CHAPTER 11

Miscellaneous Multiple-Choice Practice Questions

These questions provide further practice for Parts A and B of Section I of the examination. Answers begin on page 472.

Part A. Directions: Answer these questions *without* using your calculator.

1. Which of the following functions is continuous at $x = 0$?

(A) $f(x)\begin{cases} = \sin\frac{1}{x} & \text{for } x \neq 0 \\ = 0 & \text{for } x = 0 \end{cases}$ **(B)** $f(x) = [x]$ (greatest-integer function)

(C) $f(x)\begin{cases} = \frac{x}{x} & \text{for } x \neq 0 \\ = 0 & \text{for } x = 0 \end{cases}$ **(D)** $f(x)\begin{cases} = x\sin\frac{1}{x} & \text{for } x \neq 0 \\ = 0 & \text{for } x = 0 \end{cases}$

(E) $f(x) = \frac{x+1}{x}$

2. Which of the following statements about the graph of $y = \frac{x^2+1}{x^2-1}$ is *not* true?

(A) The graph is symmetric to the y-axis.
(B) The graph has two vertical asymptotes.
(C) There is no y-intercept.
(D) The graph has one horizontal asymptote.
(E) There is no x-intercept.

3. $\lim_{x\to 1^-} ([x] - |x|) =$

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** none of these

4. The x-coordinate of the point on the curve $y = x^2 - 2x + 3$ at which the tangent is perpendicular to the line $x + 3y + 3 = 0$ is

(A) $-\frac{5}{2}$ **(B)** $-\frac{1}{2}$ **(C)** $\frac{7}{6}$ **(D)** $\frac{5}{2}$ **(E)** none of these

5. $\lim_{x\to 1} \frac{\frac{3}{x}-3}{x-1}$ is

(A) -3 (B) -1 (C) 1 (D) 3 (E) nonexistent

6. For polynomial function p, $p''(2) = -6$, $p''(4) = 0$, and $p''(5) = 3$. Then p must:

(A) have an inflection point at $x = 4$ (B) have a minimum at $x = 4$
(C) have a root at $x = 4$ (D) be increasing on [2,5] (E) none of these

7. $\int_0^6 |x-4|\,dx =$

(A) 6 (B) 8 (C) 10 (D) 11 (E) 12

8. $\lim_{x\to\infty} \frac{3+x-2x^2}{4x^2+9}$ is

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) 3 (E) nonexistent

9. The maximum value of the function $f(x) = x^4 - 4x^3 + 6$ on $[1, 4]$ is

(A) 1 (B) 0 (C) 3 (D) 6 (E) none of these

10. Let $\begin{cases} f(x) = \frac{\sqrt{x+4}-3}{x-5} \\ f(5) = c \end{cases}$ if $x \neq 5$, and let f be continuous at $x = 5$. Then $c =$

(A) $-\frac{1}{6}$ (B) 0 (C) $\frac{1}{6}$ (D) 1 (E) 6

11. $\int_0^{\pi/2} \cos^2 x \sin x\,dx =$

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1

12. If $\sin x = \ln y$ and $0 < x < \pi$, then, in terms of x, $\frac{dy}{dx}$ equals

(A) $e^{\sin x}\cos x$ (B) $e^{-\sin x}\cos x$ (C) $\frac{e^{\sin x}}{\cos x}$

(D) $e^{\cos x}$ (E) $e^{\sin x}$

13. If $f(x) = x\cos x$, then $f'\left(\frac{\pi}{2}\right)$ equals

(A) $\frac{\pi}{2}$ (B) 0 (C) -1 (D) $-\frac{\pi}{2}$ (E) 1

14. The equation of the tangent to the curve $y = e^x \ln x$, where $x = 1$, is

(A) $y = ex$ **(B)** $y = e^x + 1$ **(C)** $y = e(x - 1)$
(D) $y = ex + 1$ **(E)** $y = x - 1$

15. If the displacement from the origin of a particle moving along the x-axis is given by $s = 3 + (t - 2)^4$, then the number of times the particle reverses direction is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** none of these

16. $\int_{-1}^{0} e^{-x} dx$ equals

(A) $1 - e$ **(B)** $\frac{1-e}{e}$ **(C)** $e - 1$ **(D)** $1-\frac{1}{e}$ **(E)** $e + 1$

17. If $f(x) = \begin{cases} x^2 & \text{for } x \le 2 \\ 4x - x^2 & \text{for } x > 2 \end{cases}$, then $\int_{-1}^{4} f(x)\, dx$ equals

(A) 7 **(B)** $\frac{23}{3}$ **(C)** $\frac{25}{3}$ **(D)** 9 **(E)** $\frac{65}{3}$

18. If the position of a particle on a line at time t is given by $s = t^3 + 3t$, then the speed of the particle is decreasing when

(A) $-1 < t < 1$ **(B)** $-1 < t < 0$ **(C)** $t < 0$
(D) $t > 0$ **(E)** $|t| > 1$

19. A rectangle with one side on the x-axis is inscribed in the triangle formed by the lines $y = x$, $y = 0$, and $2x + y = 12$. The area of the largest such rectangle is

CHALLENGE

(A) 6 **(B)** 3 **(C)** $\frac{5}{2}$ **(D)** 5 **(E)** 7

20. The x-value of the first-quadrant point that is on the curve of $x^2 - y^2 = 1$ and closest to the point (3, 0) is

(A) 1 **(B)** $\frac{3}{2}$ **(C)** 2 **(D)** 3 **(E)** none of these

21. If $y = \ln(4x + 1)$, then $\frac{d^2y}{dx^2}$ is

(A) $\frac{1}{4}$ **(B)** $\frac{-1}{(4x+1)^2}$ **(C)** $\frac{-4}{(4x+1)^2}$

(D) $\frac{-16}{(4x+1)^2}$ **(E)** $\frac{-1}{16(4x+1)^2}$

22. The region bounded by the parabolas $y = x^2$ and $y = 6x - x^2$ is rotated about the x-axis so that a vertical line segment cut off by the curves generates a ring. The value of x for which the ring of largest area is obtained is

(A) 4 **(B)** 3 **(C)** $\frac{5}{2}$ **(D)** 2 **(E)** $\frac{3}{2}$

23. $\int \frac{dx}{x \ln x}$ equals

(A) $\ln(\ln x) + C$ **(B)** $-\frac{1}{\ln^2 x} + C$ **(C)** $\frac{(\ln x)^2}{2} + C$
(D) $\ln x + C$ **(E)** none of these

24. The volume obtained by rotating the region bounded by $x = y^2$ and $x = 2 - y^2$ about the y-axis is equal to

(A) $\frac{16\pi}{3}$ **(B)** $\frac{32\pi}{3}$ **(C)** $\frac{32\pi}{15}$ **(D)** $\frac{64\pi}{15}$ **(E)** $\frac{8\pi}{3}$

25. The general solution of the differential equation $\frac{dy}{dx} = \frac{1-2x}{y}$ is a family of

(A) straight lines **(B)** circles **(C)** hyperbolas
(D) parabolas **(E)** ellipses

26. Estimate $\int_0^4 \sqrt{25-x^2}\, dx$ using the Left Rectangular Rule and two subintervals of equal width.

(A) $3+\sqrt{21}$ **(B)** $5+\sqrt{21}$ **(C)** $6+2\sqrt{21}$

(D) $8+2\sqrt{21}$ **(E)** $10+2\sqrt{21}$

27. $\int_0^7 \sin \pi x \, dx =$

(A) -2 **(B)** $-\frac{2}{\pi}$ **(C)** 0 **(D)** $\frac{1}{\pi}$ **(E)** $\frac{2}{\pi}$

BC ONLY

28. $\lim_{x\to 0} \frac{\tan 3x}{2x} =$

(A) 0 **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{2}$ **(E)** ∞

29. $\lim_{h\to 0} \frac{\tan(\pi/4+h)-1}{h} =$

(A) 0 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** ∞

CHALLENGE

30. The number of values of k for which $f(x) = e^x$ and $g(x) = k \sin x$ have a common point of tangency is

(A) 0 **(B)** 1 **(C)** 2 **(D)** large but finite **(E)** infinite

31. The curve $2x^2y + y^2 = 2x + 13$ passes through (3, 1). Use the line tangent to the curve there to find the approximate value of y at $x = 2.8$.

(A) 0.5 **(B)** 0.9 **(C)** 0.95 **(D)** 1.1 **(E)** 1.4

32. $\int \cos^3 x \, dx =$

(A) $\frac{\cos^4 x}{4} + C$ **(B)** $\frac{\sin^4 x}{4} + C$ **(C)** $\sin x - \frac{\sin^3 x}{3} + C$

(D) $\sin x + \frac{\sin^3 x}{3} + C$ **(E)** $\cos x - \frac{\cos^3 x}{3} + C$

33. The region bounded by $y = \tan x$, $y = 0$, and $x = \frac{\pi}{4}$ is rotated about the x-axis. The volume generated equals

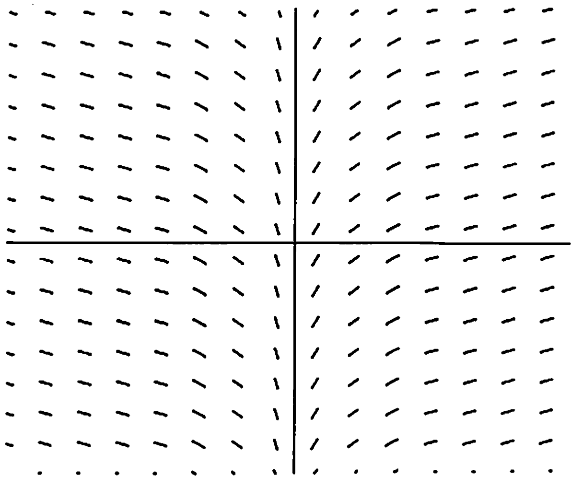
(A) $\pi - \frac{\pi^2}{4}$ **(B)** $\pi(\sqrt{2} - 1)$ **(C)** $\frac{3\pi}{4}$

(D) $\pi\left(1 + \frac{\pi}{4}\right)$ **(E)** none of these

34. $\lim_{h \to 0} \frac{a^h - 1}{h}$, for the constant $a > 0$, equals

(A) 1 **(B)** a **(C)** $\ln a$ **(D)** $\log_{10} a$ **(E)** $a \ln a$

35. Solutions of the differential equation whose slope field is shown here are most likely to be

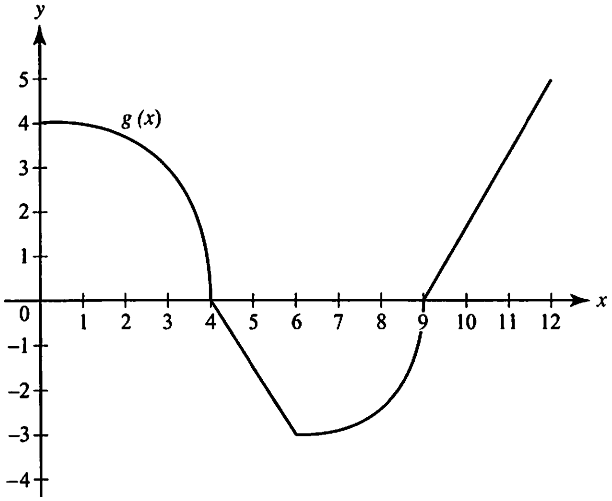


(A) quadratic **(B)** cubic **(C)** sinusoidal
(D) exponential **(E)** logarithmic

