Answer Sheet

AB PRACTICE EXAMINATION 2

Part A

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Part B

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AB Practice Examination 2

SECTION I

Part A TIME: 55 MINUTES

The use of calculators is not permitted for this part of the examination. There are 28 questions in Part A, for which 55 minutes are allowed. Because there is no deduction for wrong answers, you should answer every question, even if you need to guess.

Directions: Choose the best answer for each question.

1. $\lim_{x \to 2} \frac{x^2 - 2}{4 - x^2}$ is

(A) -2 **(B)** -1 **(C)** $\frac{1}{2}$ **(D)** 0 **(E)** nonexistent

2. $\lim_{x \to \infty} \frac{\sqrt{x} - 4}{4 - 3\sqrt{x}}$ is

(A) $-\frac{1}{3}$ **(B)** -1 **(C)** ∞ **(D)** 0 **(E)** $\frac{1}{3}$

3. If $y = \frac{e^{\ln u}}{u}$, then $\frac{dy}{du}$ equals

(A) $\frac{e^{\ln u}}{u^2}$ **(B)** $e^{\ln u}$ **(C)** $\frac{2e^{\ln u}}{u^2}$ **(D)** 1 **(E)** 0

4. Using the line tangent to $f(x) = \sqrt{9 + \sin(2x)}$ at $x = 0$, an estimate of $f(0.06)$ is

(A) 0.02 **(B)** 2.98 **(C)** 3.01 **(D)** 3.02 **(E)** 3.03

5. Air is escaping from a balloon at a rate of $R(t) = \frac{60}{1 + t^2}$ cubic feet per minute, where t is measured in minutes. How much air, in cubic feet, escapes during the first minute?

(A) 15 **(B)** 15π **(C)** 30 **(D)** 30π **(E)** $30 \ln 2$

6. If $y = \sin^3(1 - 2x)$, then $\dfrac{dy}{dx}$ is

(A) $3\sin^2(1 - 2x)$ (B) $-2\cos^3(1 - 2x)$ (C) $-6\sin^2(1 - 2x)$
(D) $-6\sin^2(1 - 2x)\cos(1 - 2x)$ (E) $-6\cos^2(1 - 2x)$

7. If $y = x^2 e^{1/x}$ $(x \neq 0)$, then $\dfrac{dy}{dx}$ is

(A) $xe^{1/x}(x + 2)$ (B) $e^{1/x}(2x - 1)$ (C) $-\dfrac{2e^{1/x}}{x}$

(D) $e^{-x}(2x - x^2)$ (E) none of these

8. A point moves along the curve $y = x^2 + 1$ so that the x-coordinate is increasing at the constant rate of $\dfrac{3}{2}$ units per second. The rate, in units per second, at which the distance from the origin is changing when the point has coordinates (1, 2) is equal to

(A) $\dfrac{7\sqrt{5}}{10}$ (B) $\dfrac{3\sqrt{5}}{2}$ (C) $3\sqrt{5}$ (D) $\dfrac{15}{2}$ (E) $\sqrt{5}$

9. $\lim\limits_{h \to 0} \dfrac{\sqrt{25+h}-5}{h}$

(A) $= 0$ (B) $= \dfrac{1}{10}$ (C) $= 1$ (D) $= 10$ (E) does not exist

10. The base of a solid is the first-quadrant region bounded by $y = \sqrt[4]{1-x^2}$. Each cross section perpendicular to the x-axis is a square with one edge in the xy-plane. The volume of the solid is

(A) $\dfrac{2}{3}$ (B) $\dfrac{\pi}{4}$ (C) 1 (D) $\dfrac{\pi}{2}$ (E) π

11. $\displaystyle\int \frac{x\,dx}{\sqrt{9-x^2}}$ equals

(A) $-\dfrac{1}{2}\ln\sqrt{9-x^2}+C$ (B) $\sin^{-1}\dfrac{x}{3}+C$ (C) $-\sqrt{9-x^2}+C$

(D) $-\dfrac{1}{4}\sqrt{9-x^2}+C$ (E) $2\sqrt{9-x^2}+C$

AB Practice Examination 2

12. $\int \frac{(y-1)^2}{2y}\,dy$ equals

(A) $\frac{y^2}{4} - y + \frac{1}{2}\ln|y| + C$ **(B)** $y^2 - y + \ln|2y| + C$ **(C)** $y^2 - 4y + \frac{1}{2}\ln|2y| + C$

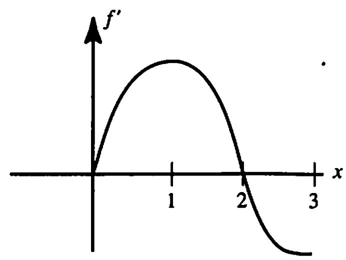
(D) $\frac{(y-1)^3}{3y^2} + C$ **(E)** $\frac{1}{2} - \frac{1}{2y^2} + C$

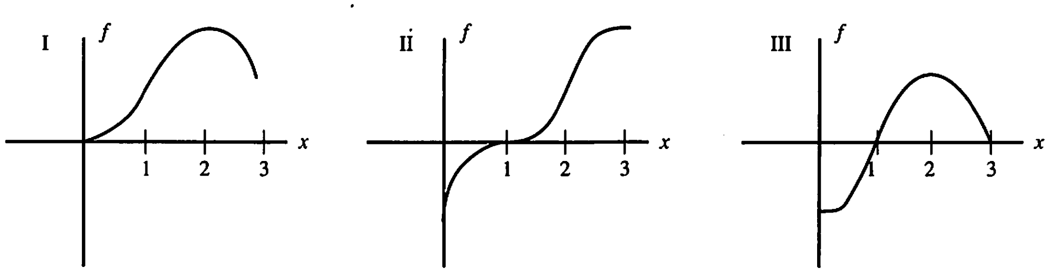
13. $\int_{\pi/6}^{\pi/2} \cot x\, dx$ equals

(A) $\ln\frac{1}{2}$ **(B)** $\ln 2$ **(C)** $-\ln(2-\sqrt{3})$

(D) $\ln(\sqrt{3}-1)$ **(E)** none of these

14. Given f' as graphed, which could be a graph of f?



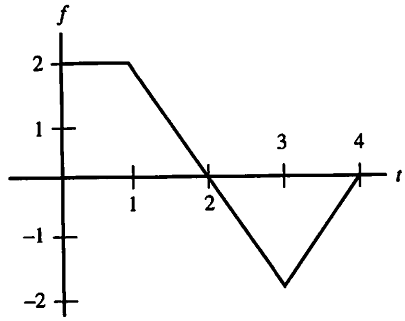


(A) I only **(B)** II only **(C)** III only
(D) I and III **(E)** none of these

15. The first woman officially timed in a marathon was Violet Piercey of Great Britain in 1926. Her record of 3:40:22 stood until 1963, mostly because of a lack of women competitors. Soon after, times began dropping rapidly, but lately they have been declining at a much slower rate. Let $M(t)$ be the curve that best represents winning marathon times in year t. Which of the following is (are) negative for $t > 1963$?

