Answer Sheet

AB PRACTICE EXAMINATION 3

Part A

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	8 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	15 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	22 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	9 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	16 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	23 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	10 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	17 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	24 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	11 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	18 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	25 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	12 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	19 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	26 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	13 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	20 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	27 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	14 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	21 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	28 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Part B

29 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	34 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	38 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	42 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
30 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	35 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	39 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	43 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
31 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	36 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	40 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	44 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
32 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	37 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	41 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	45 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
33 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ			

AB Practice Examination 3

SECTION I

Part A TIME: 55 MINUTES

The use of calculators is not permitted for this part of the examination. There are 28 questions in Part A, for which 55 minutes are allowed. Because there is no deduction for wrong answers, you should answer every question, even if you need to guess.

Directions: Choose the best answer for each question.

1. $\lim_{x\to 2}[x]$ (where $[x]$ is the greatest integer in x) is

(A) 1 **(B)** 2 **(C)** 3 **(D)** ∞ **(E)** nonexistent

2. $\lim_{h\to 0} \dfrac{\sin\left(\dfrac{\pi}{2}+h\right)-1}{h}$ is

(A) 1 **(B)** -1 **(C)** 0 **(D)** ∞ **(E)** none of these

3. If $f(x) = x \ln x$, then $f'''(e)$ equals

(A) $\dfrac{1}{e}$ **(B)** 0 **(C)** $-\dfrac{1}{e^2}$ **(D)** $\dfrac{1}{e^2}$ **(E)** $\dfrac{2}{e^3}$

4. The equation of the tangent to the curve $2x^2 - y^4 = 1$ at the point $(-1, 1)$ is
(A) $y = -x$
(B) $y = 2 - x$
(C) $4y + 5x + 1 = 0$
(D) $x - 2y + 3 = 0$
(E) $x - 4y + 5 = 0$

5. On which interval(s) does the function $f(x) = x^4 - 4x^3 + 4x^2 + 6$ increase?

(A) $x < 0$ and $1 < x < 2$ **(B)** $x > 2$ only **(C)** $0 < x < 1$ and $x > 2$
(D) $0 < x < 1$ only **(E)** $1 < x < 2$ only

6. $\int \frac{\cos x}{4+2\sin x}dx$ equals

(A) $\sqrt{4+2\sin x}+C$ (B) $-\frac{1}{2(4+\sin x)}+C$

(C) $\ln\sqrt{4+2\sin x}+C$ (D) $2\ln|4+2\sin x|+C$

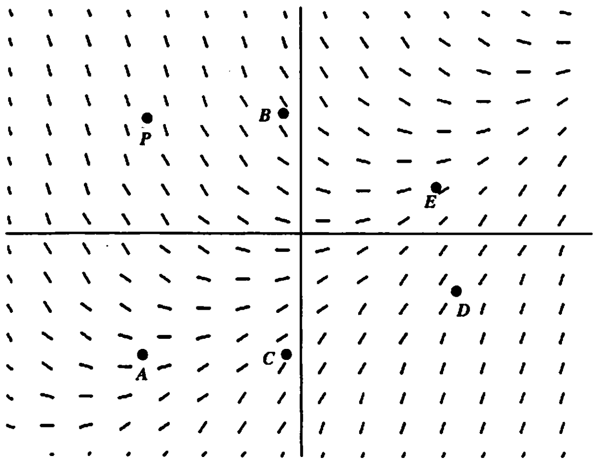
(E) $\frac{1}{4}\sin x-\frac{1}{2}\csc^2 x+C$

7. A relative maximum value of the function $y=\frac{\ln x}{x}$ is

(A) 1 (B) e (C) $\frac{2}{e}$ (D) $\frac{1}{e}$ (E) none of these

8. If a particle moves on a line according to the law $s = t^5 + 2t^3$, then the number of times it reverses direction is

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0



9. A particular solution of the differential equation whose slope field is shown above contains point P. This solution may also contain which other point?

(A) A (B) B (C) C (D) D (E) E

10. Let $F(x)=\int_5^x \frac{dt}{1-t^2}$ Which of the following statements is (are) true?

I. The domain of F is $x \neq \pm 1$.
II. $F(2) > 0$.
III. The graph of F is concave upward.

(A) none (B) I only (C) II only
(D) III only (E) II and III only

11. As the tides change, the water level in a bay varies sinusoidally. At high tide today at 8 A.M., the water level was 15 feet; at low tide, 6 hours later at 2 P.M., it was 3 feet. How fast, in feet per hour, was the water level dropping at noon today?

(A) 3 (B) $\frac{\pi\sqrt{3}}{2}$ (C) $3\sqrt{3}$ (D) $\pi\sqrt{3}$ (E) $6\sqrt{3}$

12. A smooth curve with equation $y = f(x)$ is such that its slope at each x equals x^2. If the curve goes through the point $(-1, 2)$, then its equation is

(A) $y = \frac{x^3}{3} + 7$ **(B)** $x^3 - 3y + 7 = 0$

(C) $y = x^3 + 3$ **(D)** $y - 3x^3 - 5 = 0$ **(E)** none of these

13. $\int \frac{e^u}{1 + e^{2u}}\, du$ is equal to

(A) $\ln(1 + e^{2u}) + C$ **(B)** $\frac{1}{2}\ln\left|1 + e^u\right| + C$ **(C)** $\frac{1}{2}\tan^{-1} e^u + C$

(D) $\tan^{-1} e^u + C$ **(E)** $\frac{1}{2}\tan^{-1} e^{2u} + C$

14. Given $f(x) = \log_{10} x$ and $\log_{10}(102) \approx 2.0086$, which is closest to $f'(100)$?

(A) 0.0043 **(B)** 0.0086 **(C)** 0.01 **(D)** 1.0043 **(E)** 2

15. If $G(2) = 5$ and $G'(x) = \frac{10x}{9 - x^2}$, then an estimate of $G(2.2)$ using a tangent-line approximation is

(A) 5.4 **(B)** 5.5 **(C)** 5.8 **(D)** 8.8 **(E)** 13.8

16. The area bounded by the parabola $y = x^2$ and the lines $y = 1$ and $y = 9$ equals

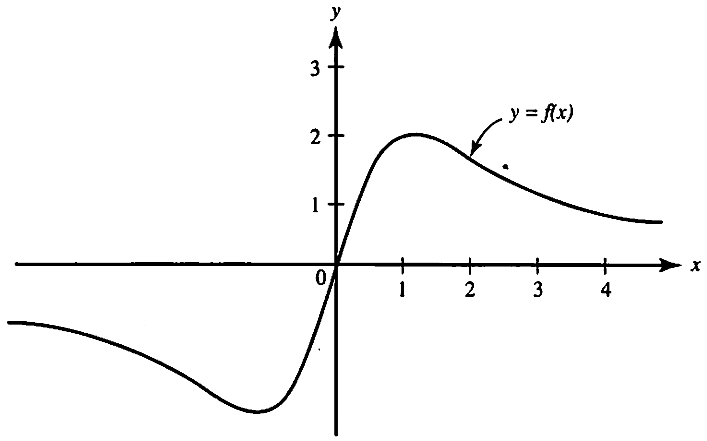
(A) 8 **(B)** $\frac{84}{3}$ **(C)** $\frac{64}{3}\sqrt{2}$ **(D)** 32 **(E)** $\frac{104}{3}$

17. Suppose $f(x) = \frac{x^2 + x}{x}$ if $x \neq 0$ and $f(0) = 1$. Which of the following statements is (are) true of f?