36. $\lim_{h \to 0} \frac{1}{h} \int_{\frac{\pi}{4}}^{\frac{\pi}{4}+h} \frac{\sin x}{x} \, dx =$

(A) 0 **(B)** 1 **(C)** $\frac{\sqrt{2}}{2}$ **(D)** $\frac{2\sqrt{2}}{\pi}$ **(E)** $\frac{2\sqrt{2}(\pi - 4)}{\pi^2}$

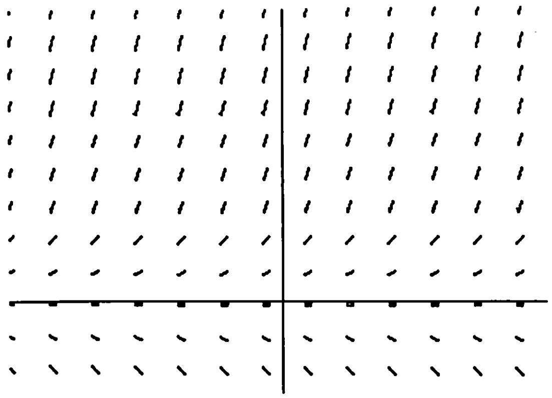
37. The graph of g, shown below, consists of the arcs of two quarter-circles and two straight-line segments. The value of $\int_0^{12} g(x)\,dx$ is



(A) $\pi + 2$ **(B)** $\frac{7\pi}{4} + \frac{9}{2}$ **(C)** $\frac{7\pi}{4} + 8$

(D) $7\pi + \frac{9}{2}$ **(E)** $\frac{25\pi}{4} + \frac{21}{2}$

38. Which of these could be a particular solution of the differential equation whose slope field is shown here?

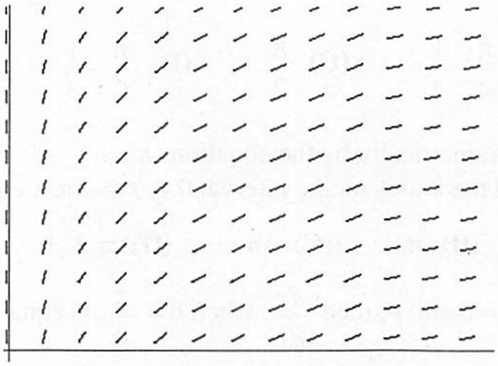


(A) $y = \frac{1}{x}$ **(B)** $y = \ln x$ **(C)** $y = e^x$ **(D)** $y = e^{-x}$ **(E)** $y = e^{x^2}$

39. What is the domain of the particular solution for $\frac{dy}{dx} = \frac{6x}{x^2 - 4}$ containing the point where $x = -1$?

(A) $x < 0$ **(B)** $x > -2$ **(C)** $-2 < x < 2$
(D) $x \neq \pm 2$ **(E)** none of these; no solution exists for $x = -1$

40. The slope field shown here is for the differential equation



(A) $y' = \frac{1}{x}$ **(B)** $y' = \ln x$ **(C)** $y' = e^x$ **(D)** $y' = y$ **(E)** $y' = -y^2$

41. If we substitute $x = \tan\theta$, which of the following is equivalent to $\int_0^1 \sqrt{1+x^2}\,dx$?

(A) $\int_0^1 \sec\theta\,d\theta$ **(B)** $\int_0^1 \sec^3\theta\,d\theta$ **(C)** $\int_0^{\pi/4} \sec\ \theta\,d\theta$

(D) $\int_0^{\pi/4} \sec^3\theta\,d\theta$ **(E)** $\int_0^{\tan 1} \sec^3\theta\,d\theta$

BC ONLY

42. If $x = 2\sin u$ and $y = \cos 2u$, then a single equation in x and y is

(A) $x^2 + y^2 = 1$ **(B)** $x^2 + 4y^2 = 4$ **(C)** $x^2 + 2y = 2$
(D) $x^2 + y^2 = 4$ **(E)** $x^2 - 2y = 2$

43. The area bounded by the lemniscate with polar equation $r^2 = 2\cos 2\theta$ is equal to

(A) 4 **(B)** 1 **(C)** $\frac{1}{4}$ **(D)** 2 **(E)** none of these

44. $\int_{-\infty}^{\infty} \frac{dx}{x^2+1} =$

(A) 0 **(B)** $\frac{\pi}{2}$ **(C)** π **(D)** 2π **(E)** none of these

45. The first four terms of the Maclaurin series (the Taylor series about $x = 0$) for

$f(x) = \frac{1}{1-2x}$ are

(A) $1 + 2x + 4x^2 + 8x^3$ **(B)** $1 - 2x + 4x^2 - 8x^3$
(C) $-1 - 2x - 4x^2 - 8x^3$ **(D)** $1 - x + x^2 - x^3$
(E) $1 + x + x^2 + x^3$

46. $\int x^2 e^{-x}\,dx =$

(A) $\frac{1}{3}x^3 e^{-x} + C$ **(B)** $-\frac{1}{3}x^3 e^{-x} + C$ **(C)** $-x^2e^{-x} + 2xe^{-x} + C$

(D) $-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$ **(E)** $-x^2e^{-x} + 2xe^{-x} - 2e^{-x} + C$

BC ONLY

47. $\int_0^{\pi/2} \sin^2 x \, dx$ is equal to

(A) $\frac{1}{3}$ **(B)** $\frac{\pi}{4} - \frac{1}{4}$ **(C)** $\frac{\pi}{2}$ **(D)** $\frac{\pi}{2} - \frac{1}{3}$ **(E)** $\frac{\pi}{4}$

48. A curve is given parametrically by the equations $x = t$, $y = 1 - \cos t$. The area bounded by the curve and the x-axis on the interval $0 \leqq t \leqq 2\pi$ is equal to

(A) $2(\pi + 1)$ **(B)** π **(C)** 4π **(D)** $\pi + 1$ **(E)** 2π

49. If $x = a \cot \theta$ and $y = a \sin^2 \theta$, then $\frac{dy}{dx}$, when $\theta = \frac{\pi}{4}$, is equal to

(A) $\frac{1}{2}$ **(B)** -1 **(C)** 2 **(D)** $-\frac{1}{2}$ **(E)** $-\frac{1}{4}$

50. Which of the following improper integrals diverges?

(A) $\int_0^{\infty} e^{-x^2} dx$ **(B)** $\int_{-\infty}^{0} e^x dx$ **(C)** $\int_0^1 \frac{dx}{x}$

(D) $\int_0^{\infty} e^{-x} dx$ **(E)** $\int_0^1 \frac{dx}{\sqrt{x}}$

51. $\int_2^4 \frac{du}{\sqrt{16 - u^2}}$ equals

(A) $\frac{\pi}{12}$ **(B)** $\frac{\pi}{6}$ **(C)** $\frac{\pi}{4}$ **(D)** $\frac{\pi}{3}$ **(E)** $\frac{2\pi}{3}$

52. $\lim_{x \to 0^+} \left(\frac{1}{x}\right)^x$ is

(A) $-\infty$ **(B)** 0 **(C)** 1 **(D)** ∞ **(E)** nonexistent

53. A particle moves along the parabola $x = 3y - y^2$ so that $\frac{dy}{dt} = 3$ at all time t. The speed of the particle when it is at position (2, 1) is equal to

(A) 0 **(B)** 3 **(C)** $\sqrt{13}$ **(D)** $3\sqrt{2}$ **(E)** none of these

54. $\lim_{x \to 0^+} \frac{\cot x}{\ln x} =$

(A) $-\infty$ **(B)** -1 **(C)** 0 **(D)** 1 **(E)** ∞

55. When rewritten as partial fractions, $\frac{3x + 2}{x^2 - x - 12}$ includes which of the following?

I. $\frac{1}{x+3}$ **II.** $\frac{1}{x-4}$ **III.** $\frac{2}{x-4}$

(A) none **(B)** I only **(C)** II only **(D)** III only **(E)** I and III

BC ONLY

56. Using two terms of an appropriate Maclaurin series, estimate $\int_0^1 \frac{1-\cos x}{x}\,dx$.

(A) $\frac{1}{96}$ **(B)** $\frac{23}{96}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{25}{96}$

(E) undefined; the integral is improper

57. The slope of the spiral $r = \theta$ at $\theta = \frac{\pi}{4}$ is

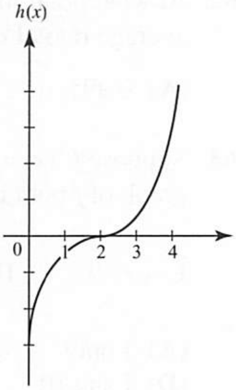
(A) $-\sqrt{2}$ **(B)** -1 **(C)** 1 **(D)** $\frac{4+\pi}{4-\pi}$ **(E)** undefined

Part B. Directions: Some of these questions require the use of a graphing calculator.

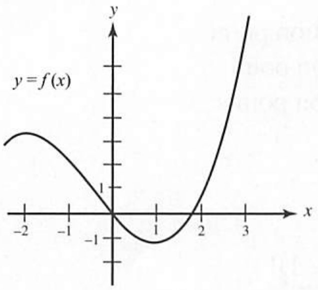
58. The graph of function h is shown here. Which of these statements is (are) true?

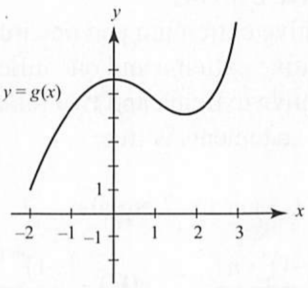
I. The first derivative is never negative.
II. The second derivative is constant.
III. The first and second derivatives equal 0 at the same point.

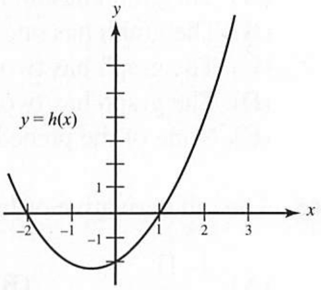
(A) I only **(B)** III only **(C)** I and II
(D) I and III **(E)** all three



59. Graphs of functions $f(x)$, $g(x)$, and $h(x)$ are shown below.







Consider the following statements:

I. $g(x) = f'(x)$
II. $f(x) = g'(x)$
III. $h(x) = g''(x)$

Which of these statements is (are) true?

(A) I only **(B)** II only **(C)** II and III only
(D) all three **(E)** none of these

60. If $y=\int_3^x \frac{1}{\sqrt{3+2t}}\,dt$, then $\frac{d^2y}{dx^2}=$

(A) $-\frac{1}{(3+2x)^{\frac{3}{2}}}$ (B) $\frac{3}{(3+2x)^{\frac{5}{2}}}$ (C) $\frac{\sqrt{3+2x}}{2}$

(D) $\frac{(3+2x)^{\frac{3}{2}}}{3}$ (E) 0

61. If $\int_{-3}^{4} f(x)\,dx=6$, then $\int_{-4}^{3} f(x+1)\,dx=$

(A) –6 (B) –5 (C) 5 (D) 6 (E) 7

62. At what point in the interval [1, 1.5] is the rate of change of $f(x) = \sin x$ equal to its average rate of change on the interval?