I. $M(t)$
II. $M'(t)$
III. $M''(t)$

(A) I only **(B)** II only **(C)** III only
(D) II and III **(E)** none of these

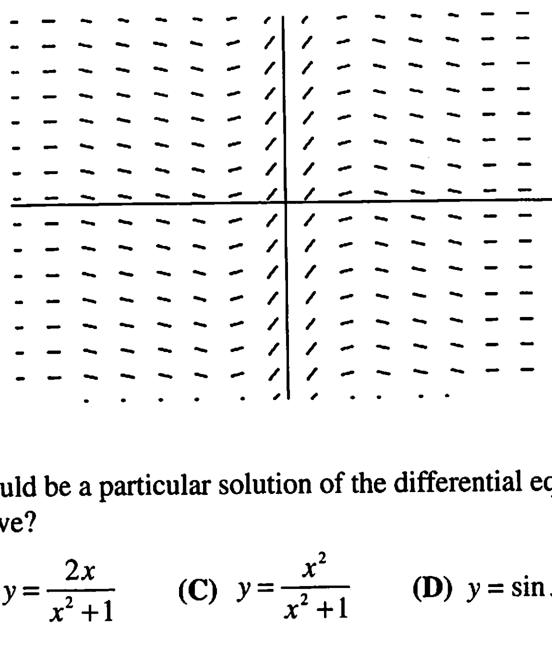


16. The graph of f is shown above. Let $G(x) = \int_0^x f(t)dt$ and $H(x) = \int_2^x f(t)\,dt$. Which of the following is true?

(A) $G(x) = H(x)$ **(B)** $G'(x) = H'(x + 2)$ **(C)** $G(x) = H(x + 2)$
(D) $G(x) = H(x) - 2$ **(E)** $G(x) = H(x) + 3$

17. The minimum value of $f(x) = x^2 + \frac{2}{x}$ on the interval $\frac{1}{2} \le x \le 2$ is

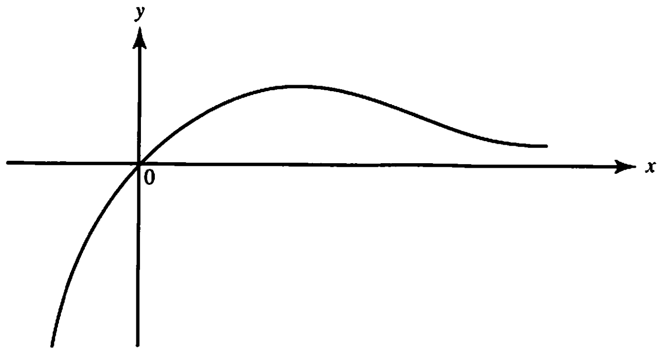
(A) $\frac{1}{2}$ **(B)** 1 **(C)** 3 **(D)** $4\frac{1}{2}$ **(E)** 5



18. Which function could be a particular solution of the differential equation whose slope field is shown above?

(A) $y = x^3$ **(B)** $y = \frac{2x}{x^2+1}$ **(C)** $y = \frac{x^2}{x^2+1}$ **(D)** $y = \sin x$ **(E)** $y = e^{-x^2}$

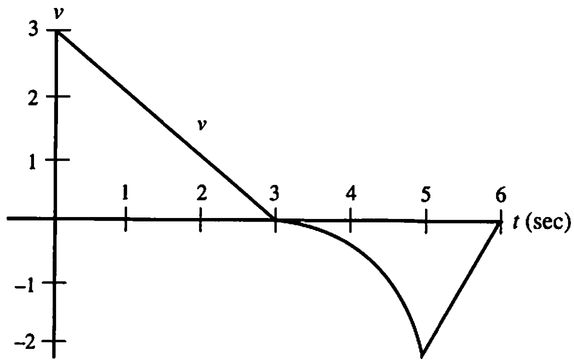
19. Which of the following functions could have the graph sketched below?



(A) $f(x) = xe^x$ (B) $f(x) = xe^{-x}$ (C) $f(x) = \frac{e^x}{x}$

(D) $f(x) = \frac{x}{x^2+1}$ (E) $f(x) = \frac{x^2}{x^3+1}$

Questions 20–22. Use the graph below, consisting of two line segments and a quarter-circle. The graph shows the velocity of an object during a 6-second interval.



20. For how many values of t in the interval $0 < t < 6$ is the acceleration undefined?

(A) none (B) one (C) two (D) three (E) four

21. During what time interval (in sec) is the speed increasing?

(A) $0 < t < 3$ (B) $3 < t < 5$ (C) $3 < t < 6$
(D) $5 < t < 6$ (E) never

22. What is the average acceleration (in units/sec²) during the first 5 seconds?

(A) $-\frac{5}{2}$ (B) -1 (C) $-\frac{1}{5}$ (D) $\frac{1}{5}$ (E) $\frac{1}{2}$

23. The curve of $y = \dfrac{2x^2}{4 - x^2}$ has

(A) two horizontal asymptotes
(B) two horizontal asymptotes and one vertical asymptote
(C) two vertical asymptotes but no horizontal asymptote
(D) one horizontal and one vertical asymptote
(E) one horizontal and two vertical asymptotes

24. Suppose

$$f(x) = \begin{cases} x^2 & \text{if } x < -2, \\ 4 & \text{if } -2 < x \leqq 1, \\ 6 - x & \text{if } x > 1. \end{cases}$$

Which statement is true?

(A) f is discontinuous only at $x = -2$.
(B) f is discontinuous only at $x = 1$.
(C) If $f(-2)$ is defined to be 4, then f will be continuous everywhere.
(D) f is continuous everywhere.
(E) f is discontinuous at $x = -2$ and at $x = 1$.

25. The function $f(x) = x^5 + 3x - 2$ passes through the point (1,2). Let f^{-1} denote the inverse of f. Then $(f^{-1})'(2)$ equals

(A) $\dfrac{1}{83}$ (B) $\dfrac{1}{8}$ (C) 1 (D) 8 (E) 83

26. $\displaystyle\int_1^e \frac{\ln^3 x}{x}\,dx =$

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{4}e$ (C) $\dfrac{1}{4}(e-1)$ (D) $\dfrac{e^4}{4}$ (E) $\dfrac{e^4 - 1}{4}$

27. Which of the following statements is (are) true about the graph of $y = \ln(4 + x^2)$?

I. It is symmetric to the y-axis.
II. It has a local minimum at $x = 0$.
III. It has inflection points at $x = \pm 2$.

(A) I only (B) II only (C) III only
(D) I and II only (E) I, II, and III

28. Let $\int_0^x f(t)\,dt = x \sin \pi x$. Then $f(3) =$

(A) -3π (B) -1 (C) 0 (D) 1 (E) 3π

STOP

END OF PART A, SECTION I

AB Practice Examination 2

Part B TIME: 50 MINUTES

Some questions in this part of the examination require the use of a graphing calculator. There are 17 questions in Part B, for which 50 minutes are allowed. Because there is no deduction for wrong answers, you should answer every question, even if you need to guess.

Directions: Choose the best answer for each question. If the exact numerical value of the correct answer is not listed as a choice, select the choice that is closest to the exact numerical answer.

