$s'(t) = -32t + 40,$$

which is zero if $t = 5/4$, and that yields maximum height, since $s''(t) = -32$.

6. (E) The velocity $v(t)$ of the car is linear, since its acceleration is constant and

$$a(t) = \frac{dv}{dt} = \frac{(60-0)\text{ mph}}{10\text{ sec}} = \frac{88\text{ ft/sec}}{10\text{ sec}} = 8.8\text{ ft/sec}^2$$

$v(t) = 8.8t + C_1$ and $v(0) = 0$, so $C_1 = 0$;
$s(t) = 4.4t^2 + C_2$ and $s(0) = 0$, so $C_2 = 0$;

$$s(10) = 4.4(10^2) = 440 \text{ ft}.$$

7. (B) Since $v = 100 - 20t$, $s = 100t - 10t^2 + C$ with $s(0) = 0$. So $s(1) = 100 - 10 = 90$ ft.

8. (A) Since $v = -32t + v_0$ and $s = -16t^2 + v_0t$, we solve simultaneously:

$$0 = -32t + v_0,$$
$$100 = -16t^2 + v_0t.$$

These yield $t = 5/2$ and $v_0 = 80$ ft/sec.

9. (A) The odometer measures the total trip distance from time $t = a$ to $t = b$ (whether the car moves forward or backward or reverses its direction one or more times from $t = a$ to $t = b$). This total distance is given exactly by $\int_a^b |v(t)|\,dt$.

10. (E) (A), (B), (C), and (D) are all true. (E) is false: see Answer 9.

11. (A) Integrating yields $\frac{y^2}{2} = \frac{x^2}{2} + C$ or $y^2 = x^2 + 2C$ or $y^2 = x^2 + C'$, where we have replaced the arbitrary constant $2C$ by C'.

12. (C) For initial point $(-2,1)$, $x^2 - y^2 = 3$. Rewriting the d.e. $y\,dy = x\,dx$ as $\frac{dy}{dx} = \frac{x}{y}$ reveals that the derivative does not exist when $y = 0$, which occurs at $x = \pm\sqrt{3}$. Since the particular solution must be differentiable in an interval containing $x = -2$, the domain is $x < -\sqrt{3}$.

13. (E) We separate variables. $\int \frac{dy}{y} = \frac{1}{2}\int x^{-\frac{1}{2}}dx$, so $\ln|y| = \sqrt{x} + c$. The initial point yields $\ln 1 = \sqrt{4} + c$; hence $c = -2$. With $y > 0$, the particular solution is $\ln y = \sqrt{x} - 2$, or $y = e^{\sqrt{x}-2}$.

14. (C) We separate variables. $\int e^{-y}dy = \int dx$, so $-e^{-y} = x + c$. The particular solution is $-e^{-y} = x - 2$.

15. (B) The general solution is $y = \frac{1}{2}\int (9 + x^2)^{-\frac{1}{2}}(2x\,dx) = \sqrt{9 + x^2} + C$; $y = 5$ when $x = 4$ yields $C = 0$.

16. (E) Since $\int \frac{dy}{y} = \int \frac{dx}{x}$, it follows that

$$\ln y = \ln x + C \qquad \text{or} \qquad \ln y = \ln x + \ln k;$$

so $y = kx$.

17. (E) $\int \frac{dy}{y} = \int dx$ yields $\ln|y| = x + c$; hence the general solution is $y = ke^x$, $k \neq 0$.

18. (A) We rewrite and separate variables, getting $y\frac{dy}{dx} = x$. The general solution is

$$y^2 = x^2 + C \qquad \text{or} \qquad f(x) = \pm\sqrt{x^2 + C}.$$

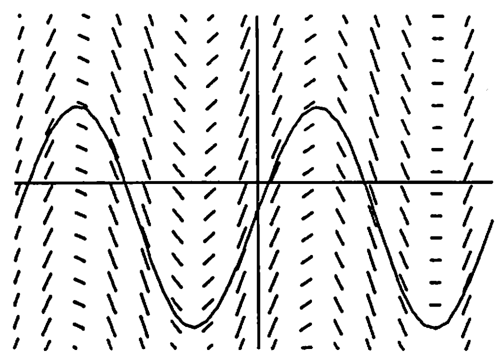
19. (C) We are given that $\frac{dy}{dx} = \frac{3y}{x}$. The general solution is $\ln|y| = 3\ln|x| + C$.

