Answer Sheet

AB PRACTICE EXAMINATION 1

Part A

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Part B

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AB Practice Examination 1

SECTION I

Part A TIME: 55 MINUTES

The use of calculators is not *permitted for this part of the examination. There are 28 questions in Part A, for which 55 minutes are allowed. Because there is no deduction for wrong answers, you should answer every question, even if you need to guess.*

Directions: Choose the best answer for each question.

1. $\lim_{x\to\infty} \frac{20x^2-13x+5}{5-4x^3}$ is

(A) -5 **(B)** ∞ **(C)** 0 **(D)** 5 **(E)** 1

2. $\lim_{h\to 0} \frac{\ln(2+h)-\ln 2}{h}$ is

(A) 0 **(B)** $\ln 2$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{\ln 2}$ **(E)** ∞

3. If $y = e^{-x^2}$, then $y''(0)$ equals

(A) 2 **(B)** -2 **(C)** $\frac{2}{e}$ **(D)** 0 **(E)** -4

Questions 4 and 5. Use the following table, which shows the values of the differentiable functions *f* and *g*.

x	f	f'	g	g'
1	2	$\frac{1}{2}$	-3	5
2	3	1	0	4
3	4	2	2	3
4	6	4	3	$\frac{1}{2}$

4. The average rate of change of function *f* on [1,4] is

(A) 7/6 **(B)** 4/3 **(C)** 15/8 **(D)** 9/4 **(E)** 8/3

5. If $h(x) = g(f(x))$ then $h'(3) =$

(A) 1/2 **(B)** 1 **(C)** 4 **(D)** 6 **(E)** 9

6. The derivative of a function *f* is given for all *x* by

$$f'(x) = x^2(x+1)^3(x-4)^2.$$

The set of *x* values for which *f* is a relative maximum is

(A) $\{0, -1, 4\}$ **(B)** $\{-1\}$ **(C)** $\{0, 4\}$
(D) $\{1\}$ **(E)** none of these

7. If $y = \dfrac{x-3}{2-5x}$, then $\dfrac{dy}{dx}$ equals

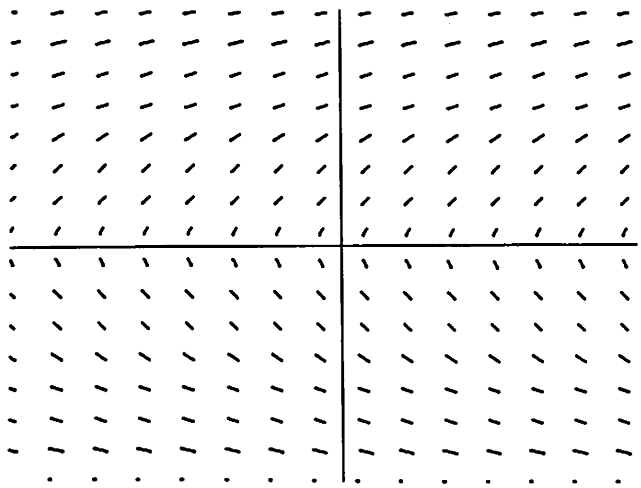
(A) $\dfrac{17-10x}{(2-5x)^2}$ **(B)** $\dfrac{13}{(2-5x)^2}$ **(C)** $\dfrac{x-3}{(2-5x)^2}$

(D) $\dfrac{17}{(2-5x)^2}$ **(E)** $\dfrac{-13}{(2-5x)^2}$

8. The maximum value of the function $f(x) = xe^{-x}$ is

(A) $\dfrac{1}{e}$ **(B)** e **(C)** 1 **(D)** -1 **(E)** none of these

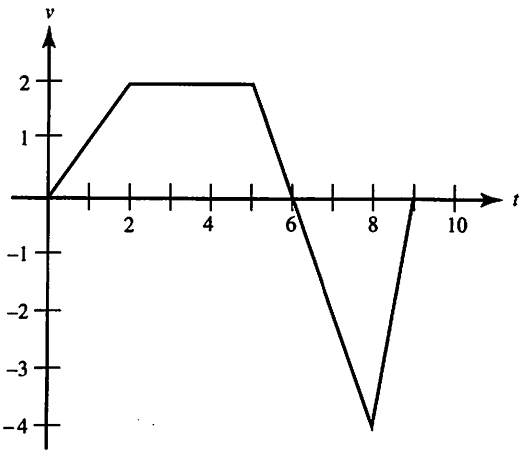
9. Which equation has the slope field shown below?



(A) $\dfrac{dy}{dx} = \dfrac{5}{y}$ **(B)** $\dfrac{dy}{dx} = \dfrac{5}{x}$ **(C)** $\dfrac{dy}{dx} = \dfrac{x}{y}$

(D) $\dfrac{dy}{dx} = 5y$ **(E)** $\dfrac{dy}{dx} = x + y$

Questions 10–12. The graph below shows the velocity of an object moving along a line, for $0 \leq t \leq 9$.



10. At what time does the object attain its maximum acceleration?

(A) $2<t<5$ **(B)** $5<t<8$ **(C)** $t=6$ **(D)** $t=8$ **(E)** $8<t<9$

11. The object is farthest from the starting point at $t =$

(A) 2 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 9

12. At $t = 8$, the object was at position $x = 10$. At $t = 5$, the object's position was $x =$

(A) −5 **(B)** 5 **(C)** 7 **(D)** 13 **(E)** 15

13. $\int_{\pi/4}^{\pi/2} \sin^3 \alpha \cos \alpha \, d\alpha$ is equal to

(A) $\frac{3}{16}$ **(B)** $\frac{1}{8}$ **(C)** $-\frac{1}{8}$ **(D)** $-\frac{3}{16}$ **(E)** $\frac{3}{4}$

14. $\int_0^1 \frac{e^x}{(3-e^x)^2} dx$ equals

(A) $3 \ln (e-3)$ **(B)** 1 **(C)** $\frac{1}{3-e}$ **(D)** $\frac{e-2}{3-e}$ **(E)** none of these

15. A differentiable function has the values shown in this table:

x	2.0	2.2	2.4	2.6	2.8	3.0
$f(x)$	1.39	1.73	2.10	2.48	2.88	3.30

Estimate $f'(2.1)$.