29. The area bounded by the curve $x = 3y - y^2$ and the line $x = -y$ is represented by

(A) $\int_0^4 (2y - y^2)\,dy$ **(B)** $\int_0^4 (4y - y^2)\,dy$ **(C)** $\int_0^3 (3y - y^2)\,dy + \int_0^4 y\,dy$

(D) $\int_0^4 (y^2 - 4y)\,dy$ **(E)** $\int_0^3 (2y - y^2)\,dy$

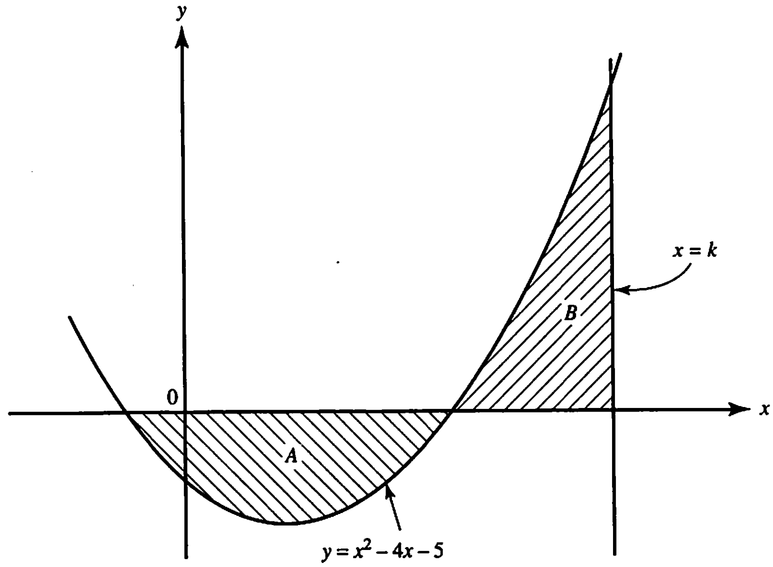
30. The region bounded by $y = e^x$, $y = 1$, and $x = 2$ is rotated about the x-axis. The volume of the solid generated is given by the integral:

(A) $\pi \int_0^2 e^{2x}\,dx$ **(B)** $2\pi \int_1^{e^2} (2 - \ln y)(y - 1)\,dy$ **(C)** $\pi \int_0^2 (e^{2x} - 1)\,dx$

(D) $2\pi \int_0^{e^2} y(2 - \ln y)\,dy$ **(E)** $\pi \int_0^2 (e^x - 1)^2\,dx$

31. A particle moves on a straight line so that its velocity at time t is given by $v = 12\sqrt{s}$, where s is its distance from the origin. If $s = 1$ when $t = 0$, then, when $t = 1$, s equals

(A) 0 **(B)** $\sqrt{7}$ **(C)** 7 **(D)** 8 **(E)** 49



(This figure is not drawn to scale.)

32. The sketch shows the graphs of $f(x) = x^2 - 4x - 5$ and the line $x = k$. The regions labeled A and B have equal areas if $k =$

(A) 5 **(B)** 7.766 **(C)** 7.899 **(D)** 8 **(E)** 11

33. Bacteria in a culture increase at a rate proportional to the number present. An initial population of 200 triples in 10 hours. If this pattern of increase continues unabated, then the approximate number of bacteria after 1 full day is

(A) 1160 **(B)** 1440 **(C)** 2408 **(D)** 2793 **(E)** 8380

34. When the substitution $x = 2t - 1$ is used, the definite integral $\int_3^5 t\sqrt{2t-1}\,dt$ may be expressed in the form $k\int_a^b (x+1)\sqrt{x}\,dx$, where $\{k, a, b\} =$

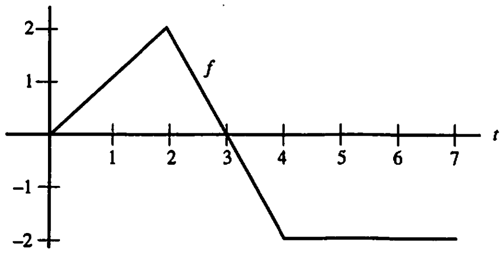
(A) $\left\{\frac{1}{4}, 2, 3\right\}$ **(B)** $\left\{\frac{1}{4}, 3, 5\right\}$ **(C)** $\left\{\frac{1}{4}, 5, 9\right\}$

(D) $\left\{\frac{1}{2}, 2, 3\right\}$ **(E)** $\left\{\frac{1}{2}, 5, 9\right\}$

35. The curve defined by $x^3 + xy - y^2 = 10$ has a vertical tangent line when $x =$

(A) 0 or $-\frac{1}{3}$ (B) 1.037 (C) 2.074 (D) 2.096 (E) 2.154

Use the graph of f shown on [0,7] for Questions 36 and 37. Let $G(x) = \int_2^{3x-1} f(t)\,dt$.



36. $G'(1)$ is

(A) 1 (B) 2 (C) 3 (D) 6 (E) undefined

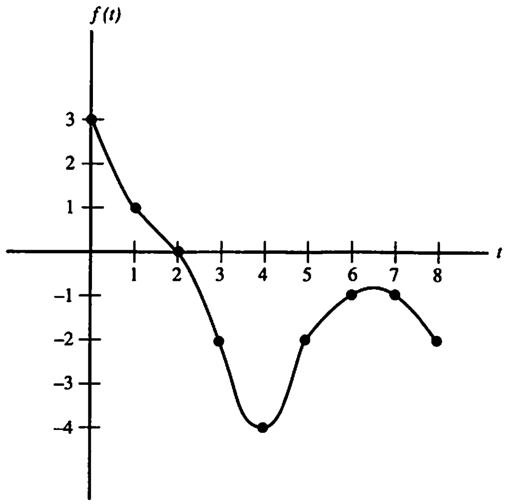
37. G has a local maximum at $x =$

(A) 1 (B) $\frac{4}{3}$ (C) 2 (D) 5 (E) 8

38. The slope of the line tangent to the curve $y = (\arctan(\ln x))^2$ at $x = 2$ is

(A) -0.563 (B) -0.409 (C) -0.342 (D) 0.409 (E) 0.563

39. Using the left rectangular method and four subintervals of equal width, estimate $\int_0^8 |f(t)|\,dt$, where f is the function graphed below.



(A) 4 (B) 5 (C) 8 (D) 15 (E) 16

40. Suppose $f(3) = 2, f'(3) = 5$, and $f''(3) = -2$. Then $\frac{d^2}{dx^2}\left(f^2(x)\right)$ at $x = 3$ is equal to

(A) -20 (B) 10 (C) 20 (D) 38 (E) 42

41. The velocity of a particle in motion along a line (for $t \geq 0$) is $v(t) = \ln(2 - t^2)$. Find the acceleration when the object is at rest.

(A) -2 (B) 0 (C) $\frac{1}{2}$ (D) 0 (E) none of these

42. Suppose $f(x) = \frac{1}{3}x^3 + x, x > 0$ and x is increasing. The value of x for which the rate of increase of f is 10 times the rate of increase of x is

(A) 1 (B) 2 (C) $\sqrt[3]{10}$ (D) 3 (E) $\sqrt{10}$

43. The rate of change of the surface area, S, of a balloon is inversely proportional to the square of the surface area. Which equation describes this relationship?

(A) $S(t) = \frac{k}{t^2}$ (B) $S(t) = \frac{k}{S^2}$ (C) $\frac{dS}{dt} = \frac{k}{S^2}$

(D) $\frac{dS}{dt} = \frac{S^2}{k}$ (E) $\frac{dS}{dt} = \frac{k}{t^2}$

44. Two objects in motion from $t = 0$ to $t = 3$ seconds have positions $x_1(t) = \cos(t^2 + 1)$ and $x_2(t) = \frac{e^t}{2t}$, respectively. How many times during the 3 seconds do the objects have the same velocity?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

45. After t years, $50e^{-0.015t}$ pounds of a deposit of a radioactive substance remain. The average amount per year *not* lost by radioactive decay during the second hundred years is

(A) 2.9 lb (B) 5.8 lb (C) 7.4 lb (D) 11.1 lb (E) none of these

STOP

END OF SECTION I

SECTION II

Part A TIME: 30 MINUTES
2 PROBLEMS

A graphing calculator is required for some of these problems.
See instructions on page 4.

