AP Calculus
2.1 Worksheet Day 2

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. When evaluating limits, what does it mean if direct substitution gives you $\frac{7}{0}$?

2. When evaluating limits, what does it mean if direct substitution gives you $\frac{0}{0}$?

3. What are the methods (options) for dealing with the result $\frac{0}{0}$?

4. Evaluate the following limits algebraically.

a) $\lim\limits_{x\to 3}\dfrac{x^2-x-6}{x-3}$

b) $\lim\limits_{x\to 0}\dfrac{\frac{1}{2+x}-\frac{1}{2}}{x}$

c) $\lim\limits_{x\to 0}\dfrac{\sqrt{2x+1}-1}{x}$

d) $\lim\limits_{x\to 4}\dfrac{\sqrt{x+5}-3}{x-4}$

e) $\lim\limits_{x\to 1}\dfrac{x-1}{x^2-1}$

f) $\lim\limits_{x\to 0}\dfrac{(4+x)^2-16}{x}$

g) $\lim\limits_{t\to 2}\dfrac{t^2-3t+2}{t^2-4}$

h) $\lim\limits_{x\to 0}\dfrac{(2+x)^3-8}{x}$

One of the limits you should know is $\lim\limits_{x\to 0}\frac{\sin x}{x} =$ ______ . This limit ONLY works when the denominator matches the inside of the sine function. If they do not match, you cannot change the inside of a sine function without a trig identity. Your goal will be to correctly show the algebra in order to use this limit.

5. Evaluate each of the following limits analytically. *Be sure to show ALL steps in your evaluation.*

a) $\lim\limits_{x\to 0}\frac{\sin x}{5x}$

b) $\lim\limits_{x\to 0}\frac{\sin 5x}{x}$

c) $\lim\limits_{x\to \pi/4}\frac{\sin\left(x-\pi/4\right)}{x-\pi/4}$

d) $\lim\limits_{x\to 0}\frac{3\sin 4x}{\sin 3x}$

6. Evaluate each of the following by combining properties of limits and your algebra skills.

a) $\lim\limits_{x\to 0}\frac{x+\sin x}{x}$

b) $\lim\limits_{x\to 0}\frac{\tan x}{x}$

c) $\lim\limits_{x\to 0}\frac{\sin x}{2x^2-x}$

d) $\lim\limits_{x\to 0}\frac{\sin^2 x}{x}$

7. Consider $\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x^2} =$

a) If you use direct substitution, what result do you get?

b) Evaluate the limit if $f(x)=2x^2+1$.

8. If $a\neq 0$, then $\lim\limits_{x\to a}\frac{x^2-a^2}{x^4-a^4} =$

9. Evaluate the following limits analytically (all mixed up):

a) $\lim\limits_{x\to 0}\dfrac{\frac{3}{4+x}-\frac{3}{4}}{x}$

b) $\lim\limits_{x\to 0}\dfrac{5x^3+8x^2}{3x^4-16x^2}$

c) $\lim\limits_{x\to 3}\dfrac{\sqrt{x+1}}{x-4}$

d) $\lim\limits_{x\to 0}\dfrac{x^2-3x}{x}$

e) $\lim\limits_{x\to 1}\dfrac{x}{x^2-x}$

f) $\lim\limits_{x\to 0}\dfrac{\sin 2x}{x}$

g) $\lim\limits_{x\to 0}\dfrac{\sin 7x}{3x}$

h) $\lim\limits_{x\to 4}\dfrac{x^2-5x+4}{x^2-2x-8}$

12. Evaluate $\lim\limits_{h\to 0}\dfrac{(x+h)^2-x^2}{h}$.

♬: *h* is going to 0 … not *x* … so treat this as if *h* is the variable … your final answer will have a *x* in it.

13. Suppose $g(x)=\begin{cases}2-x, & \text{if } x\le 1\\ \frac{x}{2}+1, & \text{if } x>1\end{cases}$.

a) $\lim\limits_{x\to 1^-} g(x)=$

b) $\lim\limits_{x\to 1^+} g(x)=$

c) $\lim\limits_{x\to 1} g(x)=$

d) $g(1)=$

AP Calculus
2.2 Worksheet

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1. Answer the following questions:

   a) How do you find horizontal asymptotes?

   b) Which of the parent functions have horizontal asymptotes? List the function(s) and asymptote(s)

   c) How do you find vertical asymptotes?

   d) Which of the parent functions have vertical asymptotes? List the function(s) and asymptote(s)

   e) When must you look for oblique (slanted) asymptotes? How do you find them?