(A) 0.34 **(B)** 0.59 **(C)** 1.56 **(D)** 1.70 **(E)** 1.91

16. If $A = \int_0^1 e^{-x}dx$ is approximated using Riemann sums and the same number of subdivisions, and if L, R, and T denote, respectively left, right, and trapezoid sums, then it follows that

(A) $R \leqslant A \leqslant T \leqslant L$ **(B)** $R \leqslant T \leqslant A \leqslant L$ **(C)** $L \leqslant T \leqslant A \leqslant R$
(D) $L \leqslant A \leqslant T \leqslant R$ **(E)** None of these is true.

17. The number of vertical tangents to the graph of $y^2 = x - x^3$ is

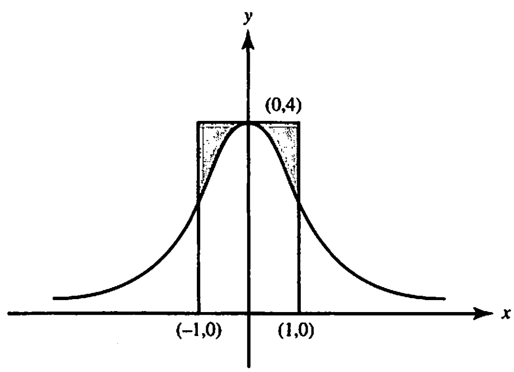
(A) 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** 0

18. $\int_0^6 f(x-1)dx =$

(A) $\int_{-1}^{7} f(x)dx$ **(B)** $\int_{-1}^{5} f(x)dx$ **(C)** $\int_{-1}^{5} f(x+1)dx$

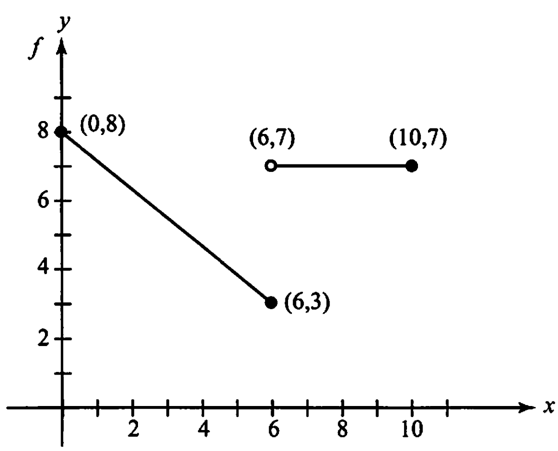
(D) $\int_{1}^{5} f(x)dx$ **(E)** $\int_{1}^{7} f(x)dx$

19. The equation of the curve shown below is $y = \dfrac{4}{1+x^2}$. What does the area of the shaded region equal?



(A) $4-\dfrac{\pi}{4}$ **(B)** $8-2\pi$ **(C)** $8-\pi$ **(D)** $8-\dfrac{\pi}{2}$ **(E)** $2\pi-4$

20. Over the interval $0 \leq x \leq 10$, the average value of the function f shown below



(A) is 6.00. **(B)** is 6.10. **(C)** is 6.25.
(D) does not exist, because f is not continuous.
(E) does not exist, because f is not integrable.

21. If $f'(x) = 2f(x)$ and $f(2) = 1$, then $f(x) =$

(A) e^{2x-4} **(B)** $e^{2x}+1-e^4$ **(C)** e^{4-2x} **(D)** e^{2x+1} **(E)** e^{x-2}

22. The table below shows values of $f''(x)$ for various values of x:

x	-1	0	1	2	3
$f''(x)$	-4	-1	2	5	8

The function f could be

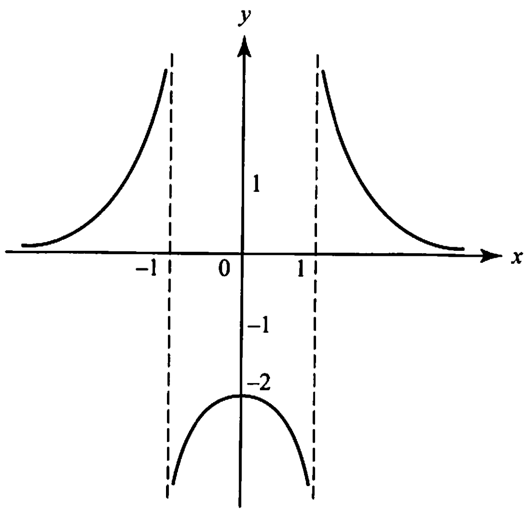
(A) a linear function **(B)** a quadratic function **(C)** a cubic function
(D) a fourth-degree function **(E)** an exponential function

23. The curve $x^3+x \tan y = 27$ passes through (3,0). Use the tangent line there to estimate the value of y at $x = 3.1$. The value is

(A) -2.7 **(B)** -0.9 **(C)** 0 **(D)** 0.1 **(E)** 3.0

24. At what value of h is the rate of increase of $\sqrt{h}$ twice the rate of increase of h?

(A) $\frac{1}{16}$ **(B)** $\frac{1}{4}$ **(C)** 1 **(D)** 2 **(E)** 4



25. The graph of a function $y = f(x)$ is shown above. Which is true?

(A) $\lim_{x\to 1} f(x) = -\infty$ **(B)** $\lim_{x\to -\infty} f(x) = \pm 1$ **(C)** $\lim_{x\to -2} f(x) = 0$

(D) $\lim_{x\to \infty} f(x) = 0$ **(E)** $\lim_{x\to 0} f(x) = \infty$

26. A function $f(x)$ equals $\dfrac{x^2 - x}{x-1}$ for all x except $x = 1$. For the function to be continuous at $x = 1$, the value of $f(1)$ must be

(A) 0 **(B)** 1 **(C)** 2 **(D)** ∞ **(E)** none of these

27. The number of inflection points on the graph of $f(x) = 3x^5 - 10x^3$ is

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** 0

28. Suppose $f(x) = \int_0^x \dfrac{4+t}{t^2+4}\,dt$. It follows that

(A) f increases for all x
(B) f increases only if $x < -4$
(C) f has a local min at $x = -4$
(D) f has a local max at $x = -4$
(E) f has no critical points

STOP

END OF PART A, SECTION I

Part B TIME: 50 MINUTES

Some questions in this part of the examination require the use of a graphing calculator. There are 17 questions in Part B, for which 50 minutes are allowed. Because there is no deduction for wrong answers, you should answer every question, even if you need to guess.

Directions: Choose the best answer for each question. If the exact numerical value of the correct answer is not listed as a choice, select the choice that is closest to the exact numerical answer.