1. Let function f be continuous and decreasing, with values as shown in the table:

x	2.5	3.2	3.5	4.0	4.6	5.0
$f(x)$	7.6	5.7	4.2	3.8	2.2	1.6

(a) Use the trapezoid method to estimate the area between f and the x-axis on the interval $2.5 \le x \le 5.0$.

(b) Find the average rate of change of f on the interval $2.5 \le x \le 5.0$.

(c) Estimate the instantaneous rate of change of f at $x = 2.5$.

(d) If $g(x) = f^{-1}(x)$, estimate the slope of g at $x = 4$.

2. Newton's law of cooling states that the rate at which an object cools is proportional to the difference in temperature between the object and its surroundings.

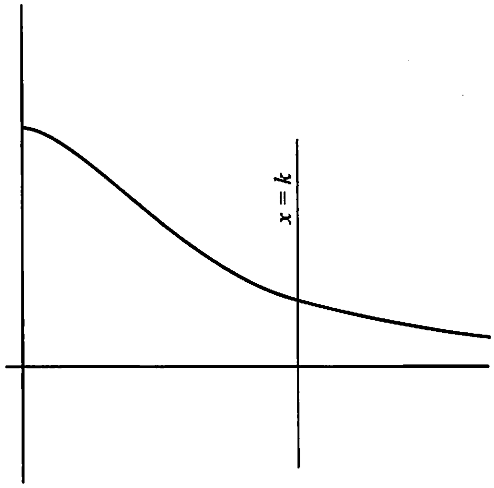
It is 9:00 P.M., time for your milk and cookies. The room temperature is 68° when you pour yourself a glass of 40° milk and start looking for the cookie jar. By 9:03 the milk has warmed to 43°, and the phone rings. It's your friend, with a fascinating calculus problem. Distracted by the conversation, you forget about the glass of milk. If you dislike milk warmer than 60°, how long, to the nearest minute, do you have to solve the calculus problem and still enjoy acceptably cold milk with your cookies?

STOP

END OF PART A, SECTION II

Part B TIME: 60 MINUTES
4 PROBLEMS

No calculator is allowed for any of these problems.
If you finish Part B before time has expired, you may return to work on Part A, but you may not use a calculator.



3. Consider the first-quadrant region bounded by the curve $y = \dfrac{18}{9 + x^2}$, the coordinate axes, and the line $x = k$, as shown in the figure above.

(a) For what value of k will the area of this region equal π?

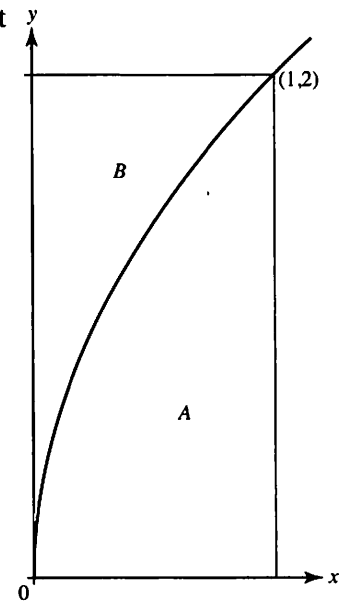
(b) What is the average value of the function on the interval $0 \le x \le k$?

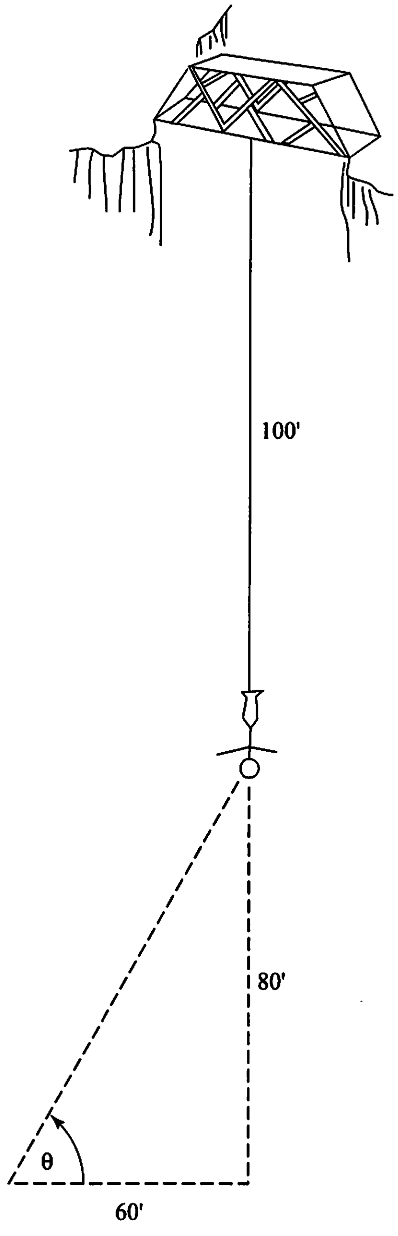
(c) What happens to the area of the region as the value of k increases?

4. The curve $y = \sqrt{8 \sin\left(\frac{\pi x}{6}\right)}$ divides a first quadrant rectangle into regions A and B, as shown in the figure.

(a) Region A is the base of a solid. Cross sections of this solid perpendicular to the x-axis are rectangles. The height of each rectangle is 5 times the length of its base in region A. Find the volume of this solid.

(b) The other region, B, is rotated around the y-axis to form a different solid. Set up but do not evaluate an integral for the volume of this solid.

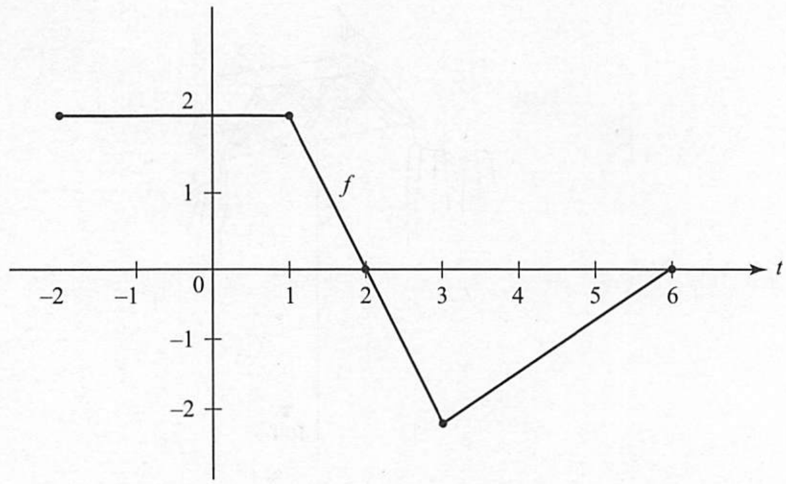




5. A bungee jumper has reached a point in her exciting plunge where the taut cord is 100 feet long with a 1/2-inch radius, and stretching. She is still 80 feet above the ground and is now falling at 40 feet per second. You are observing her jump from a spot on the ground 60 feet from the potential point of impact, as shown in the diagram above.