2. For each of the following, find (i) $\lim\limits_{x\to\infty} f(x)$ and (ii) $\lim\limits_{x\to-\infty} f(x)$. Then (iii) identify all horizontal asymptotes, if any.

   a) $f(x)=\dfrac{x-2}{2x^2+3x-5}$ b) $f(x)=\dfrac{4x^3-2x+1}{x^2-2x+1}$ c) $f(x)=\dfrac{3x^2-x+5}{x^2-4}$

   d) $f(x)=\dfrac{e^{-x}}{x}$ e) $f(x)=\dfrac{|x|}{x}$ f) $f(x)=\dfrac{\sin x}{2x^2+x}$

3. One of the functions in 2a – 2c has a slanted (oblique) asymptote. Explain why, and then find the asymptote.

4. For each of the following, (i) find the vertical asymptotes of the graph of $f(x)$ and (ii) describe the behavior of the graph of $f(x)$ to the left and right of each asymptote.

a) $f(x)=\dfrac{1}{x-3}$ b) $f(x)=\dfrac{1}{x^2-4}$ c) $f(x)=\dfrac{1-x}{2x^2-5x-3}$

5. Find the limit of $g(x)$ as (i) $x\to\infty$, (ii) $x\to-\infty$, (iii) $x\to0^-$, and (iv) $x\to0^+$

a) $g(x)=\begin{cases}\frac{1}{x} & \text{if } x<0\\ \frac{2x-3}{x+1} & \text{if } x\geq 0\end{cases}$ b) $g(x)=\begin{cases}\frac{3x}{x+1} & \text{if } x\leq 0\\ \frac{1}{x^2} & \text{if } x>0\end{cases}$

6. Sketch a **function** that satisfies the stated conditions. Include any asymptotes.

$\lim\limits_{x\to1} f(x)=2$ $\lim\limits_{x\to5^-} f(x)=\infty$

$\lim\limits_{x\to5^+} f(x)=\infty$ $\lim\limits_{x\to\infty} f(x)=-1$

$\lim\limits_{x\to-\infty} f(x)=0$ $\lim\limits_{x\to-2^-} f(x)=\infty$

$\lim\limits_{x\to-2^+} f(x)=-\infty$

7. Sketch a **function** that satisfies the stated conditions. Include any asymptotes.

$\lim\limits_{x\to2} f(x)=-1$ $\lim\limits_{x\to4^+} f(x)=-\infty$

$\lim\limits_{x\to4^-} f(x)=\infty$ $\lim\limits_{x\to\infty} f(x)=\infty$

$\lim\limits_{x\to-\infty} f(x)=2$

8. Explain why there is no value L for which $\lim\limits_{x\to\infty}\sin x=L$.

9. Let $f(x)=\dfrac{\cos x}{x}$.

a) Find the domain and range of *f*.

b) Is *f* even, odd, or neither? Justify your response.

c) Find $\lim\limits_{x\to\infty} f(x)$. Give a reason for your answer.

10. If *k* is a positive integer, then $\lim\limits_{x\to\infty}\dfrac{x^k}{e^x}=$? <u>Explain</u> your answer.
[Try letting k = 2 … what about k = 10? … what about k = 1000?]