29. Let $G(x) = [f(x)]^2$. At $x = a$, the graph of f is increasing and concave downward, while G is decreasing. Which describes the graph of G at $x = a$?

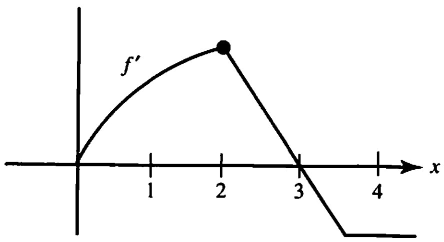
(A) concave downward **(B)** concave upward **(C)** linear
(D) point of inflection **(E)** none of these

30. The value of c for which $f(x) = x + \frac{c}{x}$ has a local minimum at $x = 3$ is

(A) -9 **(B)** -6 **(C)** -3 **(D)** 6 **(E)** 9

31. An object moving along a line has velocity $v(t) = t\cos t - \ln(t + 2)$, where $0 \leq t \leq 10$. The object achieves its maximum speed when t is approximately

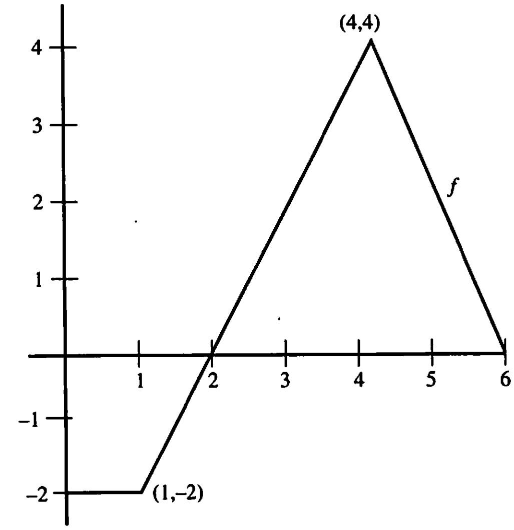
(A) 3.7 **(B)** 5.1 **(C)** 6.4 **(D)** 7.6 **(E)** 9.5



32. The graph of f', which consists of a quarter-circle and two line segments, is shown above. At $x = 2$ which of the following statements is true?

(A) f is not continuous.
(B) f is continuous but not differentiable.
(C) f has a relative maximum.
(D) The graph of f has a point of inflection.
(E) none of these

33. Let $H(x) = \int_0^x f(t)\,dt$, where f is the function whose graph appears below.



The local linearization of $H(x)$ near $x = 3$ is $H(x) \approx$

(A) $-2x + 8$ (B) $2x - 4$ (C) $-2x + 4$ (D) $2x - 8$ (E) $2x - 2$

34. The table shows the speed of an object, in feet per second, at various times during a 12-second interval.

time (sec)	0	3	6	7	8	10	12
speed (ft/sec)	15	14	11	8	7	3	0

Estimate the distance the object travels, using the midpoint method with 3 subintervals.

(A) 100 ft (B) 101 ft (C) 111 ft (D) 112 ft (E) 150 ft

35. In a marathon, when the winner crosses the finish line many runners are still on the course, some quite far behind. If the density of runners x miles from the finish line is given by $R(x) = 20[1 - \cos(1 + 0.03x^2)]$ runners per mile, how many are within 8 miles of the finish line?

(A) 30 (B) 145 (C) 157 (D) 166 (E) 195

36. Which best describes the behavior of the function $y = \arctan\left(\frac{1}{\ln x}\right)$ at $x = 1$?

(A) It has a jump discontinuity.
(B) It has an infinite discontinuity.
(C) It has a removable discontinuity.
(D) It is both continuous and differentiable.
(E) It is continuous but not differentiable.

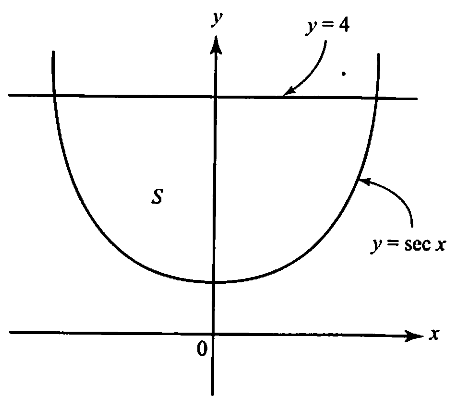
37. If $f(t) = \int_0^{t^2} \frac{1}{1+x^2}\,dx$, then $f'(t)$ equals

(A) $\frac{1}{1+t^2}$ **(B)** $\frac{2t}{1+t^2}$ **(C)** $\frac{1}{1+t^4}$ **(D)** $\frac{2t}{1+t^4}$ **(E)** $\tan^{-1} t^2$

38. $\int(\sqrt{x}-2)x^2dx$ =

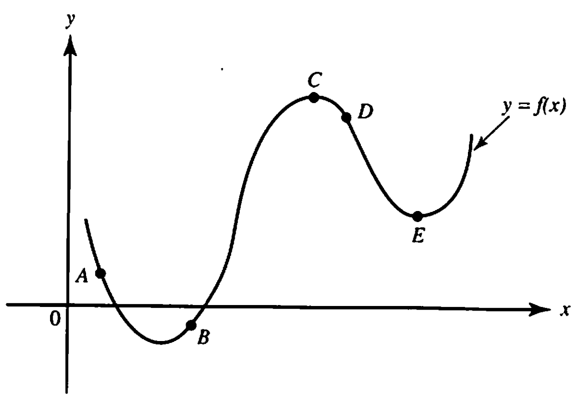
(A) $\frac{2}{3}x^{3/2} - 2x + C$ **(B)** $\frac{5}{2}x^{3/2} - 4x + C$ **(C)** $\frac{2}{3}x^{3/2} - 2x + \frac{x^3}{3} + C$

(D) $\frac{2}{5}x^{5/2} - \frac{2}{3}x^3 + C$ **(E)** $\frac{2}{7}x^{7/2} - \frac{2}{3}x^3 + C$



39. The region S in the figure shown above is bounded by $y = \sec x$ and $y = 4$. What is the volume of the solid formed when S is rotated about the x-axis?

(A) 0.304 **(B)** 39.867 **(C)** 53.126 **(D)** 54.088 **(E)** 108.177



40. At which point on the graph of $y = f(x)$ shown above is $f'(x) < 0$ and $f''(x) > 0$?

(A) A **(B)** B **(C)** C **(D)** D **(E)** E

41. Let $f(x) = x^5 + 1$, and let g be the inverse function of f. What is the value of $g'(0)$?

(A) -1 **(B)** $\frac{1}{5}$ **(C)** 1 **(D)** $g'(0)$ does not exist.