(a) Assuming the cord to be a cylinder with volume remaining constant as the cord stretches, at what rate is its radius changing when the radius is 1/2″?

(b) From your observation point, at what rate is the angle of elevation to the jumper changing when the radius is 1/2″?



6. The figure above shows the graph of f, whose domain is the closed interval $[-2,6]$. Let $F(x) = \int_1^x f(t)\,dt$.

(a) Find $F(-2)$ and $F(6)$.

(b) For what value(s) of x does $F(x) = 0$?

(c) For what value(s) of x is F increasing?

(d) Find the maximum value and the minimum value of F.

(e) At what value(s) of x does the graph of F have points of inflection? Justify your answer.

STOP

END OF TEST

Answer Key

AB PRACTICE EXAMINATION 2

Part A

1. E	**8.** B	**15.** B	**22.** B
2. A	**9.** B	**16.** E	**23.** E
3. E	**10.** B	**17.** C	**24.** E
4. D	**11.** C	**18.** B	**25.** B
5. B	**12.** A	**19.** B	**26.** A
6. D	**13.** B	**20.** C	**27.** E
7. B	**14.** D	**21.** B	**28.** A

Part B

29. B	**34.** C	**38.** D	**42.** D
30. C	**35.** C	**39.** E	**43.** C
31. E	**36.** D	**40.** E	**44.** E
32. D	**37.** B	**41.** A	**45.** B
33. D			

ANSWERS EXPLAINED

Multiple-Choice

Part A

1. (E) $\frac{x^2-2}{4-x^2} \to +\infty$ as $x \to 2$.

2. (A) Divide both numerator and denominator by $\sqrt{x}$; $\lim\limits_{x\to\infty} \frac{1-\frac{4}{\sqrt{x}}}{\frac{4}{\sqrt{x}}-3} = -\frac{1}{3}$.

3. (E) Since $e^{\ln u} = u$, $y = 1$.

4. (D) $f(0) = 3$, and $f'(x) = \frac{1}{2}(9 + \sin 2x)^{-1/2} \cdot (2\cos 2x)$, so $f'(0) = \frac{1}{3}$; $y \approx \frac{1}{3}x + 3$.

5. (B) $\int_0^1 \frac{60}{1+t^2}\,dt = 60 \arctan t \Big|_0^1 = 60 \arctan 1 = 60 \cdot \frac{\pi}{4}$.

6. (D) Here $y' = 3\sin^2(1-2x)\cos(1-2x) \cdot (-2)$.

7. (B) $\frac{d}{dx}(x^2 e^{x^{-1}}) = x^2 e^{x^{-1}}\left(-\frac{1}{x^2}\right) + 2xe^{x^{-1}}$.

8. (B) Let s be the distance from the origin: then

$$s = \sqrt{x^2+y^2} \quad \text{and} \quad \frac{ds}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}.$$

Since $\frac{dy}{dt} = 2x\frac{dx}{dt}$ and $\frac{dx}{dt} = \frac{3}{2}$, $\frac{dy}{dt} = 3x$. Substituting yields $\frac{ds}{dt} = \frac{3\sqrt{5}}{2}$.

9. (B) For $f(x) = \sqrt{x}$, this limit represents $f'(25)$.

10. (B) $V = \int_0^1 y^2\,dx = \int_0^1 \sqrt{1-x^2}\,dx$. This definite integral represents the area of a quadrant of the circle $x^2 + y^2 = 1$, hence $V = \frac{\pi}{4}$.

11. (C) $-\frac{1}{2}\int (9-x^2)^{-1/2}(-2x\,dx) = -\frac{1}{2}\frac{(9-x^2)^{\frac{1}{2}}}{1/2} + C$.

12. **(A)** The integral is rewritten as

$$\int \frac{(y-1)^2}{dy}\,dy = \frac{1}{2}\int \frac{y^2-2y+1}{y}\,dy,$$

$$= -\frac{1}{2}\int\left(y-2+\frac{1}{y}\right)dy,$$

$$= -\frac{1}{2}\left(\frac{y^2}{2}-2y+\ln|y|\right)+C.$$

13. **(B)** $\int_{\pi/6}^{\pi/2} \cot x\,dx = \ln \sin x \Big|_{\pi/6}^{\pi/2} = 0 - \ln\frac{1}{2}$.

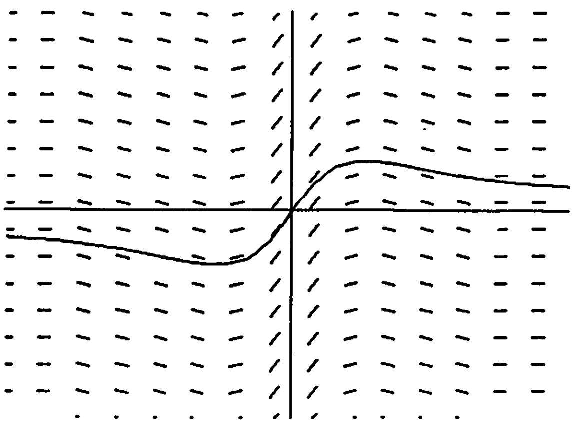
14. **(D)** Note:

```
      0                1                2                3
$f'$ { |&lt;----------positive----------&gt;|&lt;–negative–&gt;|
     { |&lt;increasing&gt;|&lt;--------decreasing---------&gt;|
$f$  { |&lt;---------increasing--------&gt;|&lt;decreasing&gt;|
     { |&lt;concave up&gt;|&lt;------concave down------&gt;|
```

15. **(B)** The winning times are positive, decreasing, and concave upward.

16. **(E)** $G(x) = H(x) + \int_0^2 f(t)\,dt$, where $\int_0^2 f(t)dt$ represents the area of a trapezoid.

17. **(C)** $f'(x) = 0$ for $x = 1$ and $f''(1) > 0$.

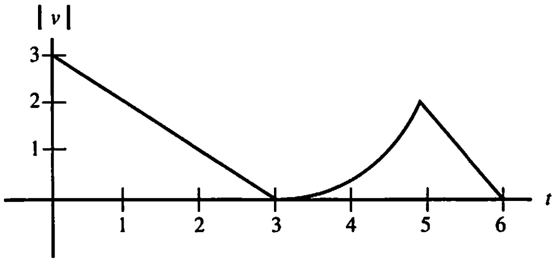


18. **(B)** Solution curves appear to represent odd functions with a horizontal asymptote. In the figure above, the curve in (B) of the question has been superimposed on the slope field.

19. (B) Note that

$$\lim_{x\to\infty} xe^x = \infty,\ \lim_{x\to\infty} \frac{e^x}{x} = \infty,\ \lim_{x\to-\infty} \frac{x}{x^2+1} = 0, \text{ and } \frac{x^2}{x^3+1} \geq 0 \text{ for } x > -1.$$

20. (C) v is not differentiable at $t = 3$ or $t = 5$.



21. (B) Speed is the magnitude of velocity; its graph is shown above.

22. (B) The average rate of change of velocity is $\frac{v(5)-v(0)}{5-0} = \frac{-2-3}{5}$.

23. (E) The curve has vertical asymptotes at $x = 2$ and $x = -2$ and a horizontal asymptote at $y = -2$.

24. (E) The function is not defined at $x = -2$; $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$. Defining $f(-2) = 4$ will make f continuous at $x = -2$, but f will still be discontinuous at $x = 1$.