I. f is defined at $x = 0$.
II. $\lim\limits_{x \to 0} f(x)$ exists.
III. f is continuous at $x = 0$.

(A) I only **(B)** II only **(C)** I and II only
(D) None of the statements is true. **(E)** All are true.

18. Which function could have the graph shown below?



(A) $y=\dfrac{x}{x^2+1}$ (B) $y=\dfrac{4x}{x^2+1}$ (C) $y=\dfrac{2x}{x^2-1}$

(D) $y=\dfrac{x^2+3}{x^2+1}$ (E) $y=\dfrac{4x}{x+1}$

19. Suppose the graph of f is both increasing and concave up on $a \leqslant x \leqslant b$. Then, using the same number of subdivisions, and with L, R, M, and T denoting, respectively, left, right, midpoint, and trapezoid sums, it follows that

(A) $R \leqslant T \leqslant M \leqslant L$ (B) $L \leqslant T \leqslant M \leqslant R$ (C) $R \leqslant M \leqslant T \leqslant L$
(D) $L \leqslant M \leqslant T \leqslant R$ (E) none of these

20. $\lim\limits_{x\to 3}\dfrac{x+3}{x^2-9}$ is

(A) $+\infty$ (B) 0 (C) $\dfrac{1}{6}$ (D) $-\infty$ (E) nonexistent

21. The only function that does not satisfy the Mean Value Theorem on the interval specified is

(A) $f(x)=x^2-2x$ on $[-3, 1]$

(B) $f(x)=\dfrac{1}{x}$ on $[1, 3]$

(C) $f(x)=\dfrac{x^3}{3}-\dfrac{x^2}{2}+x$ on $[-1, 2]$

(D) $f(x)=x+\dfrac{1}{x}$ on $[-1, 1]$

(E) $f(x)=x^{2/3}$ on $\left[\dfrac{1}{2},\dfrac{3}{2}\right]$

22. Suppose $f'(x) = x(x-2)^2(x+3)$. Which of the following is (are) true?

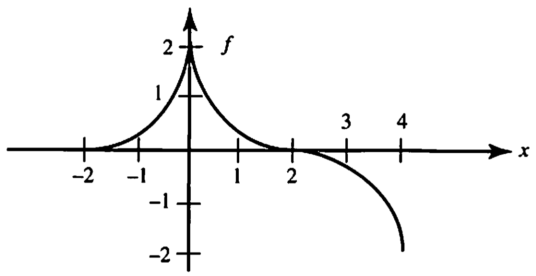
I. f has a local maximum at $x = -3$.
II. f has a local minimum at $x = 0$.
III. f has neither a local maximum nor a local minimum at $x = 2$.

(A) I only **(B)** II only **(C)** III only
(D) I and II only **(E)** I, II, and III

23. If $y = \ln\dfrac{x}{\sqrt{x^2+1}}$, then $\dfrac{dy}{dx}$ is

(A) $\dfrac{1}{x^2+1}$ **(B)** $\dfrac{1}{x(x^2+1)}$ **(C)** $\dfrac{2x^2+1}{x(x^2+1)}$

(D) $\dfrac{1}{x\sqrt{x^2+1}}$ **(E)** $\dfrac{1-x^2}{x(x^2+1)}$



24. The graph of function f shown above consists of three quarter-circles.
Which of the following is (are) equivalent to $\int_0^2 f(x)dx$?

I. $\dfrac{1}{2}\int_{-2}^{2} f(x)dx$

II. $\int_4^2 f(x)dx$

III. $\dfrac{1}{2}\int_0^4 f(x)dx$

(A) I only **(B)** II only **(C)** III only
(D) I and II only **(E)** I, II, and III

25. The base of a solid is the first-quadrant region bounded by $y = \sqrt[4]{4-2x}$, and each cross section perpendicular to the x-axis is a semicircle with a diameter in the xy-plane. The volume of the solid is

(A) $\dfrac{\pi}{2}\int_0^2 \sqrt{4-2x}\,dx$ **(B)** $\dfrac{\pi}{8}\int_0^2 \sqrt{4-2x}\,dx$ **(C)** $\dfrac{\pi}{8}\int_{-2}^2 \sqrt{4-2x}\,dx$

(D) $\dfrac{\pi}{4}\int_0^{\sqrt{2}} (4-y^4)^2\,dy$ **(E)** $\dfrac{\pi}{8}\int_0^{\sqrt[4]{4}} (4-y^4)^2\,dy$

26. The average value of $f(x)=3+|x|$ on the interval $[-2, 4]$ is

(A) $2\frac{2}{3}$ (B) $3\frac{1}{3}$ (C) $4\frac{2}{3}$ (D) $5\frac{1}{3}$ (E) 6

27. $\lim_{x\to\infty}\frac{3+x-2x^2}{4x^2+9}$ is

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) 3 (E) nonexistent

28. The area of the region in the *xy*-plane bounded by the curves $y=e^x$, $y=e^{-x}$, and $x=1$ is equal to

(A) $e+\frac{1}{e}-2$ (B) $e-\frac{1}{e}$ (C) $e+\frac{1}{e}$
(D) $2e-2$ (E) none of the these

STOP

END OF PART A, SECTION I

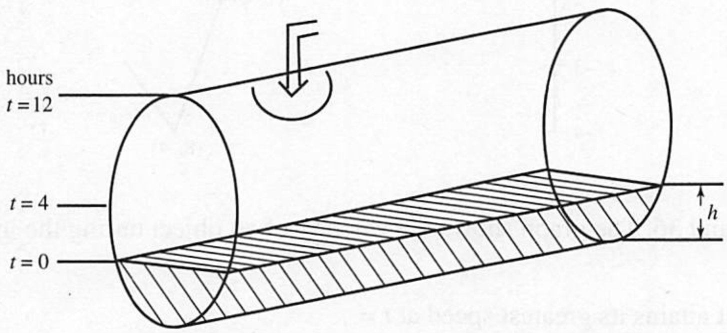
Part B TIME: 50 MINUTES

Some questions in this part of the examination require the use of a graphing calculator. There are 17 questions in Part B, for which 50 minutes are allowed. Because there is no deduction for wrong answers, you should answer every question, even if you need to guess.

Directions: Choose the best answer for each question. If the exact numerical value of the correct answer is not listed as a choice, select the choice that is closest to the exact numerical answer.

29. $f(x)=\int_0^{x^2+2}\sqrt{1+\cos t}\,dt$. Then $f'(x)=$

(A) $2x\sqrt{1+\cos(x^2+2)}$ **(B)** $2x\sqrt{1-\sin x}$ **(C)** $(x^2+2)\sqrt{1-\sin x}$
(D) $\sqrt{1+\cos(x^2+2)}$ **(E)** $\sqrt{[1+\cos(x^2+2)]\cdot 2x}$



30. A cylindrical tank, shown in the figure above, is partially full of water at time $t = 0$, when more water begins flowing in at a constant rate. The tank becomes half full when $t = 4$, and is completely full when $t = 12$. Let h represent the height of the water at time t. During which interval is $\frac{dh}{dt}$ increasing?

(A) none **(B)** $0 < t < 4$ **(C)** $0 < t < 8$ **(D)** $0 < t < 12$
(E) $4 < t < 12$

31. A particle moves on a line according to the law $s = f(t)$ so that its velocity $v = ks$, where k is a nonzero constant. Its acceleration is

(A) k^2v **(B)** k^2s **(C)** k **(D)** 0 **(E)** none of these

32. A cup of coffee placed on a table cools at a rate of $\frac{dH}{dt} = -0.05(H-70)$°F per minute, where H represents the temperature of the coffee and t is time in minutes. If the coffee was at 120°F initially, what will its temperature be 10 minutes later?

