AP Calculus AB
Chapter 2 Review

Name: (KEY)
Block: ______

1. [No Calculator] Evaluate each limit or explain why the limit does not exist.

a) $\lim\limits_{x\to 5} \operatorname{int}(x)$ $\boxed{\text{DNE}}$ b/c $\lim\limits_{x\to 5^-} \operatorname{int}(x) \neq \lim\limits_{x\to 5^+} \operatorname{int}(x)$

b) $\lim\limits_{x\to\infty} \dfrac{x^2+5x-3}{3x+2}$ $\boxed{\text{DNE}}$ grows w/o bound

c) $\lim\limits_{x\to\infty} \dfrac{x^2+5x-3}{3x^2+2} = \boxed{\dfrac{1}{3}}$

d) $\lim\limits_{x\to -2} \left(x^3-2x^2+1\right) =$ $(-2)^3-2(-2)^2+1 = -8-2(4)+1 = -8-8+1$ $\boxed{=-15}$

e) $\lim\limits_{x\to\infty} \dfrac{x^2+5x-3}{3x^3+2} = \boxed{0}$

f) $\lim\limits_{x\to 0} \dfrac{x}{\sin(2x)} \cdot \dfrac{2x}{2x} = \lim\limits_{x\to 0} \dfrac{\cancel{x}}{\cancel{2x}\cdot\frac{\sin(2x)}{2x}} = \lim\limits_{x\to 0} \dfrac{1}{2\cdot\frac{\sin(2x)}{2x}} = \dfrac{1}{2\cdot 1} = \boxed{\dfrac{1}{2}}$

g) $\lim\limits_{x\to\infty} \dfrac{\sin x}{2x} = \boxed{0}$

h) $\lim\limits_{x\to 0} \dfrac{\tan(5x)}{\sin(3x)} = \lim\limits_{x\to 0} \dfrac{\frac{\sin(5x)}{\cos(5x)}\cdot\frac{5x}{5x}}{\sin(3x)\cdot\frac{3x}{3x}} = \lim\limits_{x\to 0} \dfrac{\frac{\sin(5x)}{5x}\cdot 5\cancel{x}}{\cos(5x)\cdot\frac{\sin(3x)}{3x}\cdot 3\cancel{x}} = \dfrac{1\cdot 5}{1\cdot 1\cdot 3} = \boxed{\dfrac{5}{3}}$

i) $\lim\limits_{x\to 0} e^x \sin x = e^0\cdot\sin(0) = 1\cdot 0 = \boxed{0}$

j) $\lim\limits_{x\to 1} \dfrac{4x^2+5x}{x-3} = \dfrac{4(1)^2+5(1)}{1-3} = \boxed{-\dfrac{9}{2}}$

k) $\lim\limits_{x\to 7/2} \operatorname{int}(2x-1)$ $\boxed{\text{DNE}}$ b/c $\lim\limits_{x\to 7/2^-} \operatorname{int}(2x-1) \neq \lim\limits_{x\to 7/2^+} \operatorname{int}(2x-1)$

l) $\lim\limits_{x\to\infty} \dfrac{5x-7x^2}{4x^2+1} = \boxed{-\dfrac{7}{4}}$

m) $\lim\limits_{x\to -3} \dfrac{|x+3|}{x+3}$ $\boxed{\text{DNE}}$ b/c $\lim\limits_{x\to -3^-} \dfrac{|x+3|}{x+3} \neq \lim\limits_{x\to -3^+} \dfrac{|x+3|}{x+3}$

n) $\lim\limits_{x\to\infty} \dfrac{x^4+x^3}{2x^3+128}$ $\boxed{\text{DNE}}$ b/c grows w/o bound

o) $\lim\limits_{x\to 4} \sqrt{1-2x} = \boxed{\text{DNE}}$ b/c the function is only defined for $x \le 1/2$