25. (B) Since $\left(f^{-1}\right)'(y) = \frac{1}{f'(x)}$,

$$\left(f^{-1}\right)'(y) = \frac{1}{5x^4+3} \quad \text{and} \quad \left(f^{-1}\right)'(2) = \frac{1}{5\cdot 1+3} = \frac{1}{8}.$$

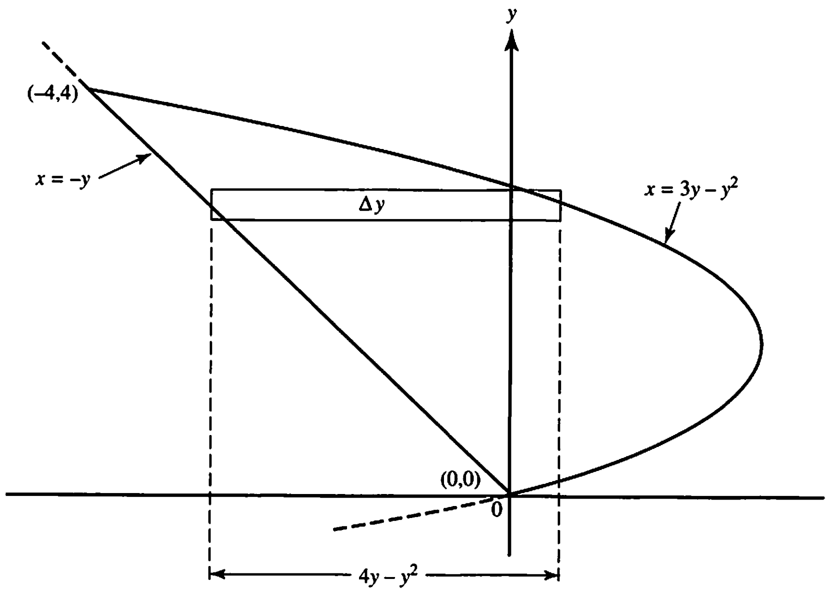
26. (A) $\int_1^e \frac{\ln^3 x}{x}\,dx = \int_1^e (\ln x)^3 \left(\frac{1}{x}dx\right) = \frac{1}{4}\ln^4 x\Big|_1^e = \frac{1}{4}(\ln^4 e - 0) = \frac{1}{4}$.

27. (E) $\ln(4 + x^2) = \ln(4 + (-x)^2)$; $y' = \frac{2x}{4+x^2}$; $y'' = \frac{-2(x^2-4)}{(4+x^2)^2}$.

28. (A) $f(x) = \frac{d}{dx}(x \sin \pi x) = \pi x \cos \pi x + \sin \pi x$.

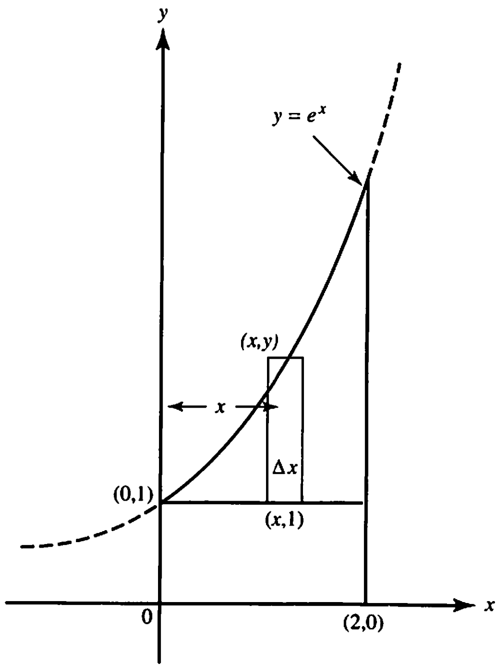
Part B

29. **(B)** See the figure below. $A = \int_0^4 [3y - y^2 - (-y)]\, dy = \int_0^4 (4y - y^2)\, dy.$



30. **(C)** See the figure below. About the x-axis: Washer. $\Delta V = \pi(y^2 - 1^2)\, \Delta x,$

$$V = \pi \int_0^2 (e^{2x} - 1)\, dx.$$



31. **(E)** We solve the differential equation $\frac{ds}{dt} = 12s^{\frac{1}{2}}$ by separation:

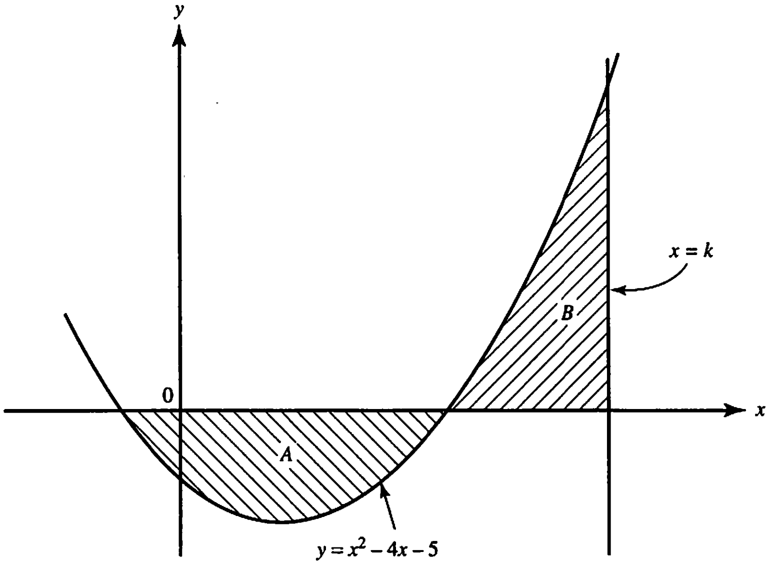
$$\int s^{-\frac{1}{2}}\,ds = 12\int dt$$

$$2s^{\frac{1}{2}} = 12t + C$$

$$\sqrt{s} = 6t + C$$

If $s = 1$ when $t = 0$, we have $C = 1$; hence, $\sqrt{s} = 6t + 1$ so $\sqrt{s} = 7$ when $t = 1$.

32. **(D)**



(This figure is not drawn to scale.)

The roots of $f(x) = x^2 - 4x - 5 = (x-5)(x+1)$ are $x = -1$ and 5. Since areas A and B are equal, therefore $\int_{-1}^{k} f(x)\,dx = 0$. Thus,

$$\left(\frac{x^3}{3} - 2x^2 - 5x\right)\Bigg|_{-1}^{k} = \left(\frac{k^3}{3} - 2k^2 - 5k\right) - \left(-\frac{1}{3} - 2 + 5\right)$$

$$= \frac{k^3}{3} - 2k^2 - 5k - \frac{8}{3} = 0.$$

Solving on a calculator gives k (or x) equal to 8.

33. **(D)** If N is the number of bacteria at time t, then $N = 200e^{kt}$. It is given that $3 = e^{10k}$. When $t = 24$, $N = 200e^{24k}$. Therefore $N = 200(e^{10k})^{2.4} = 200(3)^{2.4} \simeq 2793$ bacteria.

34. **(C)** Since $t = \frac{x+1}{2}$, $dt = \frac{1}{2}dx$. For $x = 2t - 1$, $t = 3$ yields $x = 5$ and $t = 5$ yields $x = 9$.