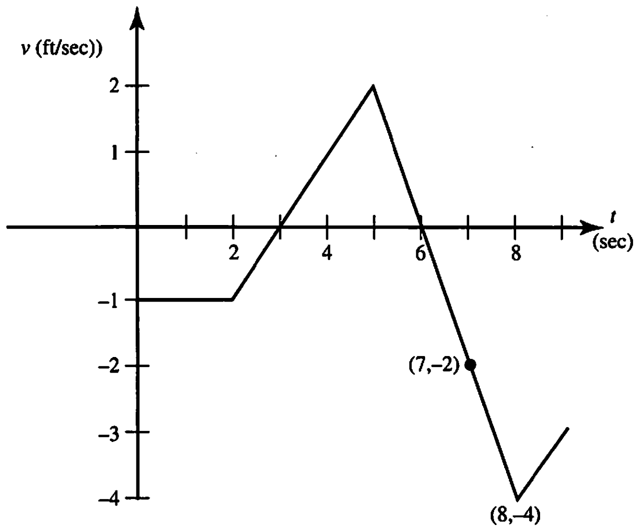
(A) 73°F **(B)** 95°F **(C)** 100°F **(D)** 118°F **(E)** 143°F

33. An investment of \$4000 grows at the rate of $320e^{0.08t}$ dollars per year after t years. Its value after 10 years is approximately

(A) \$4902 **(B)** \$8902 **(C)** \$7122
(D) \$12,902 **(E)** none of these

34. If $f(x) = (1 + e^x)$ then the domain of $f^{-1}(x)$ is

(A) $(-\infty,\infty)$ **(B)** $(0,\infty)$ **(C)** $(1,\infty)$
(D) $\{x \mid x \geqslant 1\}$ **(E)** $\{x \mid x \geqslant 2\}$



<u>Questions 35 and 36</u>. The graph shows the velocity of an object during the interval $0 \le t \le 9$.

35. The object attains its greatest speed at $t =$

(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 8

36. The object was at the origin at $t = 3$. It returned to the origin

(A) at $t = 5$ **(B)** at $t = 6$ **(C)** during $6 < t < 7$
(D) at $t = 7$ **(E)** during $7 < t < 8$

37. When the region bounded by the y-axis, $y = e^x$, and $y = 2$ is rotated around the y-axis it forms a solid with volume

(A) 0.039 **(B)** 0.386 **(C)** 0.592 **(D)** 1.214 **(E)** 4.712

38. If $\sqrt{x-2}$ is replaced by u, then $\int_3^6 \frac{\sqrt{x-2}}{x}$ is equivalent to

(A) $\int_1^2 \frac{u\,du}{u^2+2}$ **(B)** $2\int_1^2 \frac{u^2\,du}{u^2+2}$ **(C)** $\int_3^6 \frac{2u^2}{u^2+2}\,du$

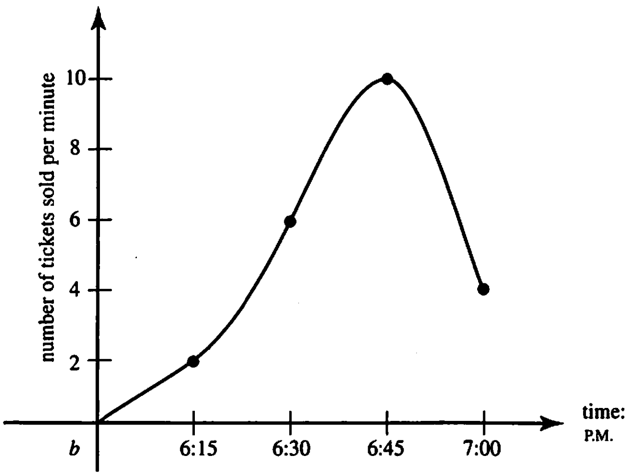
(D) $\int_3^6 \frac{u\,du}{u^2+2}$ **(E)** $\int_1^2 \frac{u^2}{u^2+2}\,du$

39. The line tangent to the graph of function f at the point (8,1) intersects the y-axis at $y = 3$. Find $f'(8)$.

(A) $-\frac{1}{4}$ (B) 0 (C) $\frac{1}{8}$ (D) 1 (E) 3

40. How many points of inflection does the function f have on the interval $0 \leqslant x \leqslant 6$ if $f''(x) = 2 - 3\sqrt{x}\cos^3 x$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5



41. The graph shows the rate at which tickets were sold at a movie theater during the last hour before showtime. Using the right-rectangle method, estimate the size of the audience.

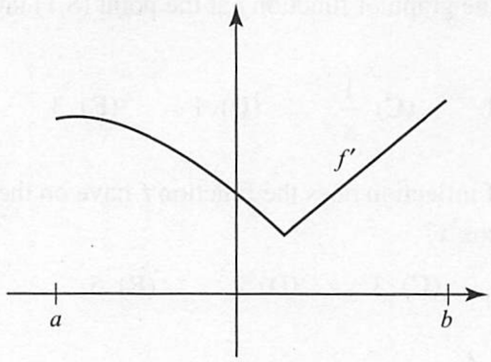
(A) 230 (B) 300 (C) 330 (D) 375 (E) 420

42. At what point of intersection of $f(x) = 4^{\sin x}$ and $g(x) = \ln(x^2)$ do their derivatives have the same sign?

(A) −5.2 (B) −4.0 (C) −1.2 (D) 2.6 (E) 7.8

43. Which statement is true?

(A) If $f(x)$ is continuous at $x = c$, then $f'(c)$ exists.
(B) If $f'(c) = 0$, then f has a local maximum or minimum at $(c, f(c))$.
(C) If $f''(c) = 0$, then the graph of f has an inflection point at $(c, f(c))$.
(D) If f is differentiable at $x = c$, then f is continuous at $x = c$.
(E) If f is continuous on (a, b), then f attains a maximum value on (a, b).



44. The graph of f' is shown above. Which statements about f must be true for $a < x < b$?

I. f is increasing.
II. f is continuous.
III. f is differentiable.

(A) I only **(B)** II only **(C)** I and II only
(D) I and III only **(E)** I, II, and III

45. After a bomb explodes, pieces can be found scattered around the center of the blast. The density of bomb fragments lying x meters from ground zero is given by $N(x) = \frac{2x}{1+x^{3/2}}$ fragments per square meter. How many fragments will be found within 20 meters of the point where the bomb exploded?

(A) 13 **(B)** 278 **(C)** 556 **(D)** 712 **(E)** 4383

STOP

END OF SECTION I

SECTION II

Part A TIME: 30 MINUTES

2 PROBLEMS

A graphing calculator is required for some of these problems.
See instructions on page 4.

1. A curve is defined by $x^2y - 3y^2 = 48$.

(a) Verify that $\dfrac{dy}{dx} = \dfrac{2xy}{6y - x^2}$.

(b) Write an equation of the line tangent to this curve at (5,3).

(c) Using your equation from part (a), estimate the y-coordinate of the point on the curve where $x = 4.93$.

(d) Show that this curve has no horizontal tangent lines.

2. The table shows the depth of water, W, in a river, as measured at 4-hour intervals during a day-long flood. Assume that W is a differentiable function of time t.

t (hr)	0	4	8	12	16	20	24
$W(t)$ (ft)	32	36	38	37	35	33	32

(a) Find the approximate value of $W'(16)$. Indicate units of measure.

(b) Estimate the average depth of the water, in feet, over the time interval $0 \le t \le 24$ hours by using a trapezoidal approximation with subintervals of length $\Delta t = 4$ days.