(E) $g'(0)$ cannot be determined from the given information.

42. The hypotenuse AB of a right triangle ABC is 5 feet, and one leg, AC, is decreasing at the rate of 2 feet per second. The rate, in square feet per second, at which the area is changing when $AC = 3$ is

(A) $\frac{25}{4}$ **(B)** $\frac{7}{4}$ **(C)** $-\frac{3}{2}$ **(D)** $-\frac{7}{4}$ **(E)** $-\frac{7}{2}$

43. At how many points on the interval $[0,\pi]$ does $f(x) = 2 \sin x + \sin 4x$ satisfy the Mean Value Theorem?

(A) none **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

44. If the radius r of a sphere is increasing at a constant rate, then the rate of increase of the volume of the sphere is

(A) constant
(B) increasing
(C) decreasing
(D) increasing for $r < 1$ and decreasing for $r > 1$
(E) decreasing for $r < 1$ and increasing for $r > 1$

45. The rate at which a purification process can remove contaminants from a tank of water is proportional to the amount of contaminant remaining. If 20% of the contaminant can be removed during the first minute of the process and 98% must be removed to make the water safe, approximately how long will the decontamination process take?

(A) 2 min **(B)** 5 min **(C)** 18 min **(D)** 20 min **(E)** 40 min

STOP

END OF SECTION I

SECTION II

Part A TIME: 30 MINUTES
2 PROBLEMS

A graphing calculator is required for some of these problems.
See instructions on page 4.

1. A function f is defined on the interval [0,4], and its derivative is $f'(x) = e^{\sin x} - 2\cos 3x$.

(a) Sketch f' in the window $[0,4] \times [-2,5]$.

(Note that the following questions refer to f.)

(b) On what interval is f increasing? Justify your answer.

(c) At what value(s) of x does f have local maxima? Justify your answer.

(d) How many points of inflection does the graph of f have? Justify your answer.

2. The rate of sales of a new software product is given by $S(t)$, where S is measured in hundreds of units per month and t is measured in months from the initial release date of January 1, 2012. The software company recorded these sales data:

t (months)	1	2	3	4	5	6	7
St (100s/mo)	1.54	1.88	2.32	3.12	3.78	4.90	6.12

(a) Using a trapezoidal approximation, estimate the number of units the company sold during the second quarter (April 1, 2012, through June 30, 2012).

(b) After looking at these sales figures, a manager suggests that the rate of sales can be modeled by assuming that the rate to be initially 120 units/month and to double every 3 months. Write an equation for S based on this model.

(c) Compare the model's prediction for total second quarter sales with your estimate from part a.

(d) Use the model to predict the average value of $S(t)$ for the entire first year. Explain what your answer means.

STOP

END OF PART A, SECTION II

Part B TIME: 60 MINUTES 4 PROBLEMS

No calculator is allowed for any of these problems.
If you finish Part B before time has expired, you may return to work on Part A, but you may not use a calculator.

3. The graph of function $y = f(x)$ passes through point (2, 5) and satisfies the differential equation $\dfrac{dy}{dx} = \dfrac{6x^2 - 4}{y}$.

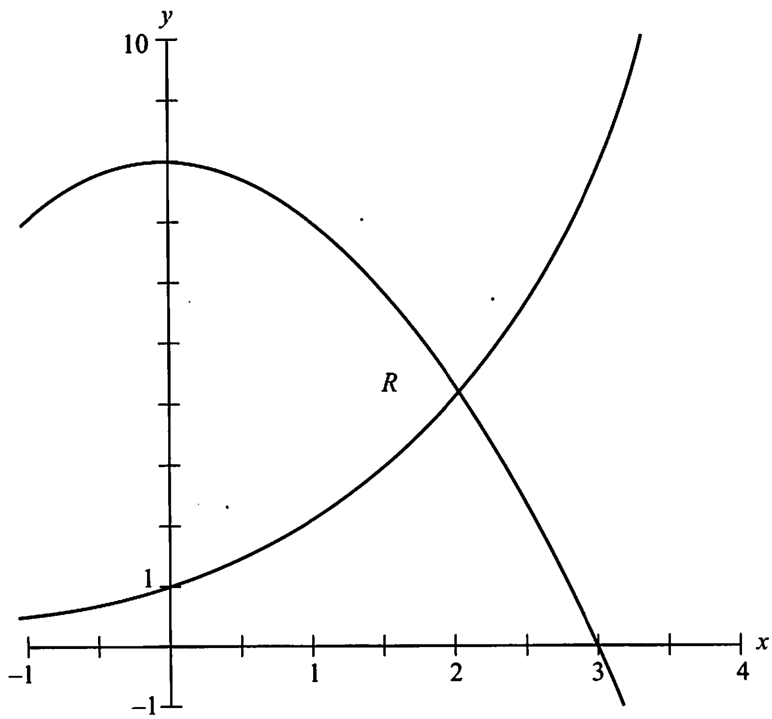
(a) Write an equation of the line tangent to f at (2, 5).

(b) Using this tangent line, estimate $f(2.1)$.

(c) Solve the differential equation, expressing f as a function of x.

(d) Using your answer to part (c), find $f(2.1)$.

4. Let R represent the first-quadrant region bounded by the y-axis and the curves $y = 2^x$ and $y = 8\cos\dfrac{\pi x}{6}$, as shown in the graph.



(a) Find the area of region R.

(b) Set up, but do not evaluate, integrals in terms of a single variable for:

(i) the volume of the solid formed when R is rotated around the x-axis,

(ii) the volume of the solid whose base is R, if all cross sections in planes perpendicular to the x-axis are squares.

5. Given the function $f(x) = e^{2x}(x^2 - 2)$:

(a) For what values of x is f decreasing?

(b) Does this decreasing arc reach a local or a global minimum? Justify your answer.

(c) Does f have a global maximum? Justify your answer.

6. (a) A spherical snowball melts so that its surface area shrinks at the constant rate of 10 square centimeters per minute. What is the rate of change of volume when the snowball is 12 centimeters in diameter?

(b) The snowball is packed most densely nearest the center. Suppose that, when it is 12 centimeters in diameter, its density x centimeters from the center is given by $d(x) = \frac{1}{1+\sqrt{x}}$ grams per cubic centimeter. Set up an integral for the total number of grams (mass) of the snowball then. Do not evaluate.