(A) 0.995 (B) 1.058 (C) 1.239 (D) 1.253 (E) 1.399

63. Suppose $f'(x) = x^2(x - 1)$. Then $f''(x) = x(3x - 2)$. Over which interval(s) is the graph of f both increasing and concave up?

I. $x < 0$ **II.** $0 < x < \frac{2}{3}$ **III.** $\frac{2}{3} < x < 1$ **IV.** $x > 1$

(A) I only (B) II only (C) II and IV
(D) I and III (E) IV only

64. Which of the following statements is true about the graph of $f(x)$ in Question 62?

(A) The graph has no relative extrema.
(B) The graph has one relative extremum and one inflection point.
(C) The graph has two relative extrema and one inflection point.
(D) The graph has two relative extrema and two inflection points.
(E) None of the preceding statements is true.

65. The nth derivative of $\ln(x + 1)$ at $x = 2$ equals

(A) $\frac{(-1)^{n-1}}{3^n}$ (B) $\frac{(-1)^n \cdot n!}{3^{n+1}}$ (C) $\frac{(-1)^{n-1}(n-1)!}{3^n}$

(D) $\frac{(-1)^{n+1} \cdot n!}{3^{n+1}}$ (E) $\frac{(-1)^{n+1}}{3^{n+1}}$

66. If $f(x)$ is continuous at the point where $x = a$, which of the following statements may be false?

(A) $\lim_{x\to a} f(x)$ exists. (B) $\lim_{x\to a} f(x) = f(a)$. (C) $f'(a)$ exists.

(D) $f(a)$ is defined. (E) $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$.

67. Suppose $\int_0^3 f(x+k)\,dx = 4$, where k is a constant. Then $\int_k^{3+k} f(x)\,dx$ equals

(A) 3 **(B)** $4 - k$ **(C)** 4 **(D)** $4 + k$ **(E)** none of these

68. The volume, in cubic feet, of an "inner tube" with inner diameter 4 ft and outer diameter 8 ft is

CHALLENGE

(A) $4\pi^2$ **(B)** $12\pi^2$ **(C)** $8\pi^2$ **(D)** $24\pi^2$ **(E)** $6\pi^2$

69. If $f(u) = \tan^{-1} u^2$ and $g(u) = e^u$, then the derivative of $f(g(u))$ is

(A) $\frac{2ue^u}{1+u^4}$ **(B)** $\frac{2ue^{u^2}}{1+u^4}$ **(C)** $\frac{2e^u}{1+4e^{2u}}$

(D) $\frac{2e^{2u}}{1+e^{4u}}$ **(E)** $\frac{2e^{2u}}{\sqrt{1-e^{4u}}}$

70. If $\sin(xy) = y$, then $\frac{dy}{dx}$ equals

(A) $\sec(xy)$ **(B)** $y\cos(xy) - 1$ **(C)** $\frac{1-y\cos(xy)}{x\cos(xy)}$

(D) $\frac{y\cos(xy)}{1-x\cos(xy)}$ **(E)** $\cos(xy)$

71. Let $x > 0$. Suppose $\frac{d}{dx}f(x) = g(x)$ and $\frac{d}{dx}g(x) = f(\sqrt{x})$; then $\frac{d^2}{dx^2}f(x^2) =$

(A) $f(x^4)$ **(B)** $f(x^2)$ **(C)** $2xg(x^2)$

(D) $\frac{1}{2x}f(x)$ **(E)** $2g(x^2) + 4x^2f(x)$

72. The region bounded by $y = e^x$, $y = 1$, and $x = 2$ is rotated about the x-axis. The volume of the solid generated is given by the integral

(A) $\pi\int_0^2 e^{2x}dx$ **(B)** $2\pi\int_1^{e^2}(2-\ln y)(y-1)\,dy$ **(C)** $\pi\int_0^2(e^{2x}-1)\,dx$

(D) $2\pi\int_0^{e^2} y(2-\ln y)\,dy$ **(E)** $\pi\int_0^2(e^x-1)^2\,dx$

73. Suppose the function f is continuous on $1 \leqq x \leqq 2$, that $f'(x)$ exists on $1 < x < 2$, that $f(1) = 3$, and that $f(2) = 0$. Which of the following statements is *not* necessarily true?

(A) The Mean-Value Theorem applies to f on $1 \leqq x \leqq 2$.

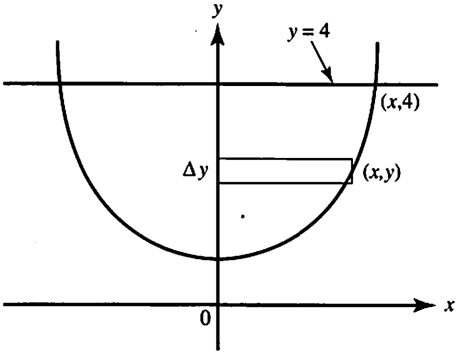
(B) $\int_1^2 f(x)\,dx$ exists.

(C) There exists a number c in the closed interval $[1, 2]$ such that $f'(c) = 0$.

(D) If k is any number between 0 and 3, there is a number c between 1 and 2 such that $f(c) = k$.

(E) If c is any number such that $1 < c < 2$, then $\lim_{x\to c} f(x)$ exists.

74. The region S in the figure is bounded by $y = \sec x$, the y–axis, and $y = 4$. What is the volume of the solid formed when S is rotated about the y-axis?



(A) 0.791 (B) 2.279 (C) 5.692 (D) 11.385 (E) 17.217

75. If 40 g of a radioactive substance decomposes to 20 g in 2 yr, then, to the nearest gram, the amount left after 3 yr is

(A) 10 (B) 12 (C) 14 (D) 16 (E) 17

76. An object in motion along a line has acceleration $a(t) = \pi t + \frac{2}{1+t^2}$ and is at rest when $t = 1$. Its average velocity from $t = 0$ to $t = 2$ is

(A) 0.362 (B) 0.274 (C) 3.504 (D) 7.008 (E) 8.497

77. Find the area bounded by $y = \tan x$ and $x + y = 2$, and above the x-axis on the interval [0, 2].

(A) 0.919 (B) 0.923 (C) 1.013 (D) 1.077 (E) 1.494

78. An ellipse has major axis 20 and minor axis 10. Rounded off to the nearest integer, the maximum area of an inscribed rectangle is

(A) 50 (B) 79 (C) 80 (D) 82 (E) 100

79. The average value of $y = x \ln x$ on the interval $1 \leqq x \leqq e$ is

(A) 0.772 (B) 1.221 (C) 1.359 (D) 1.790 (E) 2.097

80. Let $f(x) = \int_0^x (1 - 2\cos^3 t)\, dt$ for $0 \leqq x \leqq 2\pi$. On which interval is f increasing?

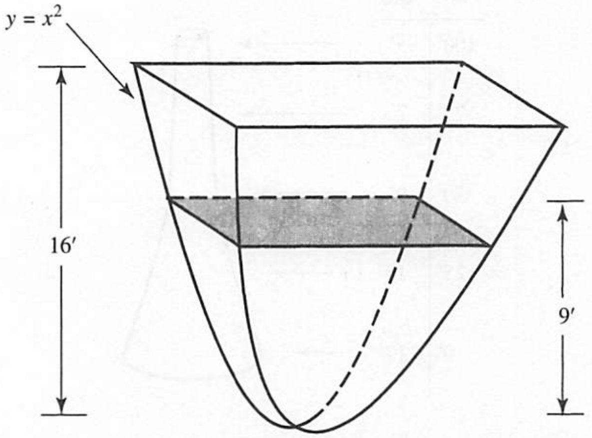
(A) $0 < x < \pi$ (B) $0.654 < x < 5.629$ (C) $0.654 < x < 2\pi$
(D) $\pi < x < 2\pi$ (E) none of these

81. The table shows the speed of an object (in ft/sec) during a 3-sec period. Estimate its acceleration (in ft/sec^2) at $t = 1.5$ sec.

time, sec	0	1	2	3
speed, ft/sec	30	22	12	0

(A) −17 (B) −13 (C) −10 (D) −5 (E) 17

82. A maple-syrup storage tank 16 ft high hangs on a wall. The back is in the shape of the parabola $y = x^2$ and all cross sections parallel to the floor are squares. If syrup is pouring in at the rate of 12 ft^3/hr, how fast (in ft/hr) is the syrup level rising when it is 9 ft deep?

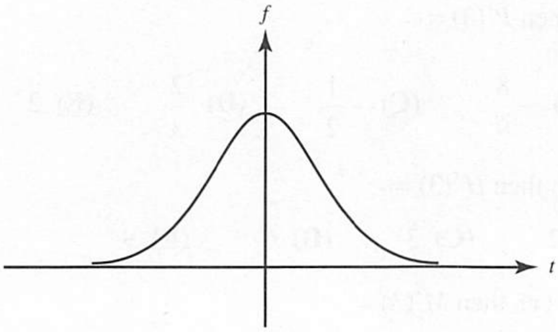


(A) $\frac{2}{27}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{4}{3}$ **(D)** 36 **(E)** 162

83. In a protected area (no predators, no hunters), the deer population increases at a rate of $\frac{dP}{dt} = k(1000 - P)$, where $P(t)$ represents the population of deer at t yr. If 300 deer were originally placed in the area and a census showed the population had grown to 500 in 5 yr, how many deer will there be after 10 yr?

(A) 608 **(B)** 643 **(C)** 700 **(D)** 833 **(E)** 892

84. Shown is the graph of $f(x) = \frac{4}{x^2 + 1}$.

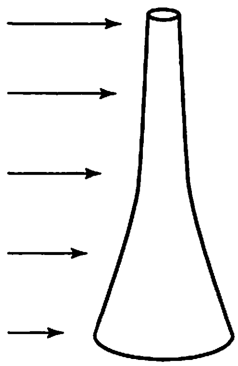


Let $H(x) = \int_0^x f(t)\,dt$. The local linearization of H at $x = 1$ is $H(x)$ equals

(A) $2x$ **(B)** $-2x - 4$ **(C)** $2x + \pi - 2$
(D) $-2x + \pi + 2$ **(E)** $2x + \ln 16 + 2$

85. A smokestack 100 ft tall is used to treat industrial emissions. The diameters, measured at 25-ft intervals, are shown in the table. Using the midpoint rule, estimate the volume of the smokestack to the nearest 100 ft^3.

ht	dia
100′	5′
75′	7′
50′	9′
25′	15′
0′	17′



(A) 8100 **(B)** 9500 **(C)** 9800 **(D)** 12,500 **(E)** 39,300

For Questions 86–90 the table shows the values of differentiable functions *f* and *g*.

x	f	f'	g	g'
1	2	$\frac{1}{2}$	-3	5
2	3	1	0	4
3	4	2	2	3
4	6	4	3	$\frac{1}{2}$

86. If $P(x) = \dfrac{f(x)}{g(x)}$, then $P'(3) =$

(A) -2 **(B)** $-\frac{8}{9}$ **(C)** $-\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** 2

87. If $H(x) = f(g(x))$, then $H'(3) =$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 9

88. If $M(x) = f(x) \cdot g(x)$, then $M'(3) =$

(A) 2 **(B)** 6 **(C)** 8 **(D)** 14 **(E)** 16

89. If $K(x) = g^{-1}(x)$, then $K'(3) =$

(A) $-\frac{1}{2}$ **(B)** $-\frac{1}{3}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** 2