(c) Scientists studying the flooding believe they can model the depth of the water with the function $F(t) = 35 - 3\cos\left(\dfrac{t+3}{4}\right)$, where $F(t)$ represents the depth of the water, in feet, after t hours. Find $F'(16)$ and explain the meaning of your answer, with appropriate units, in terms of the river depth.

(d) Use the function F to find the average depth of the water, in feet, over the time interval $0 \le t \le 24$ hours.

STOP

END OF PART A, SECTION II

Part B TIME: 60 MINUTES
4 PROBLEMS

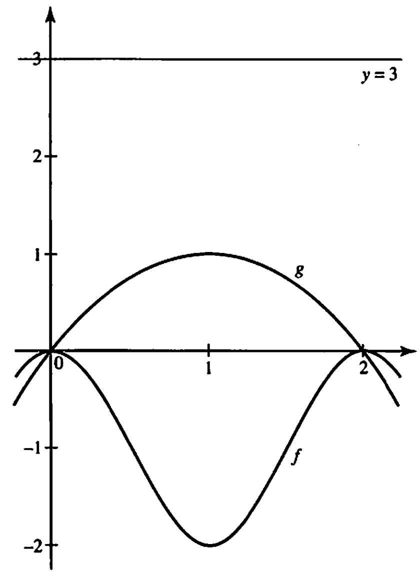
No calculator is allowed for any of these problems.
If you finish Part B before time has expired, you may return to work on Part A, but you may not use a calculator.

3. The region R is bounded by the curves $f(x) = \cos(\pi x) - 1$ and $g(x) = x(2 - x)$, as shown in the figure.

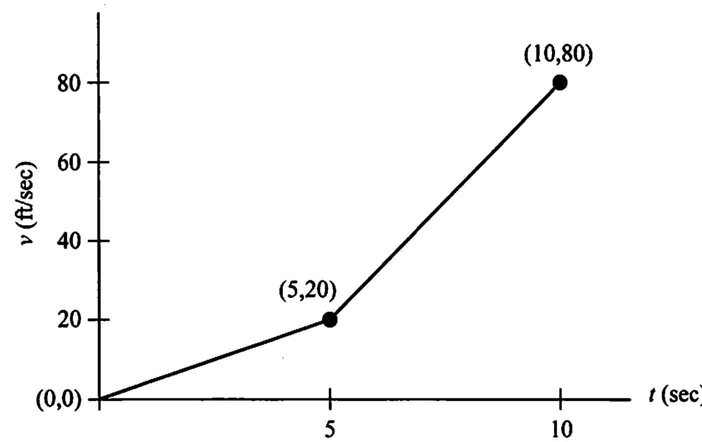
(a) Find the area of R.

(b) A solid has base R, and each cross section perpendicular to the x-axis is an isosceles right triangle whose hypotenuse lies in R. Set up, but do not evaluate, an integral for the volume of this solid.

(b) Set up, but do not evaluate, an integral for the volume of the solid formed when R is rotated around the line $y = 3$.



4. Two autos, P and Q, start from the same point and race along a straight road for 10 seconds. The velocity of P is given by $v_p(t) = 6\left(\sqrt{1+8t} - 1\right)$ feet per second. The velocity of Q is shown in the graph.



(a) At what time is P's actual acceleration (in ft/sec²) equal to its average acceleration for the entire race?

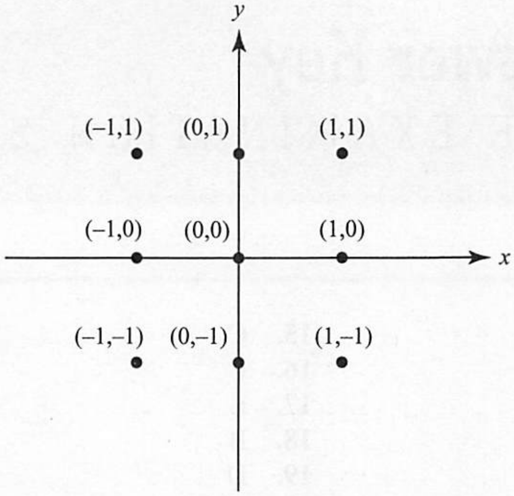
(b) What is Q's acceleration (in ft/sec²) then?

(c) At the end of the race, which auto was ahead? Explain.

5. Given the differential equation $\frac{dy}{dx} = 2x\left(y^2 + 1\right)$

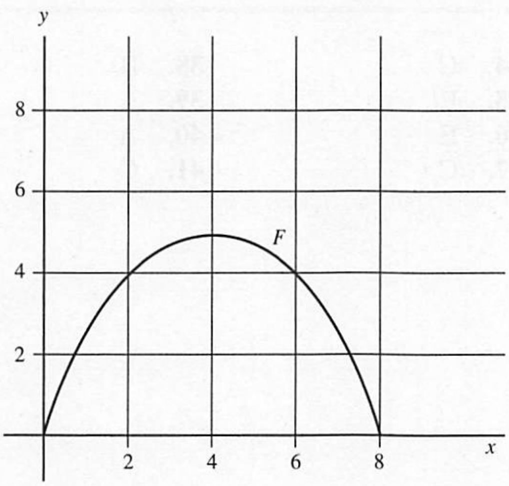
(a) Sketch the slope field for this differential equation at the points shown in the figure.

AB Practice Examination 3



(b) Let f be the particular solution to the differential equation whose graph passes through (0,1). Express f as a function of x, and state its domain.

6. The graph shown is for $F(x)=\int_0^x f(t)dt$.



(a) What is $\int_0^2 f(t)dt$?

(b) What is $\int_2^7 f(t)\,dt$?

(c) At what value of x does $f(x) = 0$?

(d) Over what interval is $f'(x)$ negative?

(e) Let $G(x)=\int_2^x f(t)dt$. Sketch the graph of G on the same axes.

STOP

END OF TEST

Answer Key

AB PRACTICE EXAMINATION 3

Part A

1. E
2. C
3. C
4. A
5. C
6. C
7. D
8. E
9. E
10. E
11. B
12. B
13. D
14. A
15. C
16. E
17. E
18. B
19. D
20. E
21. D
22. E
23. B
24. D
25. B
26. C
27. A
28. A

Part B

29. A
30. E
31. B
32. C
33. B
34. C
35. E
36. E
37. C
38. B
39. A
40. A
41. C
42. B
43. D
44. E
45. D

ANSWERS EXPLAINED

Multiple-Choice

Part A

1. (E) Here, $\lim_{x \to 2^-}[x] = 1$, while $\lim_{x \to 2^+}[x] = 2$.

2. (C) The given limit equals $f'\left(\frac{\pi}{2}\right)$, where $f(x) = \sin x$.