Thus, $|y| = c\,|x^3|$; $y = \pm c\,x^3$. Since $y = 1$ when $x = 1$, we get $c = 1$.

20. (E) The d.e. $\frac{dy}{dx} = \frac{3y}{x}$ reveals that the derivative does not exist when $x = 0$. Since the particular solution must be differentiable in an interval containing initial value $x = 1$, the domain is $x > 0$.

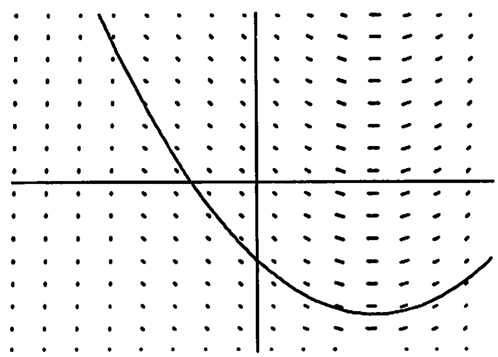
21. (E) The general solution is $y = k \ln |x| + C$, and the particular solution is $y = 2\ln|x| + 2$.

22. (D) We carefully(!) draw a curve for a solution to the d.e. represented by the slope field. It will be the graph of a member of the family $y = \sin x + C$. At the right we have superimposed the graph of the particular solution $y = \sin x - 0.5$.



$[-2\pi, 2\pi] \times [-1.5, 1.5]$

23. (B)



$[-4, 4] \times [-12, 12]$

It's easy to see that the answer must be choice (A), (B), or (C), because the slope field depends only on x: all the slope segments for a given x are parallel. Also, the solution curves in the slope field are all concave up, as they are only for choices (A) and (B). Finally, the solution curves all have a minimum at $x = 2$, which is true only for differential equation (B).

24. (E) The *solution curve* is $y = \tan x$, which we can obtain from the differential equation $y' = 1 + y^2$ with the condition $y(0) = 0$ as follows:

$$\frac{dy}{1+y^2} = dx, \qquad \tan^{-1} y = x, \qquad y = \tan x + C.$$

Since $y(0) = 0$, $C = 0$. Verify that (A) through (D) are incorrect.

NOTE: In matching slope fields and differential equations in Questions 25–29, keep in mind that if the slope segments along a vertical line are all parallel, signifying equal slopes for a fixed x, then the differential equation can be written as $y' = f(x)$. Replace "vertical" by "horizontal" and "x" by "y" in the preceding sentence to obtain a differential equation of the form $y' = g(y)$.

25. (B) The slope field for $y' = y$ must by II; it is the only one whose slopes are equal along a horizontal line.

26. (D) Of the four remaining slope fields, IV is the only one whose slopes are not equal along either a vertical or a horizontal line (the segments are *not* parallel). Its d.e. therefore cannot be either of type $y' = f(x)$ or $y' = g(y)$. The d.e. must be implicitly defined—that is, of the form $y' = F(x, y)$. So the answer here is IV.

27. (C) The remaining slope fields, I, III, and V, all have d.e.'s of the type $y' = f(x)$. The curves "lurking" in III are trigonometric curves—not so in I and V.

28. (A) Given $y' = 2x$, we immediately obtain the general solution, a family of parabolas, $y = x^2 + C$. (Trace the parabola in I through (0, 0), for example.)

29. (E) V is the only slope field still unassigned! Furthermore, the slopes "match" e^{-x^2}: the slopes are equal "about" the y-axis; slopes are very small when x is close to –2 and 2; and e^{-x^2} is a maximum at $x = 0$.

30. (A) From Answer 25, we know that the d.e. for slope field II is $y' = y$. The general solution is $y = ce^x$. For a solution curve to pass through point (0, –1), we have $-1 = ce^0$ and $c = -1$.

31. (C) Euler's method for $y' = x$, starting at (1, 5), with $\Delta x = 0.1$, yields

x	y	(SLOPE)* • Δx $= \Delta y$
1.	5	$1 \cdot (0.1) = 0.1$
1.1	5.1	$(1.1) \cdot (0.1) = 0.11$
1.2	5.21	

*The slope is x.