STOP

END OF TEST

Answer Key

AB PRACTICE EXAMINATION 1

Part A

1. C	**8.** A	**15.** D	**22.** C
2. C	**9.** A	**16.** A	**23.** B
3. B	**10.** E	**17.** B	**24.** A
4. B	**11.** C	**18.** B	**25.** D
5. B	**12.** D	**19.** B	**26.** B
6. E	**13.** A	**20.** B	**27.** B
7. E	**14.** E	**21.** A	**28.** C

Part B

29. B	**34.** D	**38.** E	**42.** D
30. E	**35.** D	**39.** E	**43.** E
31. E	**36.** A	**40.** A	**44.** B
32. D	**37.** D	**41.** B	**45.** C
33. D			

ANSWERS EXPLAINED

Multiple-Choice

Part A

1. **(C)** Use the Rational Function Theorem on page 96.

2. **(C)** Note that $\lim_{h\to 0}\frac{\ln(2+h)-\ln 2}{h}=f'(2)$, where $f(x)=\ln x$.

3. **(B)** Since $y'=-2xe^{-x^2}$, therefore $y''=-2(x\cdot e^{-x^2}\cdot(-2x)+e^{-x^2})$. Replace x by 0.

4. **(B)** $\frac{f(4)-f(1)}{4-1}=\frac{6-2}{4-1}=\frac{4}{3}$.

5. **(B)** $h'(3)=g'(f(3))\cdot f'(3)=g'(4)\cdot f'(3)=\frac{1}{2}\cdot 2$.

6. **(E)** Since $f'(x)$ exists for all x, it must equal 0 for any x_0 for which f is a relative maximum, and it must also change sign from positive to negative as x increases through x_0. For the given derivative, no x satisfies both of these conditions.

7. **(E)** By the Quotient Rule (formula (6) on page 113),

$$\frac{dy}{dx}=\frac{(2-5x)(1)-(x-3)(-5)}{(2-5x)^2}.$$

8. **(A)** Here, $f'(x)$ is $e^{-x}(1-x)$; f has maximum value when $x=1$.

9. **(A)** Note that (1) on a horizontal line the slope segments are all parallel, so the slopes there are all the same and $\frac{dy}{dx}$ must depend only on y; (2) along the x-axis (where $y=0$) the slopes are infinite; and (3) as y increases, the slope decreases.

10. **(E)** Acceleration is the derivative (the slope) of velocity v; v is largest on $8<t<9$.

11. **(C)** Velocity v is the derivative of position; because $v>0$ until $t=6$ and $v<0$ thereafter, the position increases until $t=6$ and then decreases; since the area bounded by the curve above the axis is larger than the area below the axis, the object is farthest from its starting point at $t=6$.

12. **(D)** From $t=5$ to $t=8$, the displacement (the integral of velocity) can be found by evaluating definite integrals based on the areas of two triangles: $\frac{1}{2}(1)(2)-\frac{1}{2}(2)(4)=-3$. Thus, if K is the object's position at $t=5$, then $K-3=10$ at $t=8$.

13. **(A)** The integral is of the form $\int u^3du$; evaluate $\left.\frac{1}{4}\sin^4\alpha\right|_{\pi/4}^{\pi/2}$.

14. (E) $-\int_0^1 (3-e^x)^{-2}(-e^x dx) = \left.\frac{1}{3-e^x}\right|_0^1 = \frac{e-1}{2(3-e)}$.

15. (D) $f'(2.1) \approx \frac{f(2.2)-f(2.0)}{2.2-2.0}$.

16. (A) $f(x) = e^{-x}$ is decreasing and concave upward.

17. (B) Implicit differentiation yields $2yy' = 1$; so $\frac{dy}{dx} = \frac{1-3x^2}{2y}$. At a vertical tangent, $\frac{dy}{dx}$ is undefined; y must therefore equal 0 and the numerator be non-zero. The original equation with $y = 0$ is $0 = x - x^3$, which has three solutions.

18. (B) Let $t = x - 1$; then $t = -1$ when $x = 0$, $t = 5$ when $x = 6$, and $dt = dx$.

19. (B) The required area, A, is given by the integral

$$2\int_0^1 \left(4 - \frac{4}{1+x^2}\right)dx = 2(4x - 4\tan^{-1} x)\Big|_0^1 = 2\left(4 - 4\cdot\frac{\pi}{4}\right).$$

20. (B) The average value is $\frac{1}{10-0}\int_0^{10} f(x)dx$. The definite integral represents the sum of the areas of a trapezoid and a rectangle: $\frac{1}{2}(8+3)(6) = 4(7) = 61$.

21. (A) Solve the differential equation $\frac{dy}{dx} = 2y$ by separation of variables: $\frac{dy}{y} = 2dx$ yields $y = ce^{2x}$. The initial condition yields $1 = ce^{2\cdot 2}$; so $c = e^{-4}$ and $y = e^{2x-4}$.

22. (C) Changes in values of f'' show that f''' is constant; hence f'' is linear, f' is quadratic, and f must be cubic.

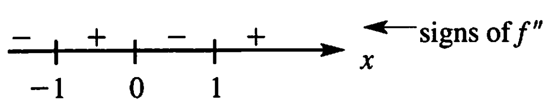
23. (B) By implicit differentiation, $3x^2 + x\sec^2 y\frac{dy}{dx} + \tan y = 0$. At (3,0), $\frac{dy}{dx} = -9$; so the equation of the tangent line at (3,0) is $y = -9(x-3)$.

24. (A) $(h^{1/2})' = 2h'$ implies $\frac{1}{2}h^{-1/2} = 2$.

25. (D) The graph shown has the x-axis as a horizontal asymptote.

26. (B) Since $\lim_{x\to 1} f(x) = 1$, to render $f(x)$ continuous at $x = 1$ $f(1)$ must be defined to be 1.