3. (C) Since $f(x) = x \ln x$,

$$f'(x) = 1 + \ln x, \quad f''(x) = \frac{1}{x}, \quad \text{and} \quad f'''(x) = -\frac{1}{x^2}.$$

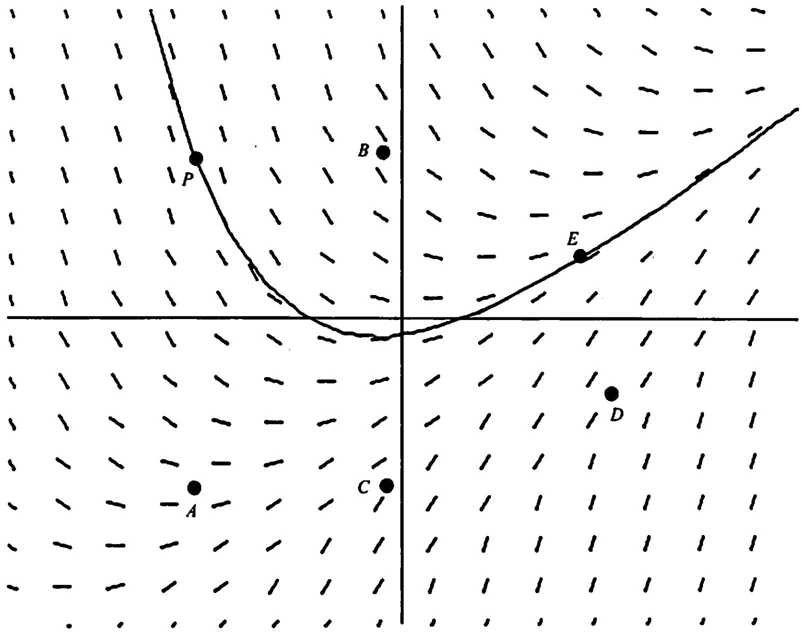
4. (A) Differentiate implicitly to get $4x - 4y^3 \frac{dy}{dx} = 0$. Substitute $(-1, 1)$ to find $\frac{dy}{dx} = -1$, the slope at this point, and write the equation of the tangent: $y - 1 = -1(x + 1)$.

5. (C) $f'(x) = 4x^3 - 12x^2 + 8x = 4x(x - 1)(x - 2)$. To determine the signs of $f'(x)$, inspect the sign at any point in each of the intervals $x < 0$, $0 < x < 1$, $1 < x < 2$, and $x > 2$. The function increases whenever $f'(x) > 0$.

6. (C) The integral is equivalent to $\frac{1}{2}\int \frac{2\cos x \, dx}{4 + 2\sin x} = \frac{1}{2}\int \frac{du}{u}$, where $u = 4 + 2\sin x$.

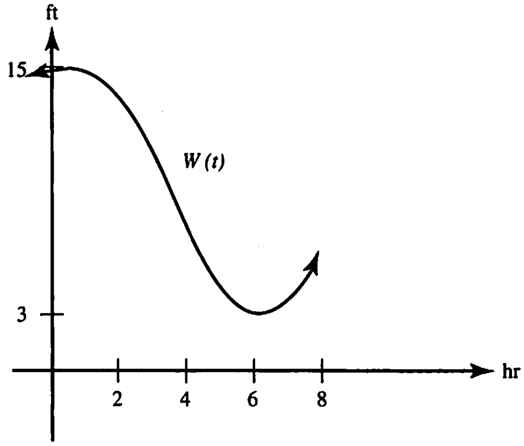
7. (D) Here $y' = \frac{1 - \ln x}{x^2}$, which is zero for $x = e$. Since the sign of y' changes from positive to negative as x increases through e, this critical value yields a relative maximum. Note that $f(e) = \frac{1}{e}$.

8. (E) Since $v = \frac{ds}{dt} = 5t^4 + 6t^2 = t^2(5t^2 + 6)$ is always positive, there are no reversals in motion along the line.



9. **(E)** The slope field suggests the curve shown above as a particular solution.

10. **(E)** Since $f(x) = F'(x) = \dfrac{1}{1-x^2}$, f is discontinuous at $x = 1$; the domain of F is therefore $x > 1$. On [2, 5] $f(x) < 0$, so $\int_5^2 f > 0$. $F''(x) = f'(x) = \dfrac{2x}{(1-x^2)^2}$, which is positive for $x > 1$.



11. **(B)** In the graph above, $W(t)$, the water level at time t, is a cosine function with amplitude 6 ft and period 12 hr:

$$W(t) = 6\cos\left(\frac{\pi}{6}t\right) + 9\text{ft},$$

$$W'(t) = -\pi\sin\left(\frac{\pi}{6}t\right)\text{ft/hr}.$$

Hence, $W'(4) = -\pi \sin\left(\dfrac{2\pi}{3}\right) = -\dfrac{\pi\sqrt{3}}{2}$ ft/hr.

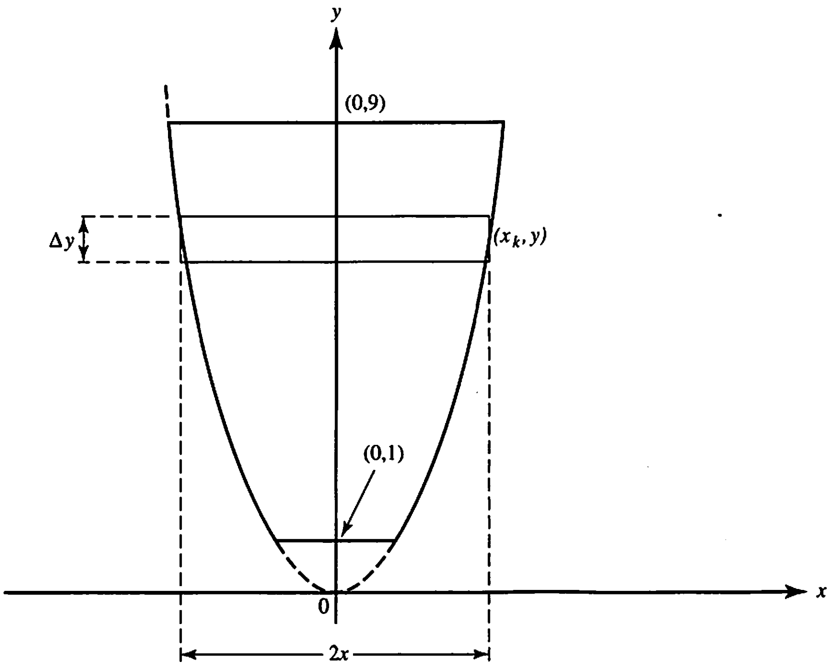
12. **(B)** Solve the differential equation $\frac{dy}{dx} = x^2$, getting $y = \frac{x^3}{3} + C$. Use $x = -1$, $y = 2$ to determine $C = \frac{7}{3}$.

13. **(D)** $\int \frac{e^u du}{1+(e^u)^2} = \tan^{-1}(e^u) + C$

14. **(A)** $f'(100) \approx \frac{f(102) - f(100)}{102 - 100} = \frac{2.0086 - 2}{2}$.

15. **(C)** $G'(2) = 4$, so $G(x) \approx 4(x - 2) + 5$.

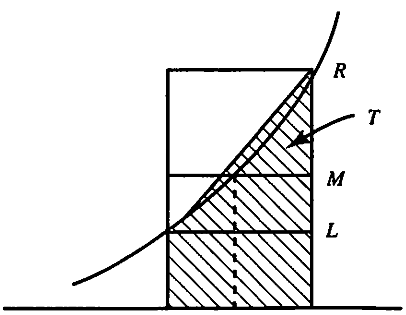
16. **(E)** $A = 2\int_1^9 x\, dy = 2\int_1^9 \sqrt{y}\, dy = \frac{104}{3}$. See the figure below.



17. **(E)** Note that $\lim_{x \to 0} f(x) = f(0) = 1$.

18. **(B)** Note that (0, 0) is on the graph, as are (1, 2) and (−1, −2). So only (B) and (E) are possible. Since $\lim_{x \to \infty} y = \lim_{x \to -\infty} y = 0$, only (B) is correct.

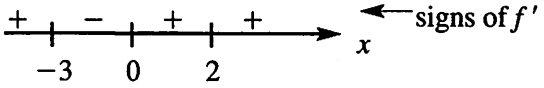
19. **(D)** See the figure.