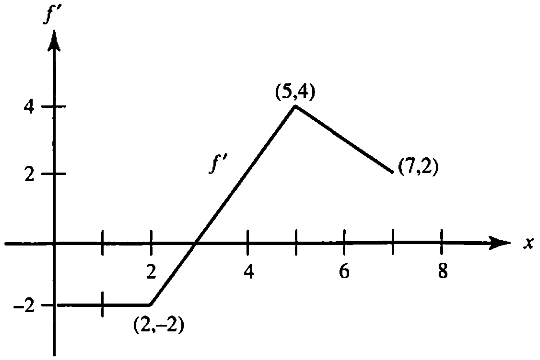
90. If $R(x) = \sqrt{f(x)}$, then $R'(3) =$

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2\sqrt{2}}$ **(C)** $\frac{1}{2}$ **(D)** $\sqrt{2}$ **(E)** 2

91. Water is poured into a spherical tank at a constant rate. If $W(t)$ is the rate of increase of the depth of the water, then W is

(A) constant **(B)** linear and increasing **(C)** linear and decreasing
(D) concave up **(E)** concave down

92. The graph of f' is shown below. If $f(7) = 3$ then $f(1) =$

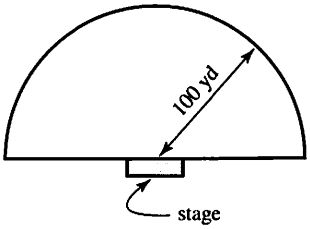


(A) -10 **(B)** -4 **(C)** -3 **(D)** 10 **(E)** 16

93. At an outdoor concert, the crowd stands in front of the stage filling a semicircular disk of radius 100 yd. The approximate density of the crowd x yd from the stage is given by

$$D(x) = \frac{20}{2\sqrt{x}+1}$$

people per square yard. About how many people are at the concert?

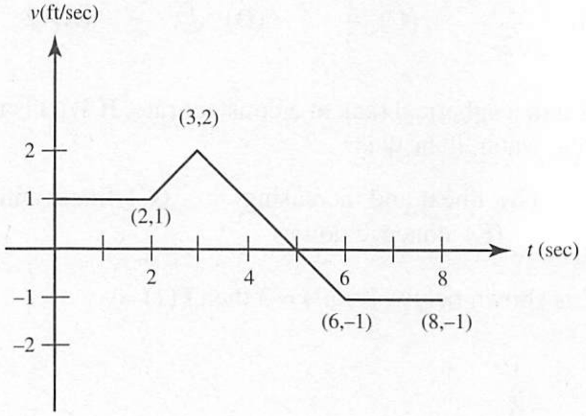


(A) 200 **(B)** 19,500 **(C)** 21,000 **(D)** 165,000 **(E)** 591,000

94. The Centers for Disease Control announced that, although more AIDS cases were reported this year, the rate of increase is slowing down. If we graph the number of AIDS cases as a function of time, the curve is currently

(A) increasing and linear
(B) increasing and concave down
(C) increasing and concave up
(D) decreasing and concave down
(E) decreasing and concave up

The graph below is for Questions 95–97. It shows the velocity, in feet per second, for $0 < t < 8$, of an object moving along a straight line.



95. The object's average speed (in ft/sec) for this 8-sec interval was

(A) 0 **(B)** $\frac{3}{8}$ **(C)** 1 **(D)** $\frac{8}{3}$ **(E)** 8

96. When did the object return to the position it occupied at $t = 2$?

(A) $t = 4$ **(B)** $t = 5$ **(C)** $t = 6$ **(D)** $t = 8$ **(E)** never

97. The object's average acceleration (in ft/sec^2) for this 8-sec interval was

(A) -2 **(B)** $-\frac{1}{4}$ **(C)** 0 **(D)** $\frac{1}{4}$ **(E)** 1

98. If a block of ice melts at the rate of $\frac{72}{2t+3}$ cm^3/min, how much ice melts during the first 3 min?

(A) 8 cm^3 **(B)** 16 cm^3 **(C)** 21 cm^3 **(D)** 40 cm^3 **(E)** 79 cm^3

BC ONLY

99. A particle moves counterclockwise on the circle $x^2 + y^2 = 25$ with a constant speed of 2 ft/sec. Its velocity vector, **v**, when the particle is at (3, 4), equals

(A) $\left\langle -\frac{8}{5},\frac{6}{5}\right\rangle$ **(B)** $\left\langle \frac{8}{5},-\frac{6}{5}\right\rangle$ **(C)** $\langle -2\sqrt{3},2\rangle$

(D) $\langle 2,-2\sqrt{3}\rangle$ **(E)** $\langle -2\sqrt{2},2\sqrt{2}\rangle$

100. Let $\mathbf{R} = a\cos kt\mathbf{i} + a\sin kt\mathbf{j}$ be the (position) vector $x\mathbf{i} + y\mathbf{j}$ from the origin to a moving point $P(x, y)$ at time t, where a and k are positive constants. The acceleration vector, **a**, equals

(A) $-k^2\mathbf{R}$ **(B)** $a^2k^2\mathbf{R}$ **(C)** $-a\mathbf{R}$
(D) $-ak^2\langle\cos t,\sin t\rangle$ **(E)** $-\mathbf{R}$

101. The length of the curve $y = 2^x$ between (0, 1) and (2, 4) is

(A) 3.141 **(B)** 3.664 **(C)** 4.823 **(D)** 5.000 **(E)** 7.199

BC ONLY

102. The position of a moving object is given by $P(t) = (3t, e^t)$. Its acceleration is

(A) undefined
(B) constant in both magnitude and direction
(C) constant in magnitude only
(D) constant in direction only
(E) constant in neither magnitude nor direction

103. Suppose we plot a particular solution of $\frac{dy}{dx} = 4y$ from initial point (0, 1) using Euler's method. After one step of size $\Delta x = 0.1$, how big is the error?

(A) 0.09 **(B)** 1.09 **(C)** 1.49 **(D)** 1.90 **(E)** 2.65

104. We use the first three terms to estimate $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$. Which of the following statements is (are) true?

I. The estimate is 0.7.
II. The estimate is too low.
III. The estimate is off by less than 0.1.

(A) I only **(B)** III only **(C)** I and II
(D) I and III **(E)** all three

105. Which of these diverges?

(A) $\sum_{n=1}^{\infty} \frac{2}{3^n}$ **(B)** $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ **(C)** $\sum_{n=1}^{\infty} \frac{2}{3n}$

(D) $\sum_{n=1}^{\infty} \frac{2}{n^3}$ **(E)** $\sum_{n=1}^{\infty} \frac{2n}{3^n}$

106. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$.

(A) 0 **(B)** $\frac{1}{e}$ **(C)** 1 **(D)** e **(E)** ∞

107. When we use $e^x \approx 1 + x + \frac{x^2}{2}$ to estimate $\sqrt{e}$, the Lagrange remainder is no greater than

(A) 0.021 **(B)** 0.034 **(C)** 0.042 **(D)** 0.067 **(E)** 0.742

108. An object in motion along a curve has position $P(t) = (\tan t, \cos 2t)$ for $0 \leqslant t \leqslant 1$. How far does it travel?

(A) 0.96 **(B)** 1.73 **(C)** 2.10 **(D)** 2.14 **(E)** 3.98

Answer Key

1. D
2. C
3. A
4. D
5. A
6. E
7. C
8. A
9. D
10. C
11. D
12. A
13. D
14. C
15. B
16. C
17. C
18. C
19. A
20. B
21. D
22. D
23. A
24. A
25. E
26. E
27. E
28. D
29. D
30. E
31. D
32. C
33. A
34. C
35. E
36. D
37. B
38. C
39. C
40. A
41. D
42. C
43. D
44. C
45. A
46. D
47. E
48. E
49. D
50. C
51. D
52. C
53. D
54. A
55. E
56. B
57. D
58. D
59. C
60. A
61. D
62. D
63. E
64. E
65. C
66. C
67. C
68. E
69. D
70. D
71. E
72. C
73. C
74. D
75. C
76. A
77. D
78. E
79. B
80. B
81. C
82. B
83. B
84. C
85. C
86. A
87. C
88. E
89. E
90. C
91. D
92. B
93. B
94. B
95. C
96. E
97. B
98. D
99. A
100. A
101. B
102. D
103. A
104. D
105. C
106. D
107. B
108. D

Answers Explained

Part A

1. **(D)** If $f(x) = x\sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$ then,

$$\lim_{x\to 0} f(x) = 0 = f(0);$$

thus this function is continuous at 0. In (A), $\lim_{x\to 0} \sin\frac{1}{x}$ does not exist; in (B), f has a jump discontinuity; in (C), f has a removable discontinuity; and in (E), f has an infinite discontinuity.

2. **(C)** To find the y-intercept, let $x = 0$; $y = -1$.

3. **(A)** $\lim_{x\to 1^-} [x] - \lim_{x\to 1^-} |x| = 0 - 1 = -1$.

4. **(D)** The line $x + 3y + 3 = 0$ has slope $-\frac{1}{3}$; a line perpendicular to it has slope 3.

The slope of the tangent to $y = x^2 - 2x + 3$ at any point is the derivative $2x - 2$. Set $2x - 2$ equal to 3.

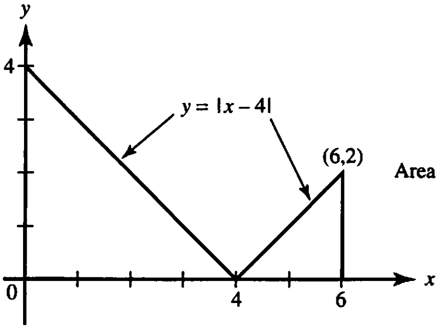
5. (A) $\lim\limits_{x\to 1}\dfrac{\frac{3}{x}-3}{x-1}$ is $f'(1)$, where $f(x)=\dfrac{3}{x}, f'(x)=-\dfrac{3}{x^2}$. Or simplify the given fraction to $\dfrac{3-3x}{x(x-1)}=\dfrac{3(1-x)}{x(x-1)}=\dfrac{-3}{x}(x\neq 1)$.

6. (E) Because $p''(2) < 0$ and $p''(5) > 0$, p changes concavity somewhere in the interval [2,5], but we cannot be sure p'' changes sign at $x = 4$.

7. (C) $\int_0^6 |x-4|\,dx=\int_0^4 (4-x)\,dx+\int_4^6 (x-4)\,dx=\left(4x-\dfrac{x^2}{2}\right)\Bigg|_0^4+\left(\dfrac{x^2}{2}-4x\right)\Bigg|_4^6$

$$= 8 + [(18-24)-(8-16)] = 8 + (-6+8) = 10.$$

Save time by finding the area under $y = |x-4|$ from a sketch!



Area = $\frac{1}{2}(4 \cdot 4) + \frac{1}{2}(2 \cdot 2) = 10$.

8. (A) Since the degrees of numerator and denominator are the same, the limit as $x\to\infty$ is the ratio of the coefficients of the terms of highest degree: $\dfrac{-2}{4}$.

9. (D) On the interval [1, 4], $f'(x) = 0$ only for $x = 3$. Since $f(3)$ is a relative minimum, check the endpoints to find that $f(4) = 6$ is the absolute maximum of the function.