27. (B) $f'(x) = 15x^4 - 30x^2$; $f''(x) = 60x^3 - 60x = 60x(x+1)(x-1)$; this equals 0 when $x = -1, 0$, or 1. Here are the signs within the intervals:



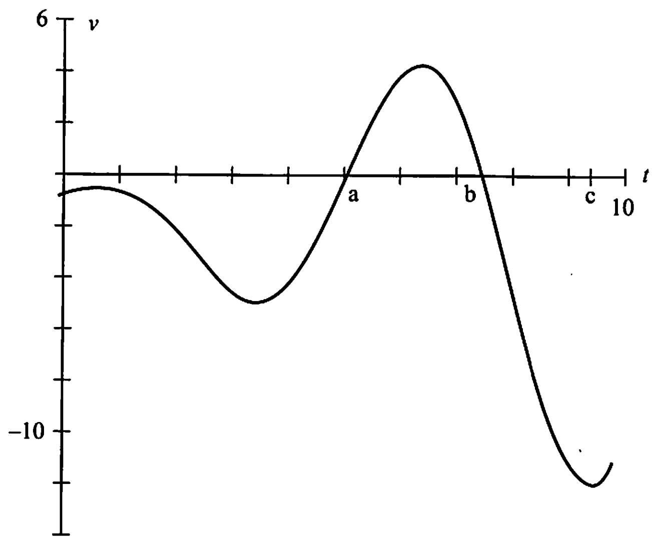
The graph of f changes concavity at $x = -1$, 0, and 1.

28. (C) Note that $f'(x) = \frac{4+x}{x^2+4}$, so f has a critical value at $x = -4$. As x passes through -4, the sign of f' changes from $-$ to $+$, so f has a local minimum at $x = -4$.

Part B

29. (B) We are given that (1) $f'(a) > 0$; (2) $f''(a) < 0$; and (3) $G'(a) < 0$. Since $G'(x) = 2f(x) \cdot f'(x)$, therefore $G'(a) = 2f(a) \cdot f'(a)$. Conditions (1) and (3) imply that (4) $f(a) < 0$. Since $G''(x) = 2[f(x) \cdot f''(x) + (f'(x))^2]$, therefore $G''(a) = 2[f(a)f''(a) + (f'(a))^2]$. Then the sign of $G''(a)$ is $2[(-) \cdot (-) + (+)]$ or positive, where the minus signs in the parentheses follow from conditions (4) and (2).

30. (E) Since $f'(x) = 1 - \frac{c}{x^2}$, it equals 0 for $x = \pm\sqrt{c}$. When $x = 3$, $c = 9$; this yields a minimum since $f''(3) > 0$.



31. (E) Use your calculator to graph velocity against time. Speed is the absolute value of velocity. The greatest deviation from $v = 0$ is at $t = c$. With a calculator, $c = 9.538$.

32. (D) Because f' changes from increasing to decreasing, f'' changes from positive to negative and thus the graph of f changes concavity.

33. (D) $H(3) = \int_0^3 f(t)dt$. We evaluate this definite integral by finding the area of a trapezoid (negative) and a triangle: $H(3) = -\frac{1}{2}(2 + 1)(2) + \frac{1}{2}(1)(2) = -2$, so the tangent line passes through the point (3,–2). The slope of the line is $H'(3) = f(3) = 2$, so an equation of the line is $y - (-2) = 2(x - 3)$.

34. **(D)** The distance is approximately 14(6) + 8(2) + 3(4).

35. **(D)** $\int_0^8 R(x)\,dx = 166.396.$

36. **(A)** Selecting an answer for this question from your calculator graph is unwise. In some windows the graph may appear continuous; in others there may seem to be cusps, or a vertical asymptote. Put the calculator aside. Find

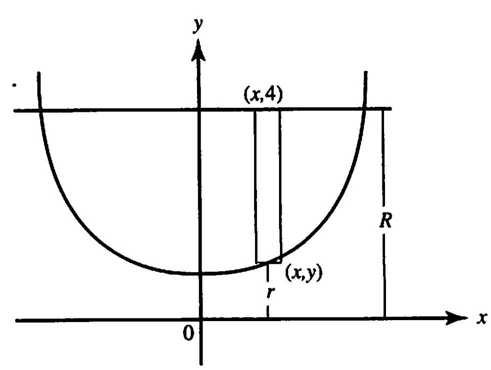
$$\lim_{x\to 1^+}\left(\arctan\left(\frac{1}{\ln x}\right)\right) = \frac{\pi}{2} \text{ and } \lim_{x\to 1^-}\left(\arctan\left(\frac{1}{\ln x}\right)\right) = -\frac{\pi}{2}.$$

These limits indicate the presence of a jump discontinuity in the function at $x = 1$.

37. **(D)** $\frac{d}{du}\int_0^u \frac{1}{1+x^2}\,dx = \frac{1}{1+u^2}$. When $u = t^2$,

$$\frac{d}{dt}\int_0^u \frac{1}{1+x^2}\,dx = \frac{1}{1+u^2}\frac{du}{dt} = \frac{1}{1+t^4}(2t).$$

38. **(E)** $\int(\sqrt{x}-2)x^2\,dx = \int(x^{5/2}-2x^2)\,dx = \frac{2}{7}x^{7/2} - \frac{2}{3}x^3 + C.$



39. **(E)** In the figure above, S is the region bounded by $y = \sec x$, the y axis, and $y = 4$. Send region S about the x-axis. Use washers; then $\Delta V = \pi(R^2 - r^2)\,\Delta x$. Symmetry allows you to double the volume generated by the first quadrant of S, so V is

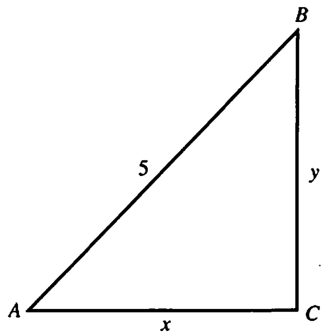
$$2\pi\int_0^{\arccos\frac{1}{4}}(16 - \sec^2 x)\,dx.$$

A calculator yields 108.177.

40. **(A)** The curve falls when $f'(x) < 0$ and is concave up when $f''(x) > 0$.