20. **(E)** $\dfrac{x+3}{x^2-9}=\dfrac{x+3}{(x+3)(x-3)}=\dfrac{1}{x-3}$; $\lim\limits_{x\to3^+}\dfrac{1}{x-3}=+\infty$; $\lim\limits_{x\to3^-}\dfrac{1}{x-3}=-\infty$.

21. **(D)** In (D), $f(x)$ is not defined at $x = 0$. Verify that each of the other functions satisfies both conditions of the Mean Value Theorem.

22. **(E)** The signs within the intervals bounded by the critical points are given below.



Since f changes from increasing to decreasing at $x = -3$, f has a local maximum at -3. Also, f has a local minimum at $x = 0$, because it is decreasing to the left of zero and increasing to the right.

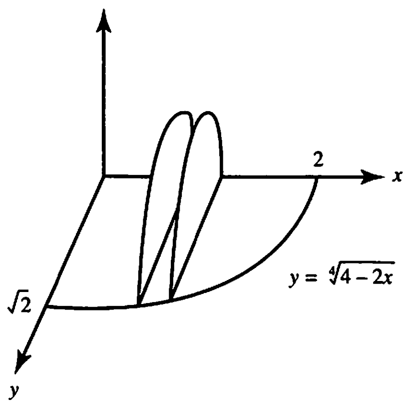
23. **(B)** Since $\ln\dfrac{x}{\sqrt{x^2+1}}=\ln x-\dfrac{1}{2}\ln(x^2+1)$, then

$$\frac{dy}{dx}=\frac{1}{x}-\frac{1}{2}\cdot\frac{2x}{x^2+1}=\frac{1}{x(x^2+1)}.$$

24. **(D)** $\int_{-2}^{0} f=\int_{0}^{2} f=-\int_{2}^{4} f$, but $\int_{0}^{4} f=0$.

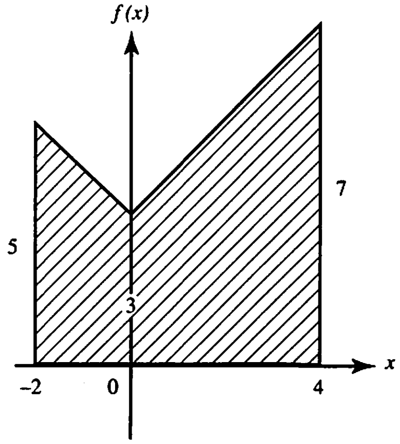
25. **(B)** As seen from the figure, $\Delta V=\dfrac{1}{2}\pi r^2\Delta x$, where $y = 2r$,

$$V=\frac{\pi}{2}\int_0^2\left(\frac{y}{2}\right)^2 dx=\frac{\pi}{8}\int_0^2\sqrt{4-2x}\,dx.$$



AB Practice Examination 3

26. **(C)** From the figure below, $\dfrac{\int_{-2}^{4} f}{6} = \dfrac{\frac{5+3}{2}\cdot 2 + \frac{3+7}{2}\cdot 4}{6} = \dfrac{28}{6}$.

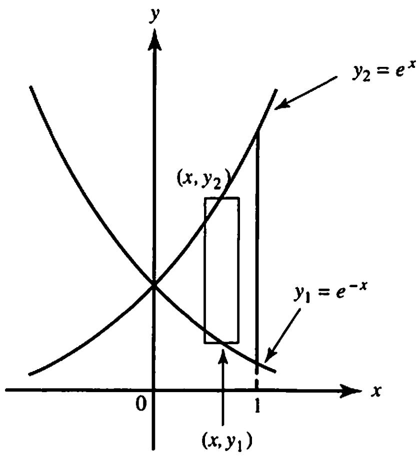


27. **(A)** Since the degrees of numerator and denominator are the same, the limit as $x \to \infty$ is the ratio of the coefficients of the terms of highest degree: $\frac{-2}{4}$.

28. **(A)** We see from the figure that $\Delta A = (y_2 - y_1)\Delta x$;

$$A = \int_0^1 (e^x - e^{-x})\, dx$$

$$= (e^x + e^{-x})\Big|_0^1 = e + \frac{1}{e} - 2.$$



Part B

29. **(A)** Let $u = x^2 + 2$. Then

$$\frac{d}{du}\int_0^u \sqrt{1+\cos t}\,dt = \sqrt{1+\cos u}$$

and

$$\frac{d}{dx}\int_0^u \sqrt{1+\cos t}\,dt = \sqrt{1+\cos u}\,\frac{du}{dx} = \sqrt{1+\cos(x^2+2)}\cdot(2x).$$

30. **(E)** $\frac{dh}{dt}$ will increase above the half-full level (that is, the height of the water will rise more rapidly) as the area of the cross section diminishes.

31. **(B)** Since $v = ks = \frac{ds}{dt}$, then $a = \frac{d^2s}{dt^2} = k\frac{ds}{dt} = kv = k^2s$.

32. **(C)** $\frac{dH}{H-70} = -0.05\,dt.\quad \ln|H-70| = -0.05t + C$

$$H - 70 = ce^{-0.05t}$$

$$H(x) = 70 + ce^{-0.05t}$$

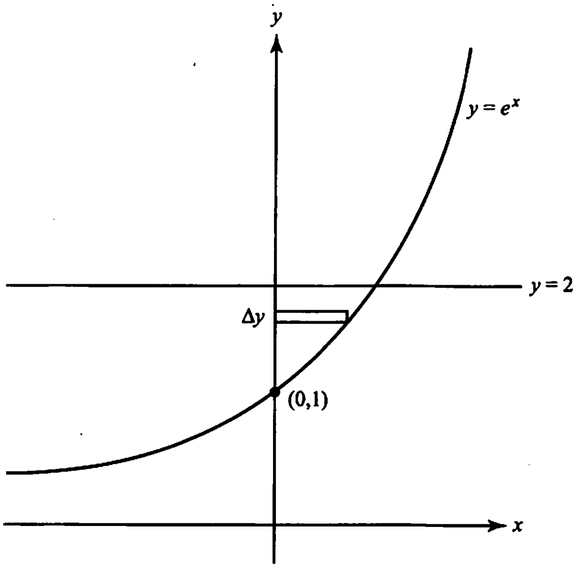
The initial condition $H(0) = 120$ shows $c = 50$. Evaluate $H(10)$.

33. **(B)** Let P be the amount after t years. It is given that $\frac{dP}{dt} = 320\,e^{0.08t}$. The solution of this differential equation is $P = 4000e^{0.08t} + C$, where $P(0) = 4000$ yields $C = 0$. The answer is $4000e^{(0.08)\cdot 10}$.

34. **(C)** The inverse of $y = 1 + e^x$ is $x = 1 + e^y$ or $y = \ln(x - 1)$; $(x - 1)$ must be positive.

35. **(E)** Speed is the magnitude of velocity: $|v(8)| = 4$.

36. **(E)** For $3 < t < 6$ the object travels to the right $\frac{1}{2}(3)(2) = 3$ units. At $t = 7$ it has returned 1 unit to the left; by $t = 8$, 4 units to the left.

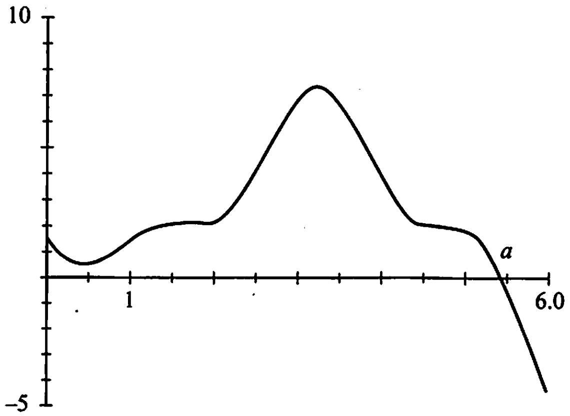


37. **(C)** Use disks: $\Delta V = \pi r^2 \Delta y = \pi x^2 \Delta y$, where $x = \ln y$. Use your calculator to evaluate $V = \pi \int_1^2 (\ln y)^2\, dy$.