10. (C) To find lim f as $x \to 5$ (if it exists), multiply f by $\dfrac{\sqrt{x+4}+3}{\sqrt{x+4}+3}$.

$$f(x)=\frac{x-5}{(x-5)\left(\sqrt{x+4}+3\right)}$$

and if $x \neq 5$ this equals $\dfrac{1}{\sqrt{x+4}+3}$. So lim $f(x)$ as $x \to 5$ is $\dfrac{1}{6}$. For f to be continuous at $x = 5$, $f(5)$ or c must also equal $\dfrac{1}{6}$.

11. (D) Evaluate $-\dfrac{1}{3}\cos^3 x\Bigg|_0^{\pi/2}$.

12. (A) $\cos x=\dfrac{1}{y}\dfrac{dy}{dx}$ and thus $\dfrac{dy}{dx}=y\cos x$. From the equation given, $y=e^{\sin x}$.

13. **(D)** If $f(x) = x \cos x$, then $f'(x) = -x \sin x + \cos x$, and

$$f'\left(\frac{\pi}{2}\right) = -\frac{\pi}{2} \cdot 1 + 0.$$

14. **(C)** If $y = e^x \ln x$, then $\frac{dy}{dx} = \frac{e^x}{x} + e^x \ln x$, which equals e when $x = 1$. Since also $y = 0$ when $x = 1$, the equation of the tangent is $y = e(x - 1)$.

15. **(B)** $v = 4(t - 2)^3$ and changes sign exactly once, when $t = 2$.

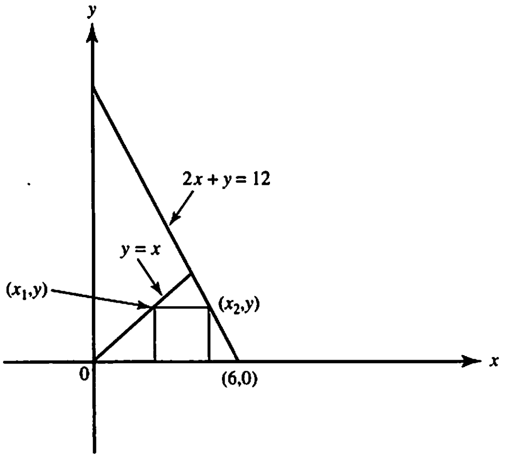
16. **(C)** Evaluate $-e^{-x}\Big|_{-1}^{0}$.

17. **(C)** $\int_{-1}^{4} f(x)\,dx = \int_{-1}^{2} x^2\,dx + \int_{2}^{4} \left(4x - x^2\right) dx = \frac{x^3}{3}\Big|_{-1}^{2} + \left(2x^2 - \frac{x^3}{3}\right)\Big|_{2}^{4}$.

18. **(C)** Since $v = 3t^2 + 3$, it is always positive, while $a = 6t$ and is positive for $t > 0$ but negative for $t < 0$. The speed therefore increases for $t > 0$ but decreases for $t < 0$.

19. **(A)** Note from the figure that the area, A, of a typical rectangle is

$$A = (x_2 - x_1) \cdot y = \left(\frac{12 - y}{2} - y\right) \cdot y = 6y - \frac{3y^2}{2}.$$



For $y = 2$, $\frac{dA}{dy} = 0$. Note that $\frac{d^2A}{dy^2}$ is always negative.

20. **(B)** If S represents the square of the distance from (3, 0) to a point (x, y) on the curve, then $S = (3 - x)^2 + y^2 = (3 - x)^2 + (x^2 - 1)$. Setting $\frac{dS}{dx} = 0$ yields the minimum distance at $x = \frac{3}{2}$.

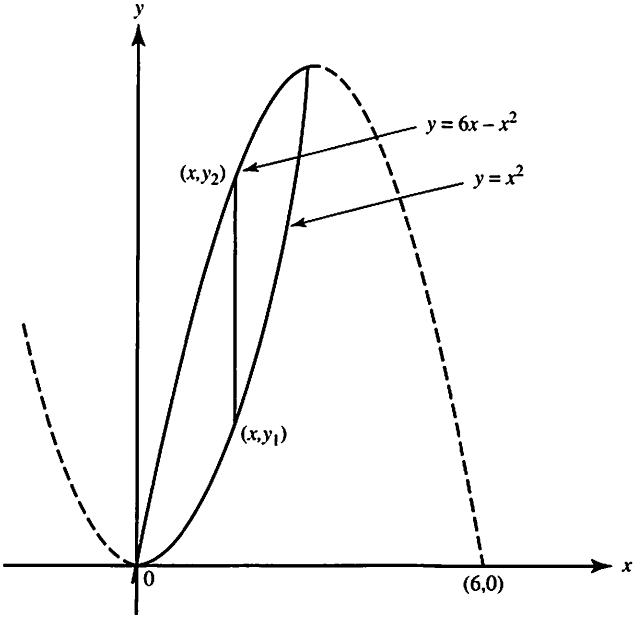
21. **(D)** $\frac{dy}{dx} = \frac{4}{4x+1} = 4(4x+1)^{-1}$, so $\frac{d^2y}{dx^2} = 4\left(-1(4x+1)^{-2} \cdot 4\right)$

22. **(D)** See the figure. Since the area, A, of the ring equals $\pi\,(y_2^{\,2} - y_1^{\,2})$,

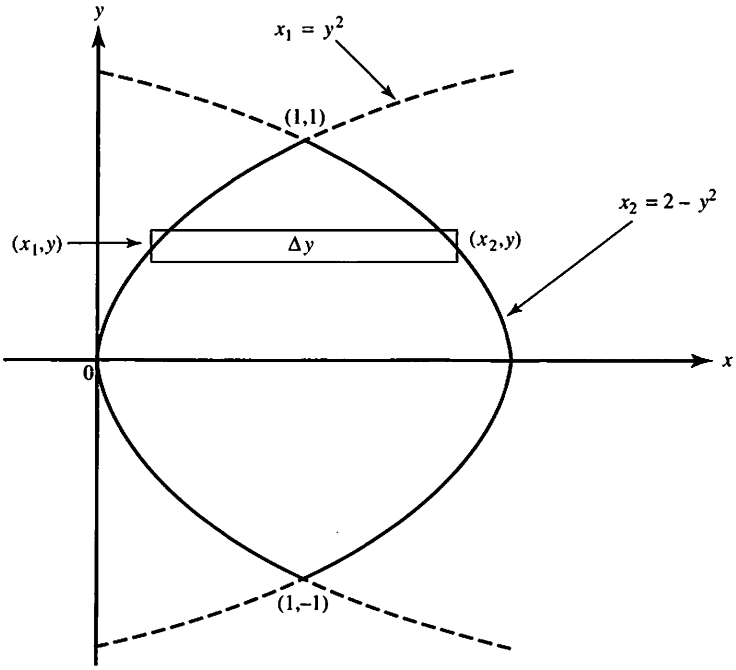
$$A = \pi\,[(6x - x^2)^2 - x^4] = \pi\,[36x^2 - 12x^3 + x^4 - x^4]$$

and $\frac{dA}{dx} = \pi\,(72x - 36x^2) = 36\pi x\,(2 - x)$.

It can be verified that $x = 2$ produces the maximum area.



23. **(A)** This is of type $\int \frac{du}{u}$ with $u = \ln x$: $\int \frac{\frac{1}{x}dx}{\ln x}$.



24. **(A)** About the y-axis; see the figure. Washer.

$$\Delta V = \pi (x_2^2 - x_1^2)\Delta y, \text{ so } V = 2\pi \int_0^1 \left[(2-y^2)^2 - y^4\right] dy \;= 2\pi \int_0^1 (4-4y^2)\, dy$$

25. **(E)** Separating variables, we get $y\,dy = (1 - 2x)\,dx$. Integrating gives

$$\frac{1}{2}y^2 = x - x^2 + C$$

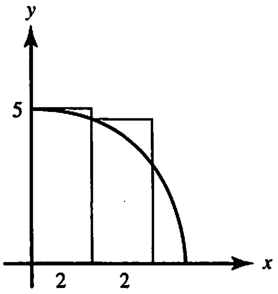
or

$$y^2 = 2x - 2x^2 + k$$

or

$$2x^2 + y^2 - 2x = k.$$

26. **(E)** $2(5) + 2\sqrt{21}$.



27. **(E)** $\frac{1}{\pi}\int_0^7 \sin \pi x\,(\pi\,dx) \;= \frac{-1}{\pi}\cos(\pi x)\Big|_0^7$.

28. **(D)** Use L'Hôpital's Rule or rewrite the expression as $\lim\limits_{x\to 0} \frac{\sin 3x}{3x} \cdot \frac{1}{\cos 3x} \cdot \frac{3}{2}$.

29. **(D)** For $f(x) = \tan x$, this is $f'\left(\frac{\pi}{4}\right) = \sec^2\left(\frac{\pi}{4}\right)$.

30. **(E)** The parameter k determines the amplitude of the sine curve. For $f = k \sin x$ and $g = e^x$ to have a common point of tangency, say at $x = q$, the curves must both go through (q, y) and their slopes must be equal at q. Thus, we must have

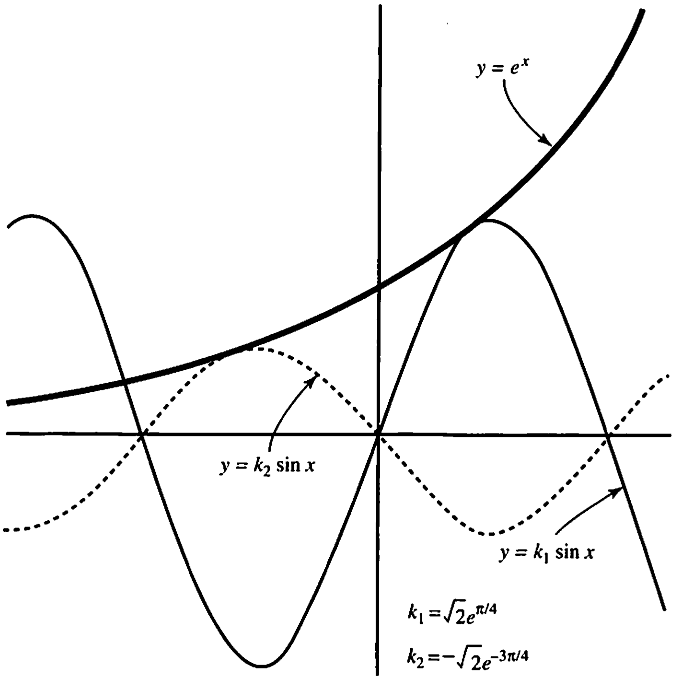
$$k \sin q = e^q \text{ and } k \cos q = e^q,$$

and therefore

$$\sin q = \cos q.$$

Thus, $q = \frac{\pi}{4} \pm n\pi$ and $k = \frac{e^q}{\sin\left(\frac{\pi}{4} \pm n\pi\right)}$.

The figure shows $k_1 = \sqrt{2}\,e^{\pi/4}$ and $k_2 = -\sqrt{2}\,e^{-3\pi/4}$.



31. **(D)** We differentiate implicitly to find the slope $\frac{dy}{dx}$:

$$2\left(x^2\frac{dy}{dx}+2xy\right)+2y\,\frac{dy}{dx}=2,$$

$$\frac{dy}{dx}=\frac{1-2xy}{x^2+y}.$$

At (3, 1), $\frac{dy}{dx}=-\frac{1}{2}$. The linearization is $y\approx-\frac{1}{2}(x-3)+1$.