41. **(B)** $g'(y) = \frac{1}{f'(x)} = \frac{1}{5x^4}$. To find $g'(0)$, find x such that $f(x) = 0$. By inspection,

$x = -1$, so $g'(0) = \frac{1}{5(-1)^4} = \frac{1}{5}.$



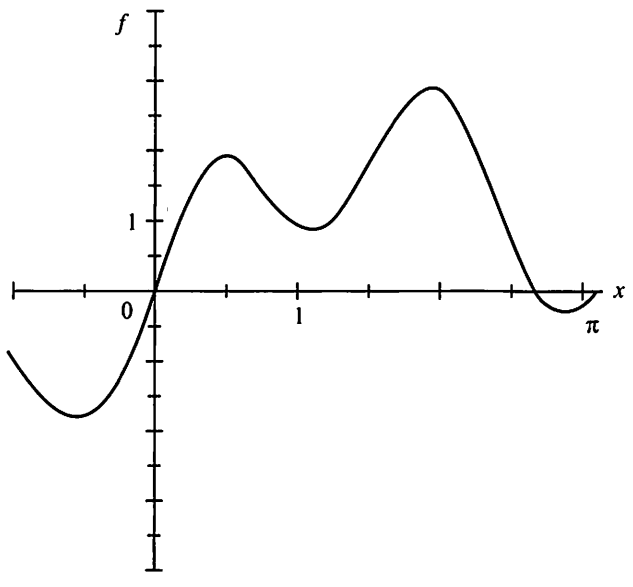
42. **(D)** It is given that $\frac{dx}{dt} = -2$; you want $\frac{dA}{dt}$, where $A = \frac{1}{2}xy$.

$$\frac{dA}{dt} = \frac{1}{2}\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right) = \frac{1}{2}\left[3 \cdot \frac{dy}{dt} + y \cdot (-2)\right].$$

Since $y^2 = 25 - x^2$, it follows that $2y\frac{dy}{dt} = -2x\frac{dx}{dt}$ and, when $x = 3$, $y = 4$

and $\frac{dy}{dt} = \frac{3}{2}$.

Then $\frac{dA}{dt} = -\frac{7}{4}$.



The function $f(x) = 2\sin x + \sin 4x$ is graphed above.

43. **(E)** Since $f(0) = f(\pi)$ and f is both continuous and differentiable, Rolle's Theorem predicts at least one c in the interval such that $f'(c) = 0$.

There are four points in $[0,\pi]$ of the calculator graph above where the tangent is horizontal.

44. **(B)** Since $\frac{dr}{dt} = k$, a positive constant, $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi r^2 k = cr^2$, where c is a positive constant. Then $\frac{d^2V}{dt^2} = 2cr\frac{dr}{dt} = 2crk$, which is also positive.

45. **(C)** If $Q(t)$ is the amount of contaminant in the tank at time t and Q_0 is the initial amount, then

$$\frac{dQ}{dt} = kQ \text{ and } Q(t) = Q_0e^{kt}.$$

Since $Q(1) = 0.8Q_0$, $0.8Q_0 = Q_0e^{k\cdot 1}$, $0.8 = e^k$, and

$$Q(t) = Q_0(0.8)^t.$$

We seek t when $Q(t) = 0.02Q_0$. Thus,

$$0.02Q_0 = Q_0(0.8)^t$$

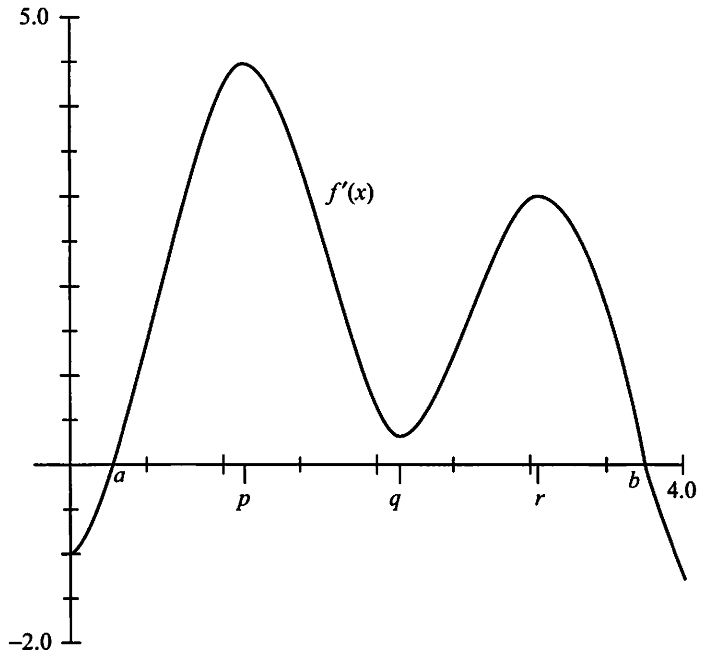
and

$$t \simeq 17.53 \text{ min.}$$

Free-Response

Part A

AB/BC 1. (a) This is the graph of $f'(x)$.



(b) f is increasing when $f'(x) > 0$. The graph shows this to be true in the interval $a < x < b$. Use the calculator to find a and b (where $e^x - 2\cos 3x = 0$); then $a = 0.283 < x < 3.760 = b$.

(c) See signs analysis.

f	0	dec	a	inc	b	dec	4
f'		$-$		$+$		$-$	

Since f decreases to the right of endpoint $x = 0$, f has a local maximum at $x = 0$. There is another local maximum at $x = 3.760$, because f changes from increasing to decreasing there.

(d) See signs analysis.

	0		p		q		r		4
f'		inc		dec		inc		dec	
f''		$+$		$-$		$+$		$-$	
f		conc up		conc down		conc up		conc down	

Since the graph of f changes concavity at p, q, and r, there are three points of inflection.

AB2. (a) Since April 1 is 3 months from January 1 and June 30 is 3 months later, we form the sum for the interval [3,6]:

$$\left(\frac{2.32+3.12}{2}\right)\cdot 1+\left(\frac{3.12+3.78}{2}\right)\cdot 1+\left(\frac{3.78+4.90}{2}\right)\cdot 1 = 10.51$$

We estimate the company sold 1051 software units during the second quarter.

(b) $S(t) = 1.2(2)^{t/3}$

(c) $\int_3^6 1.2(2^{t/3})\,dt = 10.387$. The model's estimate of 1039 sales is slightly lower, but the two are in close agreement.

(d) $\frac{1}{12}\int_0^{12} 1.2(2^{t/3})\,dt = 6.492$; the model predicts an average sales rate of 649.2 units per month from January 1, 2012, through December 31, 2012.

Part B

AB 3. (a) At (2,5), $\frac{dy}{dx} = \frac{6(2^2) - 4}{5} = 4$, so the tangent line is $y - 5 = 4(x - 2)$.

Solving for y yields $f(x) \approx 5 + 4(x - 2)$.

(b) $f(2.1) \approx 5 + 4(2.1 - 2) = 5.4$.