32. (B) We want to compare the true value of $y(1.2)$ to the estimated value of 5.21 obtained using Euler's method in Solution 31. Solving the d.e. $\frac{dy}{dx} = x$ yields $y = \frac{x^2}{2} + C$, and initial condition $y(1) = 5$ means that $5 = \frac{1^2}{2} + C$, or $C = 4.5$. Hence $y(1.2) = \frac{1.2^2}{2} + 4.5 = 5.22$. The error is $5.22 - 5.21 = 0.01$.

33. (A) Slopes depend only on the value of y, and the slope field suggests that $y' = 0$ whenever $y = 0$ or $y = -2$.

34. (D) The slope field suggests that the solution function increases (or decreases) without bound as x increases, but approaches $y = 1$ as a horizontal asymptote as x decreases.

35. (D) We separate variables to get $\csc^2\left(\frac{\pi}{2}s\right) ds = dt$. We integrate:

$-\frac{2}{\pi}\cot\left(\frac{\pi}{2}s\right) = t + C$. With $t = 0$ and $s = 1$, $C = 0$. When $s = \frac{3}{2}$, we get

$-\frac{2}{\pi}\cot\frac{3\pi}{4} = t$.

36. (B) Since $\frac{dR}{dt} = cR$, $\frac{dR}{R} = c\,dt$, and $\ln R = ct + C$. When $t = 0$, $R = R_0$; so $\ln R_0 = C$ or $\ln R = ct + \ln R_0$. Thus

$$\ln R - \ln R_0 = ct;\ \ln\frac{R}{R_0} = ct \quad \text{or} \quad \frac{R}{R_0} = e^{ct}.$$

37. (D) The question gives rise to the differential equation $\frac{dP}{dt} = kP$, where

$P = 2P_0$ when $t = 50$. We seek $\frac{P}{P_0}$ for $t = 75$. We get $\ln\frac{P}{P_0} = kt$ with $\ln 2 = 50k$; then

$$\ln\frac{P}{P_0} = \frac{t}{50}\ln 2 \quad \text{or} \quad \frac{P}{P_0} = 2^{t/50}.$$

38. (A) We let S equal the amount present at time t; using $S = 40$ when $t = 0$ yields $\ln\frac{S}{40} = kt$. Since, when $t = 2$, $S = 10$, we get

$$k = \frac{1}{2}\ln\frac{1}{4} \quad \text{or} \quad \ln\frac{1}{2} \quad \text{or} \quad -\ln 2.$$

39. (A) We replace $g(x)$ by y and then solve the equation $\frac{dy}{dx} = \pm\sqrt{y}$. We use the

constraints given to find the particular solution $2\sqrt{y} = x$ or $2\sqrt{g(x)} = x$.

40. (C) The general solution of $\frac{dy}{dx} = y$, or $\frac{dy}{y} = dx$ (with $y > 0$) is $\ln y = x + C$ or

$y = ce^x$. For a solution to pass through (2, 3), we have $3 = ce^2$ and $c = 3/e^2 \approx 0.406$.

41. (C) At a point of intersection, $y' = x + y$ and $x + y = 0$. So $y' = 0$, which implies that y has a critical point at the intersection. Since $y'' = 1 + y' = 1 + (x + y) = 1 + 0 = 1$, $y'' > 0$ and the function has a local minimum at the point of intersection. [See Figure N9–5, p. 373, showing the slope field for $y' = x + y$ and the curve $y = e^x - x - 1$ that has a local minimum at (0, 0).]

42. (A) Although there is no elementary function (one made up of polynomial, trigonometric, or exponential functions or their inverses) that is an antiderivative of $F'(x) = e^{-x^2}$, we know from the FTC, since $F(0) = 0$, that

$$F(x) = \int_0^x e^{-t^2}\,dt.$$

To approximate $\lim_{x\to\infty} F(x)$, use your graphing calculator

For upper limits of integration $x = 50$ and $x = 60$, answers are identical to 10 decimal places. Rounding to three decimal places yields 0.886.