$1-2x \ge 0$
$-2x \ge -1$
$x \le 1/2$

p) $\lim\limits_{x\to 0} \dfrac{\frac{1}{x+5}-\frac{1}{5}}{x} = \lim\limits_{x\to 0} \dfrac{5-1(x+5)}{(x+5)(5)}\cdot\dfrac{1}{x} = \lim\limits_{x\to 0} \dfrac{-\cancel{x}}{(x+5)(5)}\cdot\dfrac{1}{\cancel{x}} = \lim\limits_{x\to 0} \dfrac{-1}{(x+5)(5)} = \dfrac{-1}{(0+5)(5)} = \boxed{-\dfrac{1}{25}}$

q) $\lim\limits_{x\to 9} \dfrac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} = \lim\limits_{x\to 9} \dfrac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)} = \dfrac{1}{\sqrt{9}+3} = \boxed{\dfrac{1}{6}}$

r) $\lim\limits_{x\to 2} \dfrac{x^2-x-2}{x^2+x-6} = \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x+1)}{(x+3)\cancel{(x-2)}} = \dfrac{2+1}{2+3} = \boxed{\dfrac{3}{5}}$

2. [Calculator] Use a table of values to evaluate the following limit: $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x$

Recognize the number? … Add this to your notecards under "Limits you should know". … yeah … those things you have to hand in before your test! ☺

x	100	1,000	10,000	100,000	1,000,000	1,000,000,000
$\left(1+\frac{1}{x}\right)^x$	2.7048	2.7169	2.7181	2.7183	2.7183	2.7183

$\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

$e \approx 2.718$

3. [Calculator] Make a table of values (4 of them would work) to evaluate $\lim\limits_{x\to 2^+} \dfrac{x+3}{x-2}$.

You are only approaching 2 from the right!

x	$\frac{x+3}{x-2}$
2.1	51
2.01	501
2.001	5001
2.0001	50001

$\lim\limits_{x\to 2^+} \dfrac{x+3}{x-2}$ DNE b/c $\dfrac{x+3}{x-2}\to\infty$ (grows w/o bound)

4. [No Calculator] Suppose $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist, and

$$\lim_{x\to c}[f(x)+g(x)]=2 \Rightarrow \boxed{\lim_{x\to c} f(x)} + \boxed{\lim_{x\to c} g(x)} = 2$$
$$\lim_{x\to c}[f(x)-g(x)]=1 \quad + \left(\boxed{\lim_{x\to c} f(x)} - \boxed{\lim_{x\to c} g(x)} = 1\right)$$

Find $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$.

$$2\lim_{x\to c} f(x) = 3$$
$$\boxed{\lim_{x\to c} f(x) = 3/2}$$
$$3/2 + \lim_{x\to c} g(x) = 2$$
$$\boxed{\lim_{x\to c} g(x) = \tfrac{1}{2}}$$

5. [No Calculator] Find all asymptotes for each function and justify your response.

a) $y=\ln x$ $\boxed{x=0 \mid VA}$

Parent function of logarithm

b) $f(x)=\dfrac{(\cancel{x+2})(x-3)}{(\cancel{x+2})(x-1)}$ $\boxed{VA \mid x=1}$

Hole @ $x=-2$

$\lim_{x\to\infty} \frac{x-3}{x-1} = 1$

$\boxed{\therefore y=1 \text{ is a HA}}$

c) $f(x)=\dfrac{x-1}{x^2(x+2)}$ $\boxed{VA \mid x=0,\ x=-2}$

$\boxed{HA \mid y=0}$ $\lim_{x\to\infty} \frac{x-1}{x^2(x+2)} = \lim_{x\to\infty} \frac{x-1}{x^3+2x} = 0$

d) $f(x)=\dfrac{x^3-3x^2+x-1}{x^2+x-2}$

$f(x)=\dfrac{x^3-3x^2+x-1}{(x+2)(x-1)}$ $\boxed{VA \mid x=-2,\ x=1}$

no HA

$$\begin{array}{r} x-4 \\ x^2+x-2 \overline{\big)\, x^3-3x^2+x-1} \\ -(x^3+x^2-2x) \\ \hline -4x^2+3x-1 \\ -(-4x^2-4x+8) \\ \hline 7x-9 \end{array}$$

$\boxed{SA \mid y=x-4}$

6. [No Calculator] Let $y=\dfrac{x^2+5x-3}{x-2}$.

a) Find the End Behavior Model $= \frac{x^2}{x} = x$

b) Describe the End Behavior using limits.