(c) The differential equation $\frac{dy}{dx} = \frac{6x^2 - 4}{5}$ is separable:

$$\int y\,dy = \int (6x^2 - 4)\,dx,$$

$$\frac{y^2}{2} = 2x^3 - 4x + C,$$

$$y = \pm\sqrt{4x^2 - 8x + c}\text{, where } c = 2C.$$

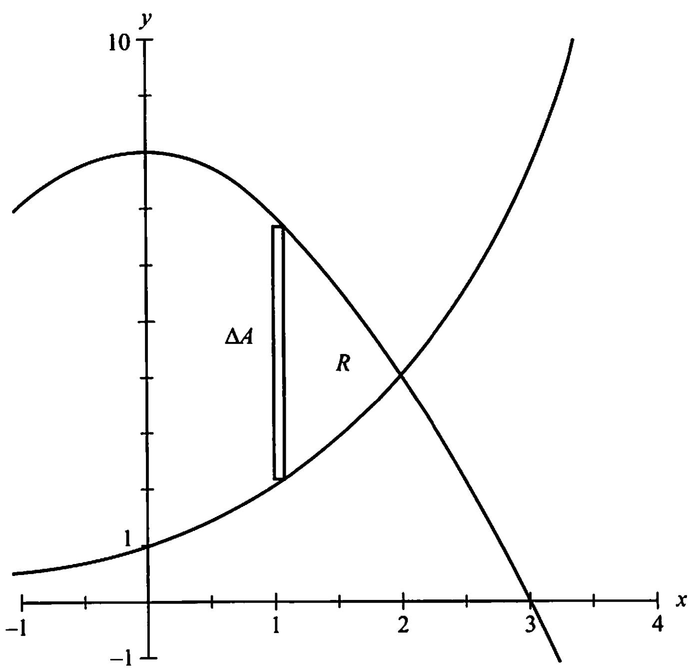
Since f passes through (2,5), it must be true that $5 = \pm\sqrt{4(2^3) - 8(2) + c}$.

Thus $c = 9$, and the positive root is used.

The solution is $f(x) = \sqrt{4x^3 - 8x + 9}$.

(d) $f(2.1) = \sqrt{4(2.1^3) - 8(2.1) + 9} = 5.408$.

AB 4. (a) Draw a vertical element of area, as shown.



$$\Delta A = (y_{\text{top}} - y_{\text{bottom}})\,\Delta x = \left(8\cos\frac{\pi x}{6} - 2^x\right)\Delta x,$$

$$A = \int_0^2 \left(8\cos\frac{\pi x}{6} - 2^x\right) dx$$

$$= \frac{6}{\pi}\cdot 8\int_0^2 \cos\frac{\pi x}{6}\,dx - \int_0^2 2^x\,dx$$

$$= \frac{48}{\pi}\cdot \sin\frac{\pi x}{6}\Bigg|_0^2 - \frac{2^x}{\ln 2}\Bigg|_0^2$$

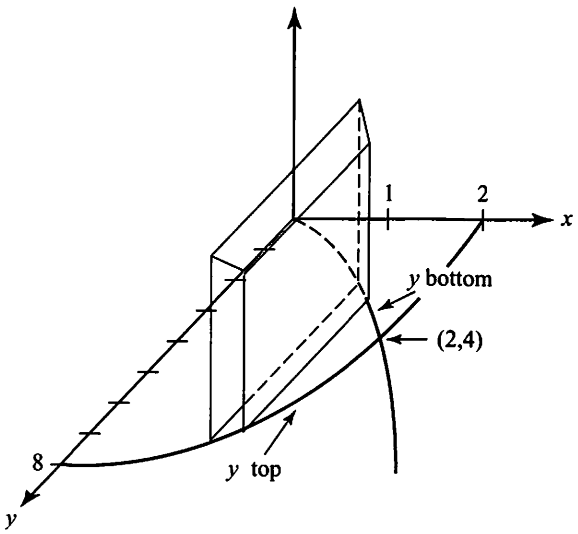
$$= \frac{48}{\pi}\left(\sin\frac{\pi}{3} - \sin 0\right) - \left(\frac{2^2}{\ln 2} - \frac{2^0}{\ln 2}\right)$$

$$= \frac{24\sqrt{3}}{\pi} - \frac{3}{\ln 2}.$$

(b) (i) Use washers; then

$$\Delta V = (r_2^2 - r_1^2)\,\Delta x = \pi(y^2_{\text{top}} - y^2_{\text{bottom}})\,\Delta x,$$

$$V = \pi\int_0^2 \left[\left(8\cos\frac{\pi x}{6}\right)^2 - (2^x)^2\right] dx.$$



(ii) See the figure above.

$$\Delta V = s^2\Delta x = (y_{\text{top}} - y_{\text{bottom}})^2\,\Delta x,$$

$$V = \int_0^2 \left(8\cos\frac{\pi x}{6} - 2^x\right)^2 dx.$$

AB/BC 5. (a) $f(x) = e^{2x}(x^2 - 2),$

$$f'(x) = e^{2x}(2x) + 2e^{2x}(x^2 - 2)$$
$$= 2e^{2x}(x + 2)(x - 1)$$
$$= 0 \text{ at } x = -2, 1.$$

f is decreasing where $f'(x) < 0$, which occurs for $-2 < x < 1$.

(b) f is decreasing on the interval $-2 < x < 1$, so there is a minimum at $(1, -e^2)$. Note that, as x approaches $\pm\infty$, $f(x) = e^{2x}(x^2 - 2)$ is always positive. Hence $(1, -e^2)$ is the global minimum.

(c) As x approaches $+\infty$, $f(x) = e^{2x}(x^2 - 2)$ also approaches $+\infty$. There is no global maximum.

AB/BC 6. (a) $S = 4\pi r^2$, so $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$. Substitute given values; then

$$-10 = 8\pi(6)\frac{dr}{dt}, \text{ so } \frac{dr}{dt} = -\frac{5}{24\pi}\text{cm/min.}$$

Since $V = \frac{4}{3}\pi r^3$, therefore $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. Substituting known values

gives $\frac{dV}{dt} = 4\pi(6^2) \bullet \frac{-5}{24\pi} = -30 \text{ cm}^3\text{/min.}$

(b) Regions of consistent density are concentric spherical shells. The volume of each shell is approximated by its surface area $(4\pi x^2)$ times its thickness (Δx). The weight of each shell is its density times its volume (g/cm^3·cm^3). If, when the snowball is 12 cm in diameter, ΔG is the weight of a spherical shell x cm from the center, then $\Delta G = \frac{1}{1 + \sqrt{x}} \bullet 4\pi x^2 \Delta x$, and the integral to find the weight of the snowball is

$$G = \int_0^6 \frac{1}{1 + \sqrt{x}} \bullet 4\pi x^2 dx.$$