32. **(C)** $\int\cos^3 x\,dx=\int\cos^2 x\cdot\cos x\,dx=\int\left(1-\sin^2 x\right)\cos x\,dx$

$$=\int\cos x\,dx-\int\sin^2 x\cdot\cos x\,dx=\sin x-\frac{\sin^3 x}{3}+C\cdot$$

33. **(A)** About the x-axis. Disk.

$$\Delta V=\pi y^2\Delta x,$$

$$V=\pi\int_0^{\pi/4}\tan^2 x\,dx=\pi\int_0^{\pi/4}\left(\sec^2 x-1\right)dx=\pi\left[\tan x-x\right]_0^{\pi/4}$$

$$=\pi\left(1-\frac{\pi}{4}\right).$$

34. **(C)** Let $f(x)=a^x$; then $\lim\limits_{h\to0}\frac{a^h-1}{h}=\lim\limits_{h\to0}\frac{a^{0+h}-a^0}{h}=f'(0)=a^0\ln a=\ln a.$

35. **(E)** $\frac{dy}{dx}$ is a function of x alone; curves appear to be asymptotic to the y-axis and to increase more slowly as $|x|$ increases.

36. **(D)** The given limit is equivalent to

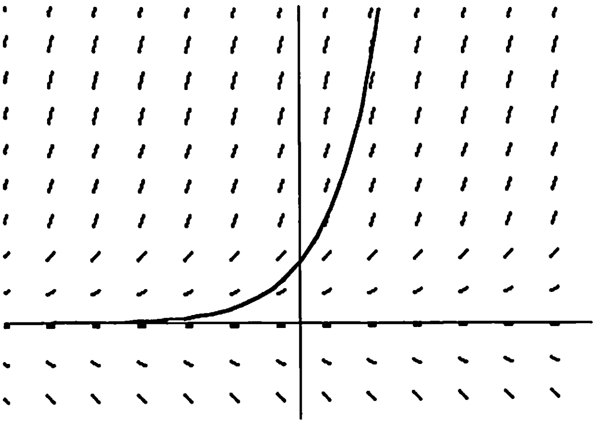
$$\lim_{h\to 0}\frac{\left(\frac{\pi}{4}+h\right)-F\left(\frac{\pi}{4}\right)}{h}=F'\left(\frac{\pi}{4}\right),$$

where $F'(x)=\frac{\sin x}{x}$. The answer is $\frac{2\sqrt{2}}{\pi}$.

37. **(B)** $\int_0^{12} g(x)\,dx = \int_0^4 g(x)\,dx+\int_4^6 g(x)\,dx+\int_6^9 g(x)\,dx+\int_9^{12} g(x)\,dx$

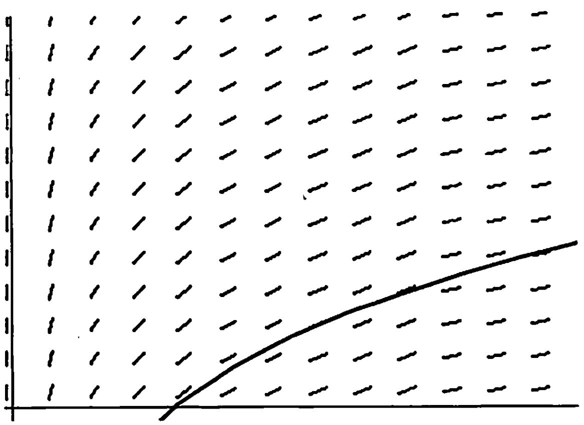
$$=4\pi - 3 - \frac{9\pi}{4}+\frac{15}{2}.$$

38. **(C)** In the figure, the curve for $y=e^x$ has been superimposed on the slope field.



39. **(C)** The general solution is $y = 3\ln|x^2 - 4| + C$. The differential equation $\frac{dy}{dx}=\frac{6x}{x^2-4}$ reveals that the derivative does not exist for $x = \pm 2$. The particular solution must be differentiable in an interval containing the initial value $x = -1$, so the domain is $-2 < x < 2$.

40. **(A)** The solution curve shown is $y = \ln x$, so the differential equation is $y'=\frac{1}{x}$.



41. **(D)** $\sqrt{1+\tan^2\theta}=\sec\theta; dx=\sec^2\theta; 0\leqslant x\leqslant 1;$ so $0\le\theta\le\frac{\pi}{4}$.

42. **(C)** The equations may be rewritten as $\frac{x}{2} = \sin u$ and $y = 1 - 2\sin^2 u$,

giving $y = 1 - 2 \cdot \frac{x^2}{2}$.

43. **(D)** Use the formula for area in polar coordinates,

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta;$$

then the required area is given by

$$4 \cdot \frac{1}{2}\int_{0}^{\pi/4} 2\cos 2\theta .$$

(See polar graph 63 in the Appendix.)

44. **(C)** $\int_{-\infty}^{\infty} \frac{dx}{x^2+1} = \lim_{b\to\infty} \tan^{-1} x \Big|_{-b}^{b} = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi.$

45. **(A)** The first three derivatives of $\frac{1}{1-2x}$ are $\frac{2}{(1-2x)^2}$, $\frac{8}{(1-2x)^3}$, and $\frac{48}{(1-2x)^4}$.

The first four terms of the Maclaurin series (about $x = 0$) are 1, $+2x$, $+\frac{8x^2}{2!}$, and $+\frac{48x^3}{3!}$.

Note also that $\frac{1}{1-2x}$ represents the sum of an infinite geometric series with first term 1 and common ratio $2x$. Hence,

$$\frac{1}{1-2x} = 1 + 2x + (2x)^2 + (2x)^3 + \cdots$$

46. **(D)** We use parts, first letting $u = x^2$, $dv = e^{-x}dx$; then $du = 2x\,dx$, $v = -e^{-x}$ and

$$\int x^2 e^{-x} dx = -x^2 e^{-x} + 2\int x e^{-x} dx$$

Now we use parts again, letting $u = x$, $dv = e^{-x}dx$; then $du = dx$, $v = -e^{-x}$ and

$$-x^2e^{-x} + 2\int xe^{-x}dx = -x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}dx\right)$$

Alternatively, we could use the Tic-Tac-Toe Method (See page 226):

u		dv
x^2	+	e^{-x}
$2x$	−	$-e^{-x}$
2	+	e^{-x}
0		$-e^{-x}$

Then $\int x^2 e^{-x} dx = x^2(-e^{-x}) - (2x)e^{-x} + 2(-e^{-x}) + C$

47. **(E)** Use formula (20) in the Appendix to rewrite the integral as

$$\frac{1}{2}\int_0^{\pi/2}(1-\cos 2x)\,dx = \frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)\Bigg|_0^{\pi/2} = \frac{1}{2}\left(\frac{\pi}{2}\right).$$

48. **(E)** The area, A, is represented by $\int_0^{2\pi}(1-\cos t) = 2\pi$.

49. **(D)** $\dfrac{dy}{dx} = \dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{2a\sin\theta\cos\theta}{-a\csc^2\theta} = -2\sin^3\theta\cos\theta.$

50. **(C)** Check to verify that each of the other improper integrals converges.

51. **(D)** Note that the integral is improper.

$$\lim_{k\to 4^-}\int_2^k \frac{du}{\sqrt{16-u^2}} = \lim_{k\to 4^-}\frac{1}{4}\int_2^k \frac{du}{\sqrt{1-\frac{u^2}{16}}} = \lim_{k\to 4^-}\frac{1}{4}\cdot 4\int_2^k \frac{\frac{1}{4}du}{\sqrt{1-\frac{u^2}{16}}} = \lim_{k\to 4^-}\sin^{-1}\frac{u}{4}\Big|_2^k = \frac{\pi}{3}$$

See Example 26, page 312.

52. **(C)** Let $y = \left(\frac{1}{x}\right)^x$. Then $\ln y = -x\ln x$ and

$$\lim_{x\to 0^+}\ln y = \lim_{x\to 0^+}\frac{-\ln x}{1/x}.$$

Now apply L'Hôpital's Rule:

$$\lim_{x\to 0^+}\ln y = \lim_{x\to 0^+}\frac{-1/x}{-1/x^2} = 0.$$

So, if $\lim_{x\to 0^+}\ln y = 0$, then $\lim_{x\to 0^+} y = 1$.

53. **(D)** The speed, $|\mathbf{v}|$, equals $\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$, and since $x = 3y - y^2$,

$$\frac{dx}{dt} = (3-2y)\,\frac{dy}{dt} = (3-2y)\cdot 3.$$

Then $|\mathbf{v}|$ is evaluated, using $y = 1$, and equals $\sqrt{(3)^2+(3)^2}$.

54. **(A)** This is an indeterminate form of type $\frac{\infty}{\infty}$; use L'Hôpital's Rule:

$$\lim_{x\to 0^+}\frac{\cot x}{\ln x} = \lim_{x\to 0^+}\frac{-\csc^2 x}{1/x} = \lim_{x\to 0^+}\frac{x}{\sin x}\cdot\frac{-1}{\sin x} = -\infty$$

55. **(E)** We find A and B such that $\dfrac{3x+2}{(x+3)(x-4)} = \dfrac{A}{x+3}+\dfrac{B}{x-4}$.

After multiplying by the common denominator, we have

$$3x+2 = A(x-4)+B(x+3).$$

Substituting $x = -3$ yields $A = 1$, and $x = 4$ yields $B = 2$; hence,

$$\frac{3x+2}{x^2-x-12} = \frac{1}{x+3}+\frac{2}{x-4}.$$

56. **(B)** Since $\cos x \simeq 1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}$, $\dfrac{1-\cos x}{x} \simeq \dfrac{x}{2}-\dfrac{x^3}{4!}$.

Then $\displaystyle\int_0^1 \frac{1-\cos x}{x}\,dx \simeq \left(\frac{x^2}{4}-\frac{x^4}{96}\right)\Bigg|_0^1 = \frac{1}{4}-\frac{1}{96}$.

Note that $\displaystyle\lim_{x\to0^+}\frac{1-\cos x}{x} = 0$, so the integral is proper.

57. **(D)** We represent the spiral as $P(\theta) = (\theta\cos\theta, \theta\sin\theta)$. So

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\theta\cos\theta+\sin\theta}{-\theta\sin\theta+\cos\theta} = \frac{\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}}{-\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}} = \frac{\pi/4+1}{-\pi/4+1}.$$

Part B

58. **(D)** Since h is increasing, $h' \ge 0$. The graph of h is concave downward for $x < 2$ and upward for $x > 2$, so h'' changes sign at $x = 2$, where it appears that $h' = 0$ also.

59. **(C)** I is false since, for example, $f'(-2) = f'(1) = 0$ but neither $g(-2)$ nor $g(1)$ equals zero.

II is true. Note that $f = 0$ where g has relative extrema, and f is positive, negative, then positive on intervals where g increases, decreases, then increases.

III is also true. Check the concavity of g: when the curve is concave down, $h < 0$; when up, $h > 0$.

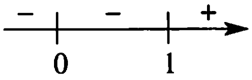
60. **(A)** If $y = \displaystyle\int_3^x \frac{1}{\sqrt{3+2t}}\,dt$, then $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{3+2x}}$, so $\dfrac{d^2y}{dx^2} = -\dfrac{1}{2}(3+2x)^{-3/2}(2)$.