$\lim_{x\to\infty} \frac{x^2+5x-3}{x-2} =$ DNE b/c $\frac{x^2+5x-3}{x-2} \to +\infty$ as $x\to\infty$

& $\frac{x^2+5x-3}{x-2} \to -\infty$ as $x \to -\infty$

c) Find all asymptotes.

$\boxed{VA \mid x=2}$ no H.A.

$$\begin{array}{r} x+7 \\ x-2 \overline{\big)\, x^2+5x-3} \\ -(x^2-2x) \\ \hline 7x-3 \\ -(7x-14) \\ \hline 11 \end{array}$$

$\boxed{SA \mid y=x+7}$

7. The number of bears in a federal wildlife reserve is given by the population $p(t)=\dfrac{200}{1+7e^{-0.1t}}$, where t is in years.

a) Find $p(0)$ and give a possible interpretation of this number.

$p(0) = \frac{200}{1+7e^{-.1(0)}} = \frac{200}{1+7} = \frac{200}{8} = 25$ There are 25 bears in the reserve in year $t=0$. (at the beginning)

b) Find $\lim_{t\to\infty} p(t)$ and give a possible interpretation of this number.

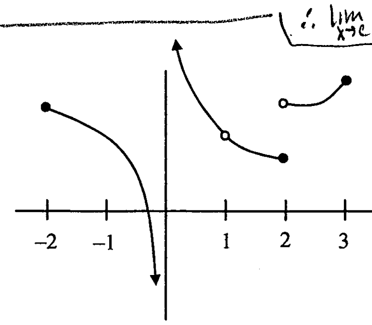
$\lim_{t\to\infty} \frac{200}{1+7e^{-.1t}} = 200$ b/c as $t\to\infty$, $e^{-.1t} \to 0$ so $\frac{200}{1+7e^{-.1t}} \to \frac{200}{1+0} = 200$.

8. Let $h(x)=\dfrac{(x-1)(\cancel{x+3})}{(\cancel{x+3})(x-2)}$. Identify all values of c where the $\lim_{x\to c} h(x)$ EXISTS.

Hole @ $x=-3$ So $\lim_{x\to -3} h(x)$ exists, but $\lim_{x\to 2} h(x)$ DNE b/c there is a VA @ $x=2$

$\boxed{\therefore \lim_{x\to c} h(x) \text{ exists for all values of } c \text{ except } c=2}$

9. The function shown to the right is defined on [–2, 3]. For what values of c, does $\lim_{x\to c} f(x)$ exist?



DNE @ $x=0$ & $x=2$

$\boxed{\lim_{x\to c} f(x) \text{ exists on } (-2,0)\cup(0,2)\cup(2,3)}$

15. [No Calculator] Find the value of the parameter that would make each function continuous. Justify your response *using the definition of continuity*.

a) $j(x)=\begin{cases} ax^2 & ;x<1 \\ 4x-2 & ;x\geq 1\end{cases}$

Need $\lim_{x\to 1^-} j(x) = \lim_{x\to 1^+} j(x) = j(1)$

$a(1)^2 = 4(1)-2 = 4(1)-2$

$\boxed{a=2}$

b) $k(x)=\begin{cases} \frac{\sin 3x}{x} & x\neq 0 \\ a & x=0\end{cases}$

Need $\lim_{x\to 0} k(x) = k(0)$

$\lim_{x\to 0} \frac{\sin(3x)\cdot 3}{x\cdot 3} = a$

$1\cdot 3 = a$

$\boxed{3=a}$

c) $f(x)=\begin{cases} \frac{x^2-2x}{x} & \text{if } x\neq 0 \\ b & \text{if } x=0\end{cases}$