35. **(C)** Using implicit differentiation on the equation

$$x^3 + xy - y^2 = 10$$

yields

$$3x^2 + x\frac{dy}{dx} + y - 2y\frac{dy}{dx} = 0,$$

$$3x^2 + y = (2y - x)\frac{dy}{dx},$$

and

$$\frac{dy}{dx} = \frac{3x^2 + y}{2y - x}.$$

The tangent is vertical when $\frac{dy}{dx}$ is undefined; that is, when $2y - x = 0$.

Replacing y by $\frac{x}{2}$ in (1) gives

$$x^3 + \frac{x^2}{2} - \frac{x^2}{4} = 10$$

or

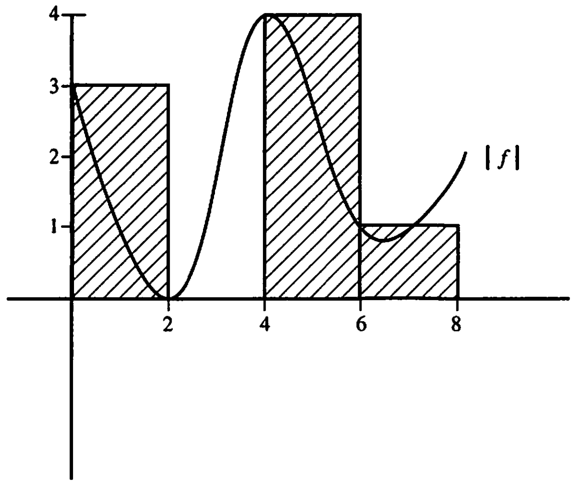
$$4x^3 + x^2 = 40.$$

Let $y_1 = 4x^3 + x^2 - 40$. Inspection of the equation $y_1 = f(x) = 0$ reveals that there is a root near $x = 2$. Solving on a calculator yields $x = 2.074$.

36. **(D)** $G'(x) = f(3x - 1) \cdot 3$.

37. **(B)** Since f changes from positive to negative at $t = 3$, G' does also where $3x - 1 = 3$.

38. **(D)** Using your calculator, evaluate $y'(2)$.



39. **(E)** $2(3) + 2(0) + 2(4) + 2(1)$.

See the figure above.

40. **(E)** $\frac{d}{dx}(f^2(x)) = 2f(x)f'(x),$

$$\frac{d^2}{dx^2}(f^2(x)) = 2[f(x)f''(x) + f'(x)f''(x)]$$

$$= 2[ff'' + (f')^2].$$

At $x = 3$, the answer is $2[2(-2) + 5^2] = 42$.

41. **(A)** The object is at rest when $v(t) = \ln(2 - t^2) = 0$; that occurs when $2 - t^2 = 1$,

so $t = 1$. The acceleration is $a(t) = v'(t) = \frac{-2t}{2-t^2}$; $a(1) = \frac{-2(1)}{2-1^2}$.

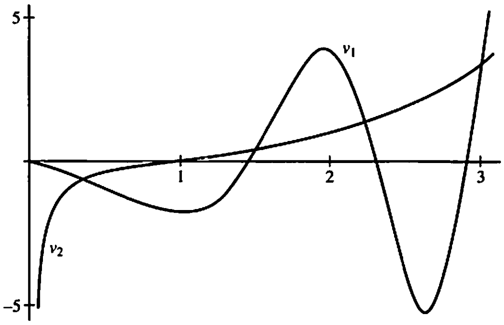
42. **(D)** $\frac{df}{dt} = (x^2 + 1)\frac{dx}{dt}$. Find x when $\frac{df}{dt} = 10\frac{dx}{dt}$.

$$10\frac{dx}{dt} = (x^2 + 1)\frac{dx}{dt}$$

implies that $x = 3$.

43. **(C)** $\frac{dS}{dt}$ represents the rate of change of the surface area; if y is inversely

proportional to x, then, $y = \frac{k}{x}$.



44. **(E)** The velocity functions are

$$v_1 = -2t \sin(t^2 + 1)$$

and

$$v_2 = \frac{2t(e^t) - 2e^t}{(2t)^2} = \frac{e^t(t-1)}{2t^2}.$$

Graph both functions in $[0, 3] \times [-5, 5]$. The graphs intersect four times during the first 3 sec, as shown in the figure above.

45. **(B)** $\frac{\int_{100}^{200} 50e^{-0.015t}\,dt}{100} \simeq 5.778$ lb.

Free-Response

Part A

AB/BC1. (a) $T=\left(\frac{7.6+5.7}{2}\right)(0.7)+\left(\frac{5.7+4.2}{2}\right)(0.3)+\left(\frac{4.2+3.8}{2}\right)(0.5)+$

$\left(\frac{3.8+2.2}{2}\right)(0.6)+\left(\frac{2.2+1.6}{2}\right)(0.4)=10.7.$

(b) $\frac{\Delta y}{\Delta x}=\frac{7.6-1.6}{2.5-5.0}=-2.4.$

(c) $f'(2.5)\approx\frac{5.7-7.6}{3.2-2.5}=-2.714.$

(d) To work with $g(x)=f^{-1}(x)$, interchange x and y:

x	7.6	5.7	4.2	3.8	2.2	1.6
$g(x)$	2.5	3.2	3.5	4.0	4.5	5.0

Now $g'(4)\approx\frac{4.0-3.5}{3.8-4.2}=-1.25$ OR $\frac{4.5-4.0}{2.2-3.8}=-0.313$ OR $\frac{4.5-3.5}{2.2-4.2}=-0.5.$

AB 2. Let M = the temperature of the milk at time t. Then

$$\frac{dM}{dt}=k(68-M).$$

The differential equation is separable:

$$\int\frac{dM}{68-M}=\int k\,dt,$$
$$-\ln|68-M|=kt+C \quad \text{(note that } 68-M>0),$$
$$\ln(68-M)=-(kt+C),$$
$$68-M=e^{-(kt+C)},$$
$$M=68-ce^{-kt},$$

where $c=e^{-C}$.
Find c, using the fact that $M=40°$ when $t=0$:

$$40=68-ce^{0} \quad \text{means} \quad c=28.$$

Find k, using the fact that $M=43°$ when $t=3$:

$$43=68-28e^{-3k},$$
$$e^{-3k}=\frac{25}{28},$$
$$k=-\frac{1}{3}\ln\frac{25}{28}.$$

Hence $M = 68 - 28e^{\frac{1}{3}\ln\frac{25}{28}t}$.

Now find t when $M = 60°$:

$$60 = 68 - 28e^{\frac{1}{3}\ln\frac{25}{28}t},$$

$$e^{\frac{1}{3}\ln\frac{25}{28}t} = \frac{8}{28},$$

$$t = \frac{\ln\frac{8}{28}}{\frac{1}{3}\ln\frac{25}{28}} = 33.163.$$

Since the phone rang at $t = 3$, you have 30 min to solve the problem.

Part B

AB 3. (a)

$$\int_0^k \frac{18}{9+x^2}\,dx = \pi,$$

$$3\cdot\frac{18}{9}\int_0^k \frac{\frac{1}{3}dx}{1+(\frac{x}{3})^2} = \pi,$$

$$6 \arctan \frac{x}{3}\Big|_0^k = \pi,$$

$$6 \arctan \frac{k}{3} - 6 \arctan \frac{0}{3} = \pi,$$

$$\frac{k}{3} = \tan\frac{\pi}{6},$$

$$k = \sqrt{3},$$

See the figure on page 556.

(b) The average value of a function on an interval is the area under the graph of the function divided by the interval width, here $\frac{\pi}{\sqrt{3}}$.