61. **(D)** $\displaystyle\int_{-4}^{3} f(x+1)\,dx$ represents the area of the same region as $\displaystyle\int_{-3}^{4} f(x)\,dx$,

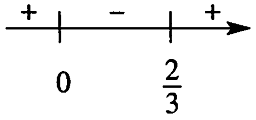
translated one unit to the left.

62. **(D)** According to the Mean Value Theorem, there exists a number c in the interval $[1,1.5]$ such that $f'(c)=\dfrac{f(1.5)-f(1)}{1.5-1}$. Use your calculator to solve the equation $\cos c=\dfrac{\sin 1.5-\sin 1}{0.5}$ for c (in radians).

63. **(E)** Here are the relevant sign lines:



signs of $f'(x)$



signs of $f''(x)$

We see that f' and f'' are both positive only if $x > 1$.

64. **(E)** Note from the sign lines in Question 63 that f changes from decreasing to increasing at $x = 1$, so f has a local minimum there.

Also, the graph of f changes from concave up to concave down at $x = 0$, then back to concave up at $x = \frac{2}{3}$; hence f has two points of inflection.

65. **(C)** The derivatives of $\ln(x + 1)$ are $\dfrac{1}{x+1}, \dfrac{-1}{(x+1)^2}, \dfrac{+2!}{(x+1)^3}, \dfrac{-(3!)}{(x+1)^4}, \ldots$

The nth derivative at $x = 2$ is $\dfrac{(-1)^{n-1}(n-1)!}{3^n}$.

66. **(C)** The absolute-value function $f(x) = |x|$ is continuous at $x = 0$, but $f'(0)$ does not exist.

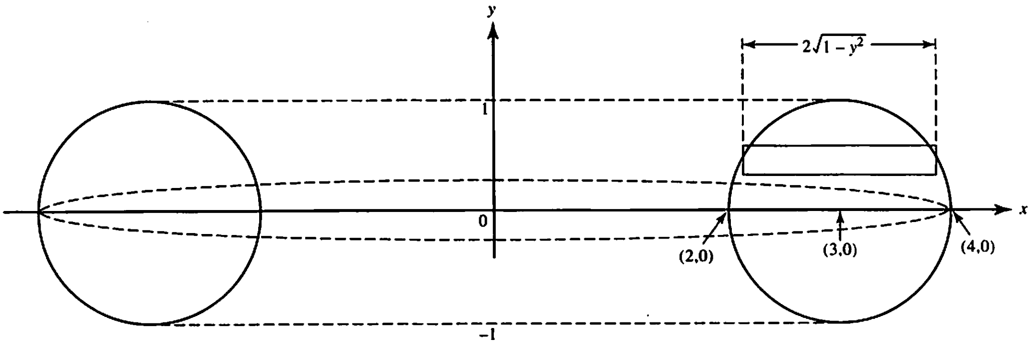
67. **(C)** Let $F'(x) = f(x)$; then $F'(x + k) = f(x + k)$;

$$\int_0^3 f(x+k)\,dx = F(3 + k) - F(k);$$

$$\int_k^{3+k} f(x)\,dx = F(3 + k) - F(k).$$

Or let $u = x + k$. Then $dx = du$; when $x = 0$, $u = k$; when $x = 3$, $u = 3 + k$.

68. **(E)** See the figure. The equation of the generating circle is $(x - 3)^2 + y^2 = 1$, which yields $x = 3\pm\sqrt{1-y^2}$.



About the y-axis: $\Delta V = 2\pi \cdot 3 \cdot 2\sqrt{1-y^2}\,\Delta y$.

$$\text{Thus, } V = 2\int_0^1 12\pi\sqrt{1-y^2}\,dy.$$

$$= 24\pi \text{ times the area of a quarter of a unit circle} = 6\pi^2.$$

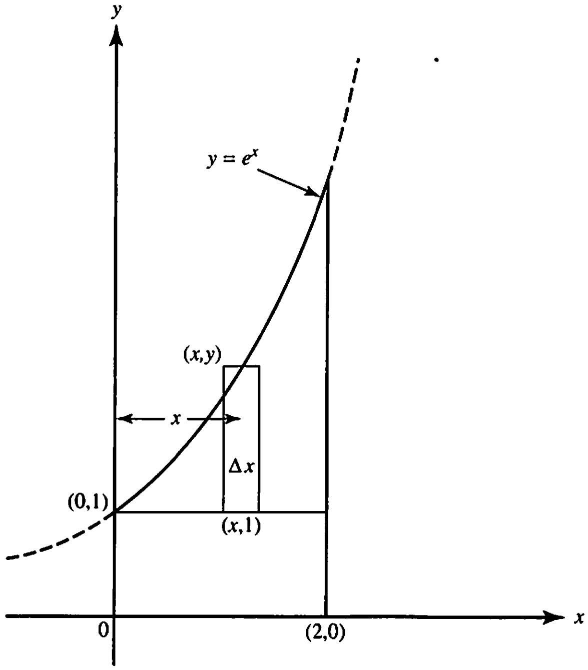
69. **(D)** Note that $f(g(u)) = \tan^{-1}(e^{2u})$; then the derivative is $\frac{1}{1+(e^{2u})^2}(2e^{2u})$.

70. **(D)** Let $y' = \frac{dy}{dx}$. Then $\cos(xy)[xy' + y] = y'$. Solve for y'.

71. **(E)** $\frac{d^2}{dx^2}f(x^2) = \frac{d}{dx}\left[\frac{d}{dx}f(x^2)\right] = \frac{d}{dx}\left[\frac{d}{dx}f(x^2)\cdot\frac{dx^2}{dx}\right] = \frac{d}{dx}\left[g(x^2)\cdot 2x\right]$

$$= g(x^2)\frac{d}{dx}(2x) + 2x\frac{d}{dx}g(x^2) = g(x^2)\cdot 2 + 2x\frac{d}{dx^2}g(x^2)\frac{dx^2}{dx}$$

$$= 2g(x^2) + 2x\cdot f\left(\sqrt{x^2}\right)\cdot 2x = 2g(x^2) + 4x^2\cdot f(x).$$



72. **(C)** About the x-axis; see the figure. Washer.

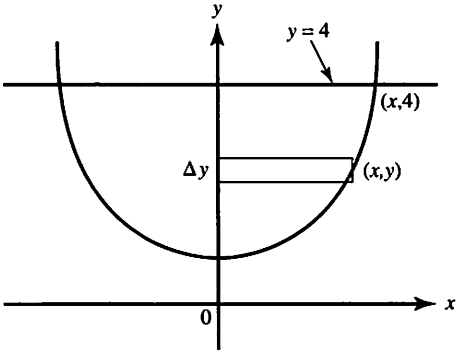
$$\Delta V = \pi\left(y^2 - 1^2\right)\Delta x,$$

$$V = \pi\int_0^2\left(e^{2x} - 1\right)dx.$$

73. **(C)** By the Mean Value Theorem, there is a number c in $[1, 2]$ such that

$$f'(c) = \frac{f(2)-f(1)}{2-1} = -3.$$

74. **(D)** The enclosed region, S, is bounded by $y = \sec x$, the y-axis, and $y = 4$. It is to be rotated about the y-axis.



Use disks; then $\Delta V = \pi R^2 H = \pi(\text{arc sec } y)^2 \, \Delta y$. Using the calculator, we find that

$$\pi\int_1^4 \left(\arccos\left(\frac{1}{y}\right)\right)^2 dx \approx 11.385.$$

75. **(C)** If Q is the amount at time t, then $Q = 40e^{-kt}$. Since $Q = 20$ when $t = 2$, $k = -0.3466$. Now find Q when $t = 3$, from $Q = 40e^{-(0.3466)3}$, getting $Q = 14$ to the nearest gram.

76. **(A)** The velocity $v(t)$ is an antiderivative of $a(t)$, where $a(t) = \pi t + \dfrac{2}{1+t^2}$. So

$v(t) = \dfrac{\pi t^2}{2} + 2 \arctan t + C$. Since $v(1) = 0$, $C = -\pi$.

$$\text{Required average velocity } = \frac{1}{2-0}\int_0^2 v(t)\,dt$$

$$= \frac{1}{2}\int_0^2 \left(\frac{\pi t^2}{2} + 2\arctan t - \pi\right) dt \approx 0.362.$$

77. **(D)** Graph $y = \tan x$ and $y = 2 - x$ in $[-1, 3] \times [-1, 3]$ as shown on page 485. Note that

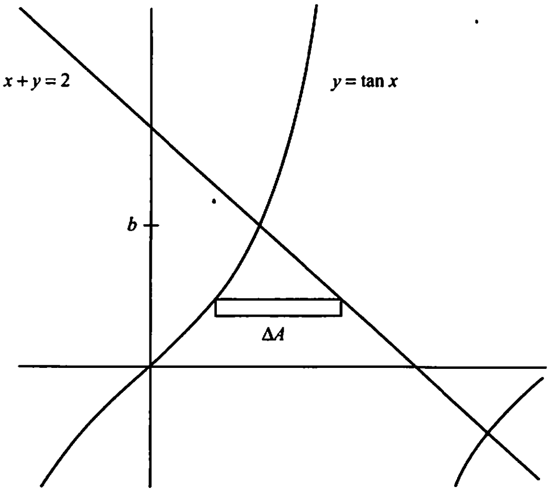
$$\Delta A = (x_{\text{line}} - x_{\text{curve}})\,\Delta y$$
$$= (2 - y - \arctan y)\,\Delta y.$$

The limits are $y = 0$ and $y = b$, where b is the ordinate of the intersection of the curve and the line. Using the calculator, solve

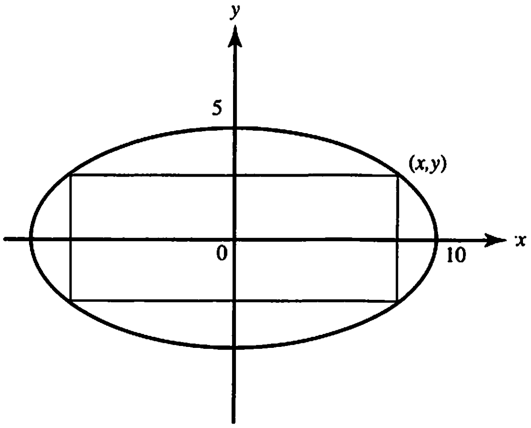
$$\arctan y = 2 - y$$

and store the answer in memory as B. Evaluate the desired area:

$$\int_0^B (2 - y - \arctan y)\,dy \approx 1.077.$$



78. **(E)** Center the ellipse at the origin and let (x, y) be the coordinates of the vertex of the inscribed rectangle in the first quadrant, as shown in the figure.