Need $\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{x^2-2x}{x} = b$

$\lim_{x\to 0} \frac{\cancel{x}(x-2)}{\cancel{x}} = b$ $\quad \boxed{-2=b}$

$0-2=b$

d) $f(x)=\begin{cases} \frac{x^2-2x-3}{x-3} & \text{if } x<3 \\ kx-2 & \text{if } x\geq 3\end{cases}$

Need $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

$\lim_{x\to 3^-} \frac{x^2-2x-3}{x-3} = k(3)-2 = k(3)-2$

$\lim_{x\to 3^-} \frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}} = 3k-2$

$3+1 = 3k-2$

$4=3k-2$

$6=3k$

$\boxed{2=k}$

16. [No Calculator] Let $k(x)=\frac{\sqrt{x}-3}{x-9}$. Write an extension to the function so that it is continuous at $x=9$.

To be continuous at $x=9$,

$\lim_{x\to 9} k(x) = k(9)$

$\lim_{x\to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} = k(9)$

$\lim_{x\to 9} \frac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)} = k(9)$

$\frac{1}{\sqrt{9}+3} = k(9)$

$\frac{1}{6} = k(9)$

$\boxed{k(x)=\begin{cases} \frac{\sqrt{x}-3}{x-9} & \text{if } x\neq 9 \\ \frac{1}{6} & \text{if } x=9\end{cases}}$

17. [No Calculator] Find the average rate of change of $f(x)=3-\sin x$ over the interval $[0,\frac{\pi}{2}]$.

slope between 2 pts

$\frac{f(\pi/2)-f(0)}{\pi/2-0} = \frac{3-2}{\pi/2} = \frac{1}{\pi/2} = \boxed{\frac{2}{\pi}}$

$f(0) = 3-\sin(0) = 3-0 = 3$

$f(\pi/2) = 3-\sin(\pi/2) = 3-1 = 2$

18. [No Calculator] Find the rate of change of the Surface Area $A=6s^2$ of a cube with respect to the edge length s when $s=3$.

INSTANTANEOUS!

$\lim_{h\to 0} \frac{A(3+h)-A(3)}{h} = \lim_{h\to 0} \frac{[6(3+h)^2]-[6(3)^2]}{h} = \lim_{h\to 0} \frac{6(9+6h+h^2)-6(9)}{h} = \lim_{h\to 0} \frac{\cancel{54}+36h+6h^2-\cancel{54}}{h}$

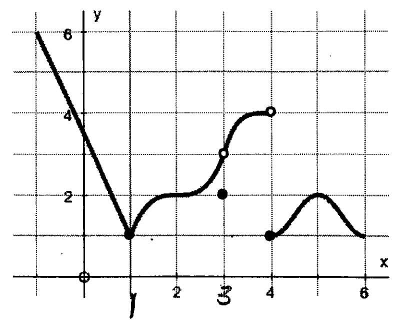
$= \lim_{h\to 0} \frac{\cancel{h}(36+6h)}{\cancel{h}} = \lim_{h\to 0} (36+6h) = 36+6(0) = \boxed{36}$

19. [No Calculator] Let $g(x)=\sqrt{x}$. Find the instantaneous slope at $x=4$.