11. ***<u>Investigate</u>*** using tables and graphs to determine the value of each limit: $\lim\limits_{x\to\infty}\dfrac{3x-2}{\sqrt{2x^2+1}}$ and $\lim\limits_{x\to-\infty}\dfrac{3x-2}{\sqrt{2x^2+1}}$

12. Evaluate each of the following limits using all methods learned from this chapter.

a) $\lim\limits_{x\to\infty}\left(\dfrac{2}{x}+1\right)\left(\dfrac{5x^2-1}{x^2}\right)$

b) $\lim\limits_{n\to\infty}\dfrac{3n^3-5n}{n^3-2n^2+1}=$

c) $\lim\limits_{x\to\infty}\left(5-\dfrac{2}{x^2}\right)=$

d) $\lim\limits_{x\to\pi/2}\sec x$

e) $\lim\limits_{x\to\infty} e^{-x}\cos x$

f) $\lim\limits_{x\to 7/2^+}\operatorname{int}(2x-1)$

g) $\lim_{x\to\infty} \dfrac{\cos\left(\frac{1}{x}\right)}{1+\frac{1}{x}}$

h) $\lim_{x\to\infty} \dfrac{4n^3}{n^2+10000n} =$

i) $\lim_{x\to 0} \dfrac{\sin 2x}{4x}$

j) $\lim_{x\to 0} \dfrac{\frac{1}{2+x}-\frac{1}{2}}{x}$

k) $\lim_{x\to\infty} \dfrac{x\sin x+2\sin x}{2x^2}$

l) $\lim_{x\to -2} \dfrac{x^2+1}{3x^2-2x+5}$

AP Calculus
2.3 Worksheet

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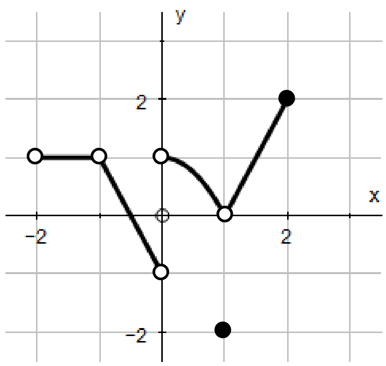
1. What is the definition of continuity?

2. Sketch a possible graph for each function described.

a) $f(5)$ exists, but $\lim_{x \to 5} f(x)$ does not exist.

b) The $\lim_{x \to 5} f(x)$ exists, but $f(5)$ does not exist.

3. Use the function $g(x)$ defined and graphed below to answer the following questions.



$$g(x) = \begin{cases} 1 & \text{if } -2 < x < -1 \\ -2x - 1 & \text{if } -1 < x < 0 \\ 1 - x^2 & \text{if } 0 < x < 1 \\ -2 & \text{if } x = 1 \\ 2x - 2 & \text{if } 1 < x \leq 2 \end{cases}$$

a) Does $g(1)$ exist?

b) Does $\lim_{x \to 1} g(x)$ exist?

c) Does $\lim_{x \to 1} g(x) = g(1)$?

d) Is g continuous at $x = 1$?

e) Is g defined at $x = –1$?

f) Is g continuous at $x = –1$?

g) For what values of x is g continuous?

h) What value should be assigned to $g(–1)$ to make the extended function continuous at $x = –1$?

i) What new value should be assigned to $g(1)$ to make the new function continuous at $x = 1$?

j) Is it possible to extend g to be continuous at $x = 0$? If so, what value should the extended function have there? If not, why not?

4. Let $f(x)=\begin{cases} x^2-1 & ;x<3 \\ 2ax & ;x\geq 3 \end{cases}$. Find a value of *a* so that the function f is continuous.

Using the definition of continuity, justify your response.

5. Let $f(x)=\begin{cases} 2x+3 & ;x\leq 2 \\ kx+1 & ;x>2 \end{cases}$. Find a value of *k* so that the function f is continuous.

Using the definition of continuity, justify your response.

6. Let $f(x)=\begin{cases} x^2-a^2x & ;x<2 \\ 4-2x^2 & ;x\geq 2 \end{cases}$. Find all values of *a* that make f continuous at 2.

Using the definition of continuity, justify your response.

7. If $f(x)=\begin{cases} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} & \text{if } x\neq 2 \\ k+3 & \text{if } x=2 \end{cases}$, and if f is continuous at $x = 2$, then $k = ?$

Using the definition of continuity, justify your response.

8. If the function f is continuous for all real numbers and if $f(x)=\dfrac{x^2-4}{x+2}$, when $x\neq -2$, then $f(-2) =$

Use the definition of continuity to justify your response.