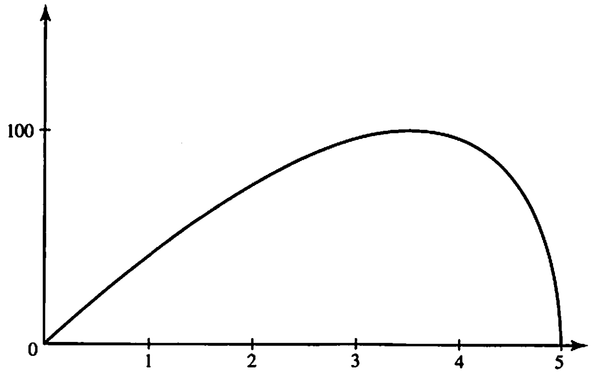
$$\frac{x^2}{100}+\frac{y^2}{25}=1$$

To maximize the rectangle's area $A = 4xy$, solve the equation of the ellipse, getting

$$x=\sqrt{100-4y^2} = 2\sqrt{25-y^2}.$$

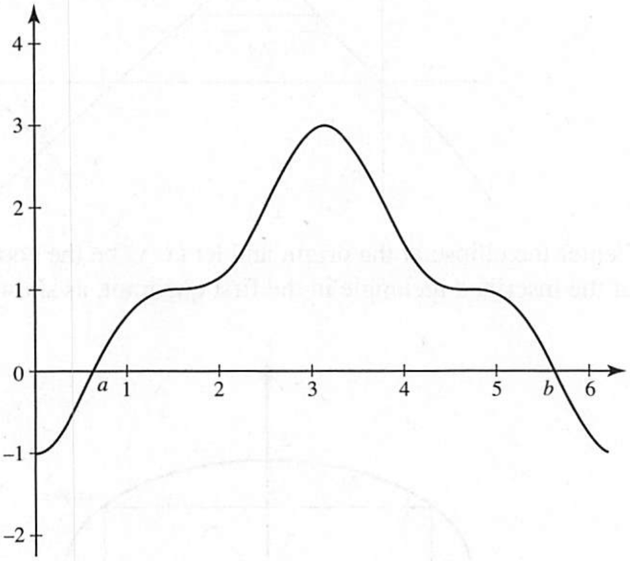
So $A = 8y\sqrt{25-y^2}$. Graph $y = 8x\sqrt{(25-x^2)}$ in the window $[0,5] \times [0,150]$.

The calculator shows that the maximum area (the y-coordinate) equals 100.



79. **(B)** $\dfrac{\int_1^e x \ln x \, dx}{e-1} \simeq 1.221.$

80. **(B)** When f' is positive, f increases. By the Fundamental Theorem of Calculus, $f'(x) = 1 - 2(\cos x)^3$. Graph f' in $[0, 2\pi] \times [-2, 4]$. It is clear that $f' > 0$ on the interval $a < x < b$. Using the calculator to solve $1 - 2(\cos x)^3 = 0$ yields $a = 0.654$ and $b = 5.629$.



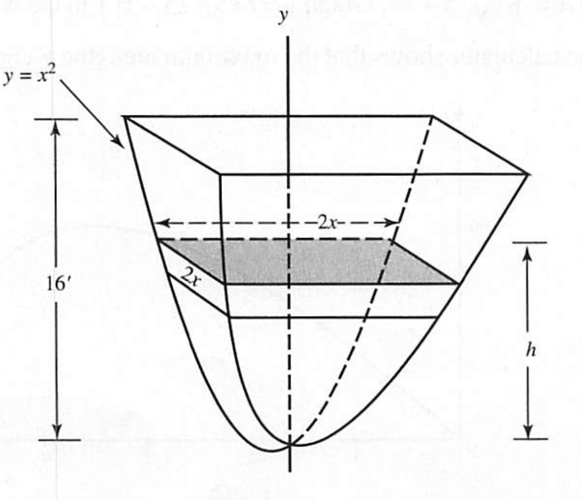
81. **(C)** $a(1.5) \simeq \dfrac{v(2)-v(1)}{2-1} = \dfrac{12-22}{1}.$

82. **(B)** The volume is composed of elements of the form $\Delta V = (2x)^2 \Delta y$. If h is the depth, in feet, then, after t hr,

$$V(h) = 4\int_0^h y\, dy \text{ and } \frac{dV}{dt} = 4h\frac{dh}{dt}.$$

$$\text{Thus, } 12 = 4(9)\frac{dh}{dt}$$

$$\text{and } \frac{dh}{dt} = \frac{1}{3} \text{ ft/hr.}$$



83. **(B)** Separating variables yields

$$\frac{dP}{1000-P} = k\,dt,$$

$$-\ln(1000 - P) = kt + C,$$

$$1000 - P = ce^{-kt}.$$

Then

$$P(t) = 1000 - ce^{-kt}.$$

$P(0) = 300$ gives $c = 700$. $P(5) = 500$ yields $500 = 1000 - 700e^{-5k}$, so $k \simeq +\,0.0673$. Now $P(10) = 1000 - 700e^{-0.673} \simeq 643$.

84. **(C)** $H(1) = \int_0^1 \frac{4}{x^2+1}\,dx = 4 \arctan 1 = \pi$. $H'(1) = f(1) = 2$.

The equation of the tangent line is $y - \pi = 2(x - 1)$.

85. **(C)** Using midpoint diameters to determine cylinders, estimate the volume to be

$$V \simeq \pi \cdot 8^2 \cdot 25 + \pi \cdot 6^2 \cdot 25 + \pi \cdot 4^2 \cdot 25 + \pi \cdot 3^2 \cdot 25.$$

86. **(A)** $\left(\frac{f}{g}\right)'(3) = \frac{g(3) \cdot f'(3) - f(3) \cdot g'(3)}{(g(3))^2} = \frac{2(2)-4(3)}{2^2}.$

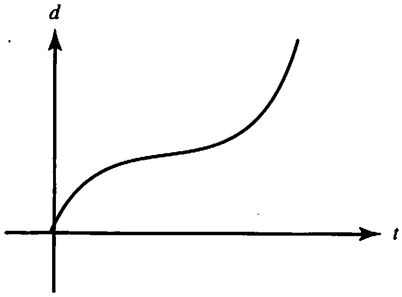
87. **(C)** $H'(3) = f'(g(3)) \cdot g'(3) = f'(2) \cdot g'(3).$

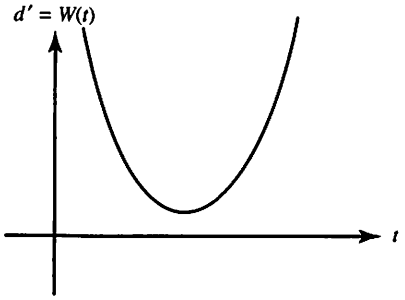
88. **(E)** $M'(3) = f(3) \cdot g'(3) + g(3) \cdot f'(3) = 4 \cdot 3 + 2 \cdot 2.$

89. **(E)** $K'(3) = \frac{1}{g'(K(3))} = \frac{1}{g'(g^{-1}(3))} = \frac{1}{g'(4)} = \frac{1}{\frac{1}{2}}.$

90. **(C)** $R'(3) = \frac{1}{2}(f(3))^{-1/2} \cdot f'(3).$

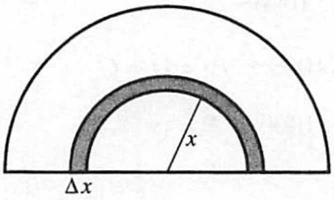
91. **(D)** Here are the pertinent curves, with d denoting the depth of the water:

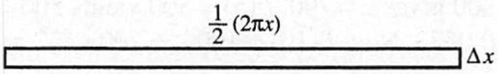




92. **(B)** Use areas; then $\int_1^7 f' = -3 + 10 = 7$. Thus, $f(7) - f(1) = 7$.

93. **(B)** The region x units from the stage can be approximated by the semicircular ring shown; its area is then the product of its circumference and its width.





The number of people standing in the region is the product of the area and the density:

$$\Delta P = (\pi x\ \Delta x)\left(\frac{20}{2\sqrt{x}+1}\right).$$

To find the total number of people, evaluate

$$20\pi\int_0^{100}\frac{x}{2\sqrt{x}+1}\,dx$$

94. **(B)** $\frac{dy}{dt}$ is positive, but decreasing; hence $\frac{dy^2}{dt^2}<0$.

95. **(C)** Average speed $= \frac{\text{total distance}}{\text{elapsed time}} = \frac{\text{total area}}{8} = \frac{8}{8}$.

96. **(E)** On $2 \leqslant t \leqslant 5$, the object moved $3\frac{1}{2}$ ft to the right; then on $5 \leqslant t \leqslant 8$, it moved only $2\frac{1}{2}$ ft to the left.

97. **(B)** Average acceleration $= \frac{\Delta v}{\Delta t} = \frac{v(8)-v(0)}{8-0} = \frac{-1-1}{8}$.

98. **(D)** Evaluate $\int_0^3 \frac{72}{2t+3}\,dt = 36 \ln\left(2t+3\right)\Big|_0^3 = 36 \ln 3$.

99. **(A)** $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ and $\frac{dy}{dt} = -\frac{3}{4}\frac{dx}{dt}$ at the point (3, 4).

Use, also, the facts that the speed is given by $|\mathbf{v}| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ and that the point moves counterclockwise; then $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 4$, yielding $\frac{dx}{dt} = -\frac{8}{5}$ and $\frac{dy}{dt} = +\frac{6}{5}$ at the given point. The velocity vector, **v**, at (3, 4) must therefore be $\left\langle -\frac{8}{5}, \frac{6}{5}\right\rangle$.

100. **(A)** $\mathbf{v} = \langle -ak \sin kt, ak \cos kt\rangle$, and

$\mathbf{a} = \langle -ak^2 \cos kt, ak^2 \sin kt\rangle = -k^2\mathbf{R}$.

101. **(B)** The formula for length of arc is

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx.$$

Since $y = 2^x$, we find

$$L = \int_0^2 \sqrt{1+\left(2^x \ln 2\right)^2}\, dx \approx 3.664.$$

102. **(D)** $\mathbf{a}(t) = (0, e^t)$; the acceleration is always upward.

103. **(A)** At (0, 1), $\frac{dy}{dx} = 4$, so Euler's method yields $(0.1, 1+0.1(4)) = (0.1, 1.4)$.

$\frac{dy}{dx} = 4y$ has particular solution $y = e^{4x}$; the error is $e^{4(0.1)} - 1.4$.

104. **(D)** $1 - \frac{1}{2} + \frac{1}{5} = 0.7$. Note that the series converges by the Alternating Series

Test. Since the first term dropped in the estimate is $-\frac{1}{10}$, the estimate is too high, but within 0.1 of the true sum.

105. **(C)** $\sum_{n=1}^{\infty} \frac{2}{3n} = \frac{2}{3}\sum_{n=1}^{\infty} \frac{1}{n}$, which equals a constant times the harmonic series.

106. **(D)** We seek x such that

$$\lim_{n\to\infty}\left|\frac{(n+1)! \bullet x^{n+1}}{(n+1)^{n+1}} \bullet \frac{n^n}{n! \bullet x^n}\right| < 1$$

or such that $|x| \bullet \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n < 1$

or such that $|x| < \dfrac{1}{\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n}$.

The fraction equals $\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.

Then $|x| < e$ and the radius of convergence is e.

107. **(B)** The error is less than the maximum value of $\frac{e^c}{3!}x^3$ for $0 \le x \le \frac{1}{2}$.

This maximum occurs at $c = x = \frac{1}{2}$.

108. **(D)** Distance = $\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt.$

$$= \int_0^1 \sqrt{\left(\sec^2 t\right)^2 + (-2\sin 2t)^2}\ dt.$$

Note that the curve is traced exactly once by the parametric equations from $t = 0$ to $t = 1$.