$\lim_{h\to 0} \frac{g(4+h)-g(4)}{h} = \lim_{h\to 0} \frac{\sqrt{4+h}-\sqrt{4}}{h} = \lim_{h\to 0} \frac{(\sqrt{4+h}-2)(\sqrt{4+h}+2)}{(h)(\sqrt{4+h}+2)} = \lim_{h\to 0} \frac{(4+h)-(4)}{h(\sqrt{4+h}+2)} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$

$= \lim_{h\to 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{\sqrt{4+0}+2} = \boxed{\frac{1}{4}}$

For questions 10 - 13, use the function shown to the right. **The domain is [–1, 6].**



10. Evaluate each of the following limits. If they do not exist, explain why.

a) $\lim_{x\to 1} f(x) = 1$

b) $\lim_{x\to 3^-} f(x) = 3$

c) $\lim_{x\to 3^+} f(x) = 3$

d) $\lim_{x\to 3} f(x) = 3$

e) $\lim_{x\to 4^+} f(x) = 1$

f) $\lim_{x\to 4^-} f(x) = 4$

g) $\lim_{x\to 4} f(x)$ DNE b/c the $\lim_{x\to 4^-} f(x) \neq \lim_{x\to 4^+} f(x)$

h) $\lim_{x\to 0} f(x) = 3.5$

11. For what values of x is the function continuous?

$(-1, 3) \cup (3, 4) \cup (4, 6)$

Continuous on the Right at $x = -1$
Continuous on the Left at $x = 6$

12. For what values of x is the function not continuous?

$\boxed{x = 3 \ \& \ x = 4}$

13. Are any of the values you used to answer question ~~11~~ $\boxed{12}$ removable? If so, describe how you would make the function continuous at that point?

Removable at $x = 3$

Redefine $f(3) = 3$ so that the function would be continuous at $x = 3$.

14. [No Calculator] Let $f(x) = \begin{cases} 2 & \text{if } x \leq -1 \\ -x+1 & \text{if } -1 < x < 0 \\ 2 & \text{if } x = 0 \\ -x+1 & \text{if } 0 < x < 1 \\ 2 & \text{if } x \geq 1 \end{cases}$.

a) Find the right-hand and left-hand limits of f at $x = -1$, 0, and 1.

$\lim_{x\to -1^-} f(x) = 2$

$\lim_{x\to -1^+} f(x) = -(-1) + 1 = 2$

$\lim_{x\to 0^-} f(x) = -(0) + 1 = 1$

$\lim_{x\to 0^+} f(x) = -(0) + 1 = 1$

$\lim_{x\to 1^-} f(x) = -(1) + 1 = 0$

$\lim_{x\to 1^+} f(x) = 2$

b) Does f have a limit as x approaches –1? 0? 1? If so, what is it? If not, why not?

$\lim_{x\to -1} f(x) = 2$ $\quad \lim_{x\to 0} f(x) = 1$ $\quad$ BUT $\lim_{x\to 1} f(x)$ DNE b/c the Left and Right limits are not equal.

c) Is f continuous at $x = -1$? 0? 1? Explain.

$f(-1) = 2$ $\quad f(0) = 2$ $\quad f(1) = 2$

Continuous at $x = -1$ since $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) = f(-1)$

NOT continuous at $x = 0$ b/c $\lim_{x\to 0} f(x) \neq f(0)$

Not continuous at $x = 1$ b/c $\lim_{x\to 1} f(x)$ DNE

$$y(a+h) = (a+h)^3 - 4(a+h)$$
$$= a^3 + 3a^2h + 3ah^2 + h^3 - 4a - 4h$$

20. [No Calculator] Let $y = x^3 - 4x$.

a) Find the instantaneous slope for any value of $x = a$.

$$\lim_{h\to 0} \frac{y(a+h) - y(a)}{h} = \lim_{h\to 0} \frac{(\cancel{a^3} + 3a^2h + 3ah^2 + h^3 - \cancel{4a} - 4h) - (\cancel{a^3} - \cancel{4a})}{h} = \lim_{h\to 0} \frac{3a^2h + 3ah^2 + h^3 - 4h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3a^2 + 3ah + h^2 - 4)}{\cancel{h}} = \lim_{h\to 0}(3a^2 + 3ah + h^2 - 4) = 3a^2 + 3a(0) + (0)^2 - 4 = \boxed{3a^2 - 4}$$

"Slope at $x = a$"

b) Use your answer in part *a* to find the slope at $x = -1$.