9. Let f be the function defined by the following:

$$f(x)=\begin{cases}\sin x, & x<0\\ x^2, & 0\le x<1\\ 2-x, & 1\le x<2\\ x-3, & x\ge 2\end{cases}$$

For what values of x is f NOT continuous? Use the definition of continuity to explain why.

10. Determine the points of discontinuity and identify their type for each of the following functions:

a) $y=\dfrac{1}{(x+2)^2}$ b) $y=\dfrac{x-1}{x^2-4x+3}$ c) $y=\dfrac{|x|}{x}$

11. Write an extended function so that the given function is continuous at the indicated point.

a) $h(x)=\dfrac{\sin(5x)}{x}$ at $x=0$ b) $k(x)=\dfrac{x-4}{\sqrt{x}-2}$ at $x=4$

12. Let f be the function given by $f(x)=\dfrac{(x-1)(x^2-4)}{x^2-a}$. For what positive values of a is f continuous for all real numbers?

A) None B) 1 only C) 2 only D) 4 only E) 1 and 4

13. Let $g(x)=\dfrac{x^2+5x+6}{x^2+7x+10}$.

a) Find the domain of $g(x)$.

b) Find the $\lim_{x\to c} g(x)$ for all values of c where $g(x)$ is not defined.

c) Find any horizontal asymptotes and justify your response.

d) Find any vertical asymptotes and justify your response.

e) Write an extension to the function so that $g(x)$ is continuous at $x = -2$.
<u>Use the definition of continuity</u> to justify your response.

14. Without using a picture, give a <u>written</u> explanation of why the function $f(x)=x^2-4x+3$ has a zero in the interval [2, 4].

15. Without using a picture, give a <u>written</u> explanation of why the function $f(x)=x^2+2x-3$ must equal 3 at least once in the interval [0, 2].

16. Let $h(x)=\begin{cases} 3x^2-4, & \text{if } x\le 2 \\ 5+4x, & \text{if } x>2 \end{cases}$.

a) What is $h(0)$?

b) What is $h(4)$?

c) On the interval [0, 4], there is no value of x such that $h(x) = 10$ even though $h(0) < 10$ and $h(4) > 10$. Explain why this result does not contradict the IVT.

AP Calculus
2.4 Worksheet

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1. What is a difference quotient?

2. How do you find the slope of a curve (aka slope of the tangent line to a curve) when $x = a$?

3. What is a *normal line* ?

4. What is the difference between the AVERAGE RATE OF CHANGE and INSTANTANEOUS RATE OF CHANGE?

5. Find the average rate of change of each function over the indicated interval.

   a) $h(x) = e^x$ on $[-2, 0]$

   b) $k(x) = 2 + \sin x$ on $[-\pi/2, \pi/2]$

   c) $f(x) = x^2 - x$ on $[1, 3]$

6. Let $f(x) = x^3$.

   a) Write and simplify an expression for $f(a+h)$.

   b) Find the slope of the curve at $x = a$.

   c) When does the slope equal 12?

   d) Write the equation of the tangent line to the curve at $x = 4$ ______________________________

   e) Write the equation of the normal line to the curve at $x = 4$ ______________________________

7. Let $g(x) = \sqrt{x}$

a) Find the average rate of change from $x = 4$ to $x = 9$.

b) Find the instantaneous rate of change at $x = 9$.

c) Write the equation of the tangent line when $x = 9$

d) Write the equation of the normal line when $x = 9$.

8. Let $y = \dfrac{1}{x-1}$. Find the slope of the curve at $x = 2$. Using this slope, write the equation of the tangent line and the equation of the normal line at that point.

9. Let $y = x^2 - 3x - 2$. Find the slope of the curve at $x = 0$. Using this slope, write the equation of the tangent line and the equation of the normal line at that point.

10. Find an equation of the tangent line to the graph of $f(x) = \frac{3}{x}$ at $x = 1$.

11. An object is dropped from the top of a 150-m tower. It's height above the ground after t seconds is $150 - 4.9t^2$ m. How fast is the object falling 2 seconds after it is dropped?

12. What is the rate of change of the area of a circle with respect to the radius when the radius is 4 in?

13. At what point is the tangent line to $h(x) = x^2 - 6x + 1$ horizontal?