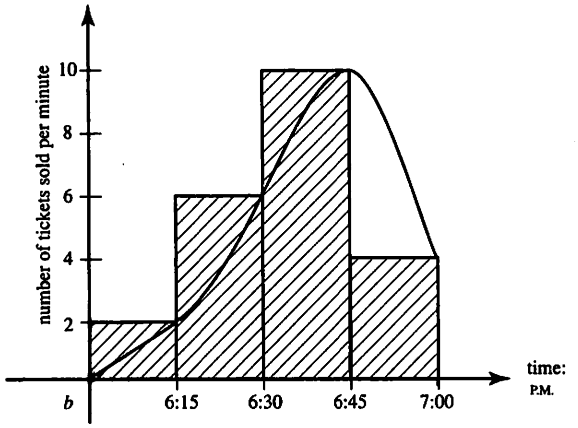
38. **(B)** If $u = \sqrt{x-2}$, then $u^2 = x - 2$, $x = u^2 + 2$, $dx = 2u\, du$. When $x = 3$, $u = 1$; when $x = 6$, $u = 2$.

39. **(A)** The tangent line passes through points (8,1) and (0,3). Its slope, $\frac{1-3}{8-0}$, is $f'(8)$.

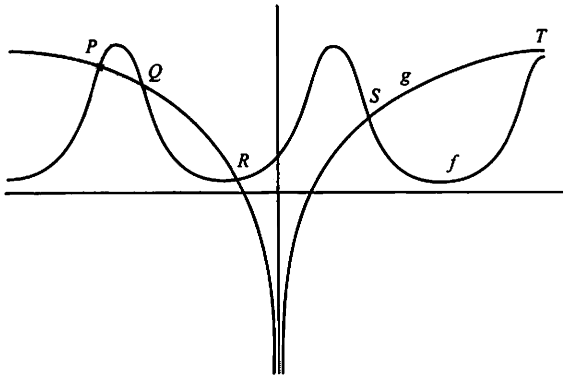
40. **(A)** Graph f'' in $[0,6] \times [-5,10]$. The sign of f'' changes only at $x = a$, as seen in the figure.



41. **(C)** In the graph below, the first rectangle shows 2 tickets sold per minute for 15 min, or 30 tickets. Similarly, the total is $2(15) + 6(15) + 10(15) + 4(15)$.



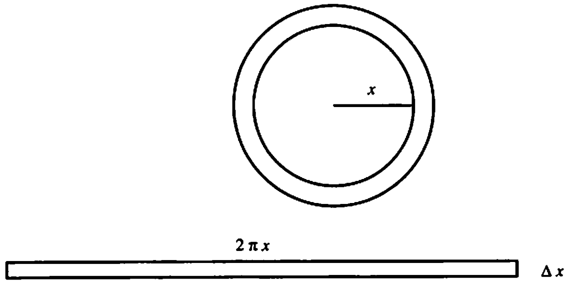
42. **(B)** Graph both functions in $[-8,8] \times [-5,5]$. At point of intersection Q, both are decreasing. Tracing reveals $x \simeq -4$ at Q. If you zoom in on the curves at $x = T$, you will note that they do not actually intersect there.



43. **(D)** Counterexamples are, respectively, for (A), $f(x) = |x|$, $c = 0$; for (B), $f(x) = x^3$, $c = 0$; for (C), $f(x) = x^4$, $c = 0$; for (E), $f(x) = x^2$ on $(-1, 1)$.

44. **(E)** $f'(x) > 0$; the curve shows that f' is defined for all $a < x < b$, so f is differentiable and therefore continuous.

45. **(D)** Consider the blast area as a set of concentric rings; one is shown in the figure. The area of this ring, which represents the region x meters from the center of the blast, may be approximated by the area of the rectangle shown. Since the number of particles in the ring is the area times the density, $\Delta P = 2\pi x \cdot \Delta x \cdot N(x)$. To find the total number of fragments within 20 m of the point of the explosion, integrate: $2\pi \int_0^{20} x \frac{2x}{1+x^{3/2}}\,dx \simeq 711.575$.



Free-Response

Part A

AB1. (a) Since $x^2y - 3y^2 = 48$,

$$x^2\frac{dy}{dx} + 2xy - 6y\frac{dy}{dx} = 0,$$
$$(x^2 - 6y)\frac{dy}{dx} = -2xy,$$
$$\frac{dy}{dx} = \frac{2xy}{6y - x^2}.$$

(b) At (5,3), $\frac{dy}{dx} = \frac{2 \cdot 5 \cdot 3}{6 \cdot 3 - 5^2} = -\frac{30}{7}$, so the equation of the tangent line is

$$y - 3 = -\frac{30}{7}(x - 5).$$

(c) $y - 3 = -\frac{30}{7}(4.93 - 5) = 0.3$, so $y = 3.3$.

(d) Horizontal tangent lines have $\frac{dy}{dx} = \frac{2xy}{6y - x^2} = 0$. This could happen only if $2xy = 0$, which means that $x = 0$ or $y = 0$.
If $x = 0$, $0y - 3y^2 = 48$, which has no real solutions.
If $y = 0$, $x^2 \cdot 0 - 3 \cdot 0^2 = 48$, which is impossible. Therefore, there are no horizontal tangents.

AB/BC2. (a) $W'(16) \approx \frac{W(20) - W(16)}{20 - 16} = \frac{33 - 35}{4} = -\frac{1}{2}$ ft/hr

$\left(\text{OR } \frac{W(16) - W(12)}{16 - 12} = \frac{35 - 37}{4} \text{ OR } \frac{W(20) - W(12)}{20 - 12} = \frac{33 - 37}{8}\right).$

(b) The average value of a function is the integral across the given interval divided by the interval width. Here $\text{Avg}(W) = \frac{\int_0^{24} W(t)\,dt}{24 - 0}$. Estimate the value of the integral using trapezoid rule T with values from the table and $\Delta t = 4$:

$$T = \frac{\Delta t}{2}(W(0) + 2W(4) + 2W(8) + 2W(12) + 2W(16) + 2W(20) + W(24))$$
$$= \frac{4}{2}(32 + 2\times 36 + 2\times 38 + 2\times 37 + 2\times 35 + 2\times 33 + 32)$$
$$= 844.$$

Hence

$$\text{Avg}(W) \approx \frac{844}{24} = 35.167 \text{ ft}.$$

(c) For $F(t)=35-3\cos\left(\frac{t+3}{4}\right)$, use your calculator to evaluate $F'(16)\approx -0.749$. After 16 hr, the river depth is dropping at the rate of 0.749 ft/hr.