(c) From part (a) you know that the area of the region is given by $\int_0^k \frac{18}{9+x^2}\,dx = 6 \arctan\frac{k}{3}$. Since $\lim_{k\to\infty} 6 \arctan \frac{k}{3} = 6\left(\frac{\pi}{2}\right) = 3\pi$, as k increases the area of the region approaches 3π.

AB/BC 4. (a) The rectangular slices have base y, height $5y$, and thickness along the x-axis:

$$\Delta V = (y)(5y)\Delta x = 5y^2\Delta x = 5\left(\sqrt{8\sin\left(\frac{\pi x}{6}\right)}\right)\Delta x$$

$$V = 40\int_0^1 \sin\left(\frac{\pi x}{6}\right)dx = 40\frac{6}{\pi}\int_0^1 \sin\left(\frac{\pi x}{6}\right)\left(\frac{\pi}{6}dx\right)$$

$$= -\frac{240}{\pi}\cos\left(\frac{\pi x}{6}\right)\Big|_0^1 = -\frac{240}{\pi}\left(\cos\left(\frac{\pi}{6}\right) - \cos(0)\right)$$

$$= -\frac{240}{\pi}\left(\frac{\sqrt{3}}{2} - 1\right)$$

(b) The disks have radius x and thickness along the y-axis:

$$\Delta V = \pi x^2 \Delta y, \text{ so } V = \pi\int_0^2 x^2\, dy$$

Now we solve for x in terms of y:

$$y = \sqrt{8\sin\left(\frac{\pi x}{6}\right)}, \text{ so } y^2 = 8\sin\left(\frac{\pi x}{6}\right) \text{ and } \frac{y^2}{8} = \sin\left(\frac{\pi x}{6}\right).$$

Then $\arcsin\frac{y^2}{8} = \left(\frac{\pi x}{6}\right)$, which gives us $x = \frac{6}{\pi}\arcsin\frac{y^2}{8}$.

Therefore $V = \pi\int_0^2 \left(\frac{6}{\pi}\arcsin\frac{y^2}{8}\right)^2 dy$.

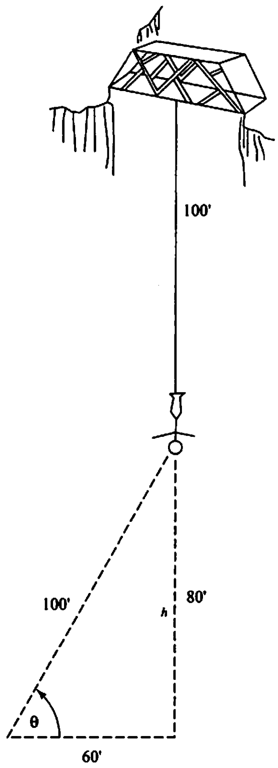
(NOTE: Although the shells method is not a required AP topic, another correct integral for this volume is $V = 2\pi\int_0^1 x\left(2 - \sqrt{8\sin\frac{\pi x}{6}}\right)dx$.)

AB/BC 5. (a) The volume of the cord is $V = \pi r^2 h$. Differentiate with respect to time, then substitute known values. (Be sure to use consistent units; here, all measurements have been converted to inches.)

$$\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + 2rh\frac{dr}{dt}\right),$$

$$0 = \pi\left(\left(\tfrac{1}{2}\right)^2\cdot 480 + 2\cdot\tfrac{1}{2}\cdot 1200\frac{dr}{dt}\right),$$

$$\frac{dr}{dt} = -\frac{1}{10}\text{ in/sec.}$$



(b) Let θ represent the angle of elevation and h the height, as shown.

$$\tan\theta = \frac{h}{60}$$

$$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{60}\frac{dh}{dt}$$

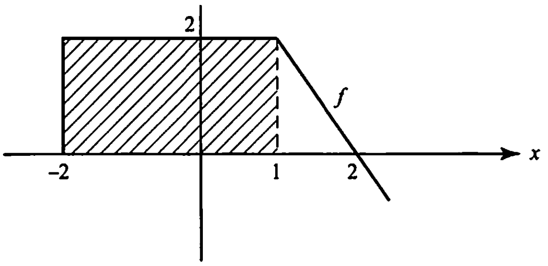
When $h = 80$, your distance to the jumper is 100 ft, as shown.

Then

$$\left(\frac{100}{60}\right)^2\frac{d\theta}{dt} = \frac{1}{60}(-40),$$

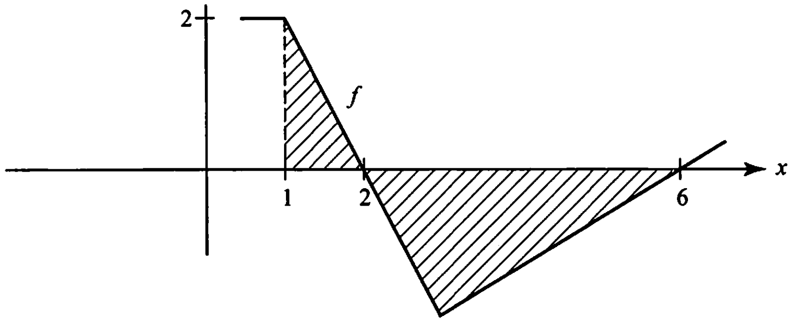
$$\frac{d\theta}{dt} = -\frac{6}{25}\text{ rad/sec.}$$

AB/BC 6. (a) $F(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt =$ the negative of the area of the shaded rectangle in the figure. Hence $F(-2) = -(3)(2) = -6$.

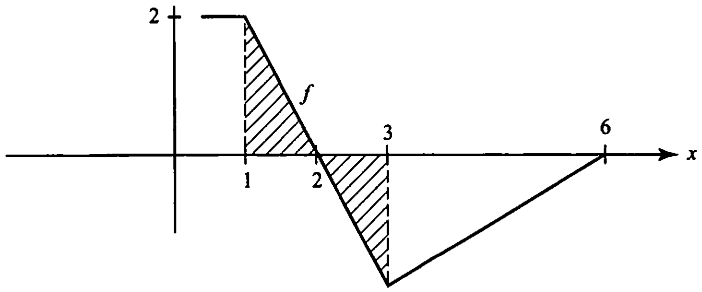


$F(6) = \int_1^6 f(t)\,dt$ is represented by the shaded triangles in the figure.

$$\int_1^6 f(t)\,dt = \int_1^2 f(t)\,dt + \int_2^6 f(t)\,dt$$
$$= \tfrac{1}{2}(1)(2) - \tfrac{1}{2}(4)(2) = -3.$$



(b) $\int_1^1 f(t)\,dt = 0$, so $F(x) = 0$ at $x = 1$. $\int_1^3 f(t)\,dt = 0$ because the regions above and below the x-axis have the same area. Hence $F(x) = 0$ at $x = 3$.



(c) F is increasing where $F' = f$ is positive: $-2 \le x < 2$.

(d) The maximum value of F occurs at $x = 2$, where $F' = f$ changes from positive to negative. $F(2) = \int_1^2 f(t)\,dt = \tfrac{1}{2}(1)(2) = 1$.

The minimum value of F must occur at one of the endpoints. Since $F(-2) = -6$ and $F(6) = -3$, the minimum is at $x = -2$.

(e) F has points of inflection where F'' changes sign, as occurs where $F' = f$ goes from decreasing to increasing, at $x = 3$.