Slope at $x = -1$ } $= 3(-1)^2 - 4 = 3 - 4 = \boxed{-1}$

c) Find the equation of the tangent line when $x = -1$.

Point $(-1, 3)$ & Slope at $x = -1 \to -1$

$y(-1) = (-1)^3 - 4(-1) = -1 + 4 = 3$

$\boxed{y - 3 = -1(x+1)}$

d) Find the equation of the normal line when $x = -1$.

$\perp$ Slope $= +1$ $\boxed{y - 3 = +1(x+1)}$

21. [No Calculator] Let $k(x) = x^2 + 3x - 1$.

$k(a+h) = (a+h)^2 + 3(a+h) - 1 = a^2 + 2ah + h^2 + 3a + 3h - 1$

a) Find the slope of the curve at $x = 1$.

$$\lim_{h\to 0} \frac{k(a+h) - k(a)}{h} = \lim_{h\to 0} \frac{(\cancel{a^2} + 2ah + h^2 + \cancel{3a} + 3h - \cancel{1}) - (\cancel{a^2} + \cancel{3a} - \cancel{1})}{h} = \lim_{h\to 0} \frac{2ah + h^2 + 3h}{h} = \lim_{h\to 0} \frac{\cancel{h}(2a + h + 3)}{\cancel{h}}$$

$$= \lim_{h\to 0}(2a + h + 3) = 2a + 3$$

b) Find the equation of the tangent line at the point when $x = 1$.

Slope when $x = 1 > 2(1) + 3 = 5$

Point $(1, 3)$ $\boxed{y - 3 = 5(x-1)}$

c) Find the equation of the normal line at the point when $x = 1$.

$\perp$ Slope $= -\frac{1}{5}$ $\boxed{y - 3 = -\frac{1}{5}(x-1)}$

d) At what point(s), if any, are the tangents to the graph of $k(x)$ horizontal? … [Use Calculus!]

Slope $= 0$

$2a + 3 = 0$

$2a = -3$

$a = -3/2$

when $x = -3/2$, $k(-3/2) = (-\frac{3}{2})^2 + 3(\frac{3}{2}) - 1 = \frac{9}{4} + \frac{9}{2} - 1 = \frac{9 + 18 - 4}{4} = 23/4$

$\boxed{\text{The graph of } k(x) \text{ is horizontal at the point } (-3/2, 23/4)}$

22. [No Calculator] Let $g(x) = \dfrac{x^2 + 5x + 6}{x^2 + 3x + 2} = \dfrac{(x+2)(x+3)}{(x+2)(x+1)}$

$x \neq -2$
$x \neq -1$
Hole @ $x = -2$

a) Find the domain of $g(x)$.

$(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$

b) Find the $\lim_{x\to c} g(x)$ for all values of c where $g(x)$ is not defined. (aka… $x = -2$ & $x = -1$)

$$\lim_{x\to -2} \frac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x+1)} = \frac{-2+3}{-2+1} = \frac{1}{-1} = \boxed{-1}$$

But $\lim_{x\to -1} \frac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x+1)} = \frac{-1+3}{-1+1} = \frac{2}{0}$ DNE

"goes w/o bound"
VA @ $x = -1$

c) Find all asymptotes and justify your response.

VA $x = -1$ HA $y = 1$ b/c $\lim_{x\to\infty} \frac{x^2 + 5x + 6}{x^2 + 3x + 2} = 1$

d) Write an extension to the function so that $g(x)$ is continuous for all $x < -1$.

$$g(x) = \begin{cases} \dfrac{x^2 + 5x + 6}{x^2 + 3x + 2} & \text{if } x \neq -2 \text{ \& } x \neq -1 \\ -1 & \text{if } x = -2 \end{cases}$$