43. **(C)** Logistic growth is modeled by equations of the form $\frac{dP}{dt} = kP(L - P)$, where L is the upper limit. The graph shows $L = 1000$, so the differential equation must be $\frac{dF}{dt} = kF(1000 - F) = 1000kF - kF^2$. Only equation C is of this form ($k = 0.003$).

44. **(D)** We start with $x = 3$ and $y = 100$. At $x = 3$, $\Delta y \approx \frac{dy}{dx} \cdot \Delta x = (2.5)(2) = 5$, moving us to $x = 3 + 2 = 5$ and $y = 100 + 5 = 105$. From there
$\Delta y \approx \frac{dy}{dx} \cdot \Delta x = (-0.5)(2) = -1$, so when $x = 5 + 2 = 7$ we estimate
$y = 105 + (-1) = 104$.

45. **(C)** We separate the variables in the given d.e., then solve:

$$\frac{dy}{y - 68} = -0.11\,dt,$$

$$\ln(y - 68) = -0.11t + c.$$

Since $y(0) = 180$, $\ln 112 = c$. Then

$$\ln\frac{y - 68}{112} = -0.11t,$$

$$y = 68 + 112e^{-0.11t}.$$

When $t = 10$, $y = 68 + 112e^{-1.1} \simeq 105°\text{F}$.

46. **(E)** The solution of the d.e. in Question 45, where y is the temperature of the coffee at time t, is

$$y = 68 + 112e^{-0.11t}.$$

We find t when $y = 75°\text{F}$:

$$75 = 68 + 112e^{-0.11t},$$

$$\frac{7}{112} = e^{-0.11t},$$

$$\frac{\ln 7 - \ln 112}{-0.11} = t \simeq 25 \text{ min}.$$

47. **(D)** If Q is the concentration at time t, then $\frac{dQ}{dt} = -0.30Q$. We separate variables and integrate:

$$\frac{dQ}{Q} = -0.30\,dt \quad \rightarrow \quad \ln Q = -0.30t + C.$$

We let $Q(0) = Q_0$. Then

$$\ln Q = -0.30t + \ln Q_0 \quad \rightarrow \quad \ln \frac{Q}{Q_0} = -0.30t \quad \rightarrow \quad \frac{Q}{Q_0} = e^{-0.30t}.$$

We now find t when $Q = 0.1Q_0$:

$$0.1 = e^{-0.30t},$$

$$t = \frac{\ln 0.1}{-0.3} \approx 7\frac{2}{3}\text{ hr.}$$

48. **(E)** See pages 386–387 for the characteristics of the logistic model.

49. **(D)** (A), (B), (C), and (E) are all of the form $y' = ky(a - y)$.

50. **(B)** The rate of growth, $\frac{dP}{dt}$, is greatest when its derivative is 0 and the curve of $y' = \frac{dP}{dt}$ is concave down. Since

$$\frac{dP}{dt} = 2P - 0.002P^2,$$

therefore

$$\frac{d^2P}{dt^2} = 2 - 0.004P,$$

which is equal to 0 if $y'' = \frac{2}{0.004}$, or 500, animals. The curve of y' is concave down for all P, since

$$\frac{d}{dt}\left(\frac{d^2P}{dt^2}\right) = -0.004,$$

so $P = 500$ is the maximum population.

51. **(A)** The description of temperature change here is an example of Case II (page 383): the rate of change is proportional to the amount or magnitude of the quantity present (i.e., the temperature of the corpse) minus a fixed constant (the temperature of the mortuary).

52. **(C)** Since (A) is the correct answer to Question 51, we solve the d.e. in (A) given the initial condition $T(0) = 32$:

$$\frac{dT}{T-10} = -k\ dt,$$

$$\ln(T-10) = -kt + C.$$

Using $T(0) = 32$, we get $\ln(22) = C$, so

$$\ln(T-10) = -kt + \ln 22,$$

$$T - 10 = 22e^{-kt}.$$

To find k, we use the given information that $T(1) = 27$:

$$27 - 10 = 22e^{-k},$$

$$\frac{17}{22} = e^{-k},$$

$$k = 0.258.$$

Therefore $T = 10 + 22e^{-0.258t}$.