(d) $\text{Avg}(F)=\dfrac{\int_0^{24}F(t)\,dt}{24-0}\approx 35.116\text{ft}.$

Part B

AB/BC 3. (a) $\Delta A=(g(x)-f(x))\,\Delta x$, so:

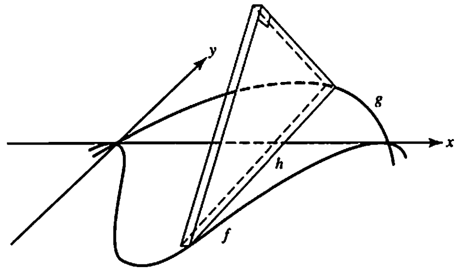
$$A=\int_0^2(g(x)-f(x))dx=\int_0^2(x(2-x)-(\cos\pi x-1)dx$$

$$=\int_0^2(2x-x^2-\cos\pi x+1)\,dx=\left(x^2-\frac{x^3}{3}-\frac{1}{\pi}\sin\pi x+x\right)\Big|_0^2$$

$$=\left(2^2-\frac{2^3}{3}-\frac{1}{\pi}\sin 2\pi+2\right)-\left(0^2-\frac{0^3}{3}-\frac{1}{\pi}\sin 0+0\right)$$

$$=\frac{10}{3}.$$

(b) Let h = the hypotenuse of an isosceles right triangle, as shown in the figure. Then each leg of the triangle is $\frac{h}{\sqrt{2}}$ and its area is $\frac{1}{2}\cdot\frac{h}{\sqrt{2}}\cdot\frac{h}{\sqrt{2}}=\frac{h^2}{4}$.



An element of volume is

$$\Delta V=\frac{h^2}{4}\Delta x=\frac{(g(x)-f(x))^2}{4}\Delta x,$$

and thus $V=\frac{1}{4}\int_0^2(x(2-x)-(\cos\pi x-1))^2dx$.

(c) Washers; $\Delta V=\pi(r_1^2-r_2^2)\Delta x$ where:

$r_1=3-f(x)=3-(\cos\pi x-1)$,

$r_2=3-g(x)=3-x(2-1)$.

So $V=\pi\int_0^2[(3-(\cos\pi x-1))^2-(3-x(2-x))^2]dx$.

AB/BC 4. (a) $v_P(t)=6\left(\sqrt{1+8t}-1\right)$, so $v(0)=0$ and $v(10)=48$.

The average acceleration is $\frac{\Delta v}{\Delta t}=\frac{48-0}{10-0}=\frac{24}{5}\text{ft/sec}^2$.

Acceleration $a(t)=v'(t)=6\cdot\frac{1}{2}(1+8t)^{-\frac{1}{2}}(8)\text{ft/sec}^2$.

$$\frac{24}{\sqrt{1+8t}}=\frac{24}{5}\text{ when } t=3\text{sec}.$$

(b) Since Q's acceleration, for all t in $0\le t\le 5$, is the slope of its velocity graph, $a=\frac{20-0}{5-0}=4\text{ ft/sec}^2$.

(c) Find the distance each auto has traveled. For P, the distance is

$$\int_0^{10} 6\left(\sqrt{1+8t}-1\right)$$

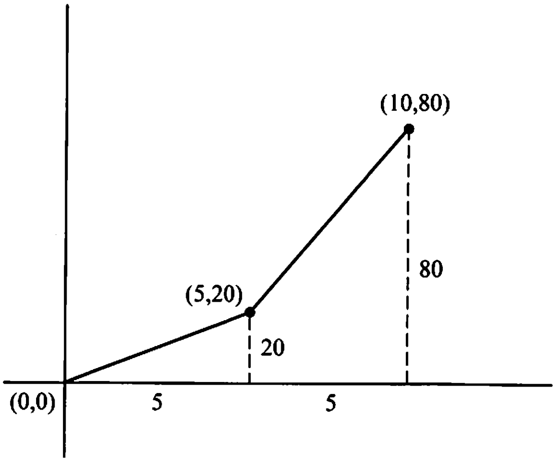
$$6\left[\frac{1}{8}\int_0^{10}\sqrt{1+8t}\cdot 8dt-\int_0^{10}dt\right]$$

$$6\left(\frac{1}{8}\cdot\frac{2}{3}(1+8t)^{\frac{3}{2}}-t\right)\Bigg|_0^{10},$$

$$6\left(\frac{1}{12}\left(81^{\frac{3}{2}}-1^{\frac{3}{2}}\right)-10\right)=304\text{ ft.}$$

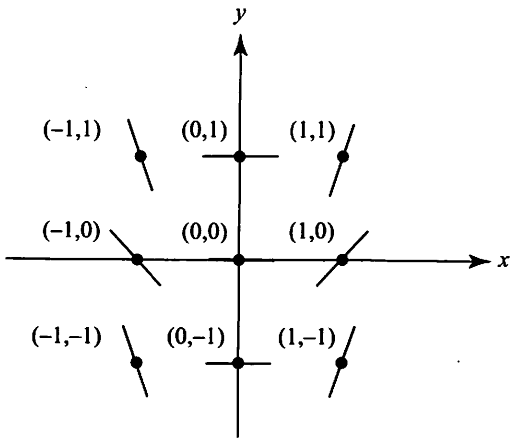
For auto Q, the distance is the total area of the triangle and trapezoid under the velocity graph shown below, namely,

$$\frac{1}{2}(5\cdot 20)+\frac{1}{2}(20+80)(5)=300\text{ft.}$$



Auto P won the race.

AB5. (a) Using the differential equation, evaluate the derivative at each point, then sketch a short segment having that slope. For example, at (–1,–1), $\frac{dy}{dx} = 2(-1)((-1)^2 + 1) - 4$; draw a steeply decreasing segment at (–1,–1). Repeat this process at each of the other points. The result follows.



(b) The differential equation $\frac{dy}{dx} = 2x(y^2 + 1)$ is separable.

$$\int \frac{dy}{y^2+1} = \int 2x\,dx$$
$$\arctan(y) = x^2 + c$$
$$y = \tan(x^2 + c)$$

It is given that f passes through (0,1), so $1 = \tan(0^2 + c)$ and $c = \frac{\pi}{4}$.

The solution is $f(x) = \tan\left(x^2 + \frac{\pi}{4}\right)$.

The particular solution must be differentiable on an interval containing the initial point (0,1). The tangent function has vertical asymptotes at $x = \pm\frac{\pi}{2}$, hence:

$-\frac{\pi}{2} < x^2 + \frac{\pi}{4} < \frac{\pi}{2}$. (Since $x^2 \geq 0$, we ignore the left inequality.)

$$x^2 < \frac{\pi}{4}$$

$$|x| < \frac{\sqrt{\pi}}{2}$$

AB 6. (a) $\int_0^2 f(t)\,dt = F(2) = 4$.

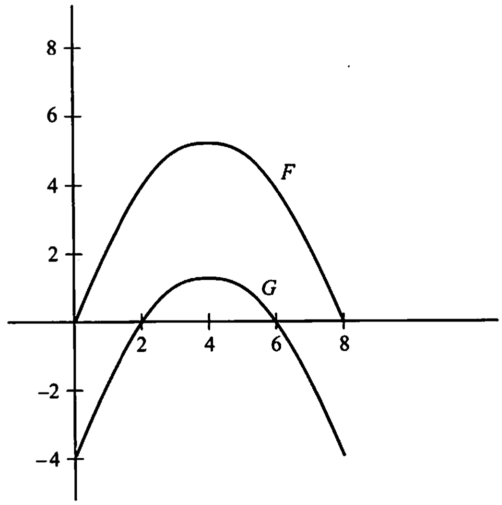
(b) One estimate might be $\int_2^7 f(t)\,dt = F(7) - F(2) = 2 - 4 = -2$.

(c) $f(x) = F'(x)$; $F'(x) = 0$ at $x = 4$.

(d) $f'(x) = F''(x)$. F'' is negative when F is concave downward, which is true for the entire interval $0 < x < 8$.

(e) $G(x) = \int_2^x f(t)\,dt$
$$= \int_0^x f(t)\,dt - \int_0^2 f(t)\,dt$$
$$= F(x) - 4.$$

Then the graph of G is the graph of F translated downward 4 units.



AB Practice Examination 3