AP Calculus
4.1 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

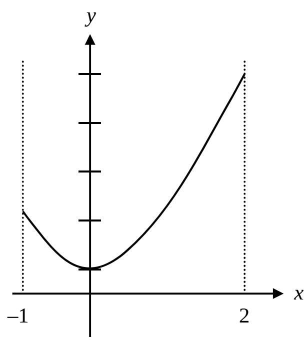
1. Earlier this year we had the Intermediate Value Theorem (IVT) and now we have the Extreme Value Theorem (EVT). The *hypothesis* of each theorem is what is needed to apply each theorem.

a) What is the hypothesis of the IVT (i.e. what is needed to apply the IVT) ?

b) What is the hypothesis of the EVT?

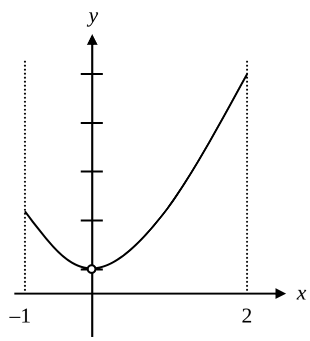
2. Using the graphs provided, find the minimum and maximum values on the given interval. *If there is no maximum or minimum value*, explain which part of the hypothesis of the Extreme Value Theorem is not satisfied.

(a) [–1, 2]



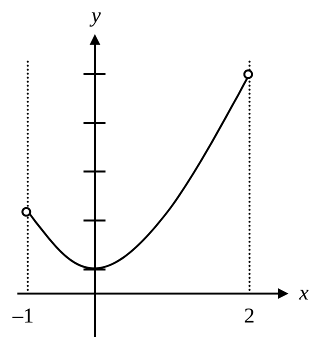
Max: __________
Min: __________

(b) [–1, 2]



Max: __________
Min: __________

(c) (–1, 2)



Max: __________
Min: __________

3. When looking for extrema, where do you find the candidates for the “candidates test”?

Each of the following statements is NOT ALWAYS TRUE. Explain/Show why each statement is false.

4. If $f'(5)=0$, then there is a maximum or a minimum at $x = 5$.

5. If $x = 2$ is a critical number, then $f'(2)=0$.

6. An extrema occurs at every critical number.

7. If m is a local minimum and M is a local maximum of a continuous function, then $m < M$.

8. If f is a continuous, decreasing function on [0, 10] with a critical point at (4, 2), which of the following statements MUST BE FALSE?

A $f(10)$ is an absolute minimum of f on [0, 10]
B $f(4)$ is neither a relative maximum nor a relative minimum
C $f'(4)$ does not exist.
D $f'(4)=0$
E $f'(4)<0$

9. Find the extrema on each interval and where they occur … Use a "candidates test".

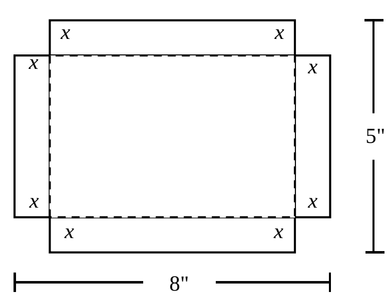
a) $f(x)=\frac{1}{x}+\ln x$ when $0.5\le x\le 4$

b) $g(x)=\ln(x+1)$ when $0\le x\le 3$

c) $k(x)=x^{2/5}$ when $-3\le x<1$

10. Find the extrema of $h(\theta)=2\sin\theta-\cos(2\theta)$ for $0\le\theta\le 2\pi$.

11. [No Calculator] An open-top box is to be made by cutting congruent squares of side length x from the corners of a 5- by 8-inch sheet of tin and bending up the sides (see figure below).



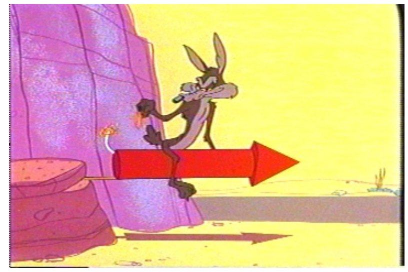
a) Write an equation for the Volume of the box.

b) What is the domain of this function? ____________________

c) How large should the squares be to maximize the volume?

d) What are the dimensions of the box with maximum volume? What is the maximum volume?

12. [Calculator Allowed] Wile E. is after Road Runner again! This time he's got it figured out. Sitting on his ACME rocket he hides behind a hill anxiously awaiting the arrival that "beeping" bird. In his excitement to light the rocket he tips the rocket up. Instead of thrusting himself parallel to the ground where he can catch the Road Runner, he sends himself widely into the air following a path given by function



$$h(t) = .1t^3 - 1.3t^2 + 4.2t + 2,$$

where h is the height of the rocket after t seconds. The rocket fuel lasts for 10 seconds. At that point, Wile E. Coyote stops suddenly and falls straight down to the ground.

a) What is the domain of this function?

b) What is the highest point reached by Wile E. Coyote?

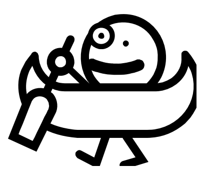
13. [No Calculator] A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$.

a) Draw a sketch the rectangle inscribed under the parabola.

b) Write a formula for the area of the inscribed rectangle as a function of x. What is the domain of this function?

c) What is the largest area the rectangle can have, and what are its dimensions?

14. [Challenge … Calculator Allowed] You are in a rowboat on Long lake, 2 miles from a straight shoreline taking your potential in-laws for a boat ride. Six miles down the shoreline from the nearest point on shore is an outhouse. You suddenly feel the need for its use. It is November, so the water is too cold to go in, and besides, your in-laws are already pretty unimpressed with your "yacht". It wouldn't help matters to jump over the side and relieve your distended bladder. Also, the shoreline is populated with lots of houses, all owned by people who already have restraining orders against you (apparently you've been out here before!). If you can row at 2 mph and run at 6 mph (you can run faster when you don't have to keep your knees together), for what point along the shoreline should you aim in order to minimize the amount of time it will take you to get to the outhouse? (…And you thought calculus wasn't useful!)



AP Calculus
4.2 Worksheet

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1. State the hypothesis of each of the following theorems:

a) IVT

b) EVT

c) MVT

2. State the MVT two different ways …

a) … in words

b) … algebraically

3. For each of the following, (a) state whether or not the function satisfies the hypotheses of the MVT on the given interval, and (b) if it does, find the value of *c* that the MVT guarantees.

a) $f(x) = -2x^2 + 14x - 12$ on the interval [1, 6]

b) $h(x) = x^{1/3}$ on [–1, 1]

4. When a trucker came to his second toll booth in a 169-mile stretch of road, he handed in a ticket stub that was stamped 2 hours earlier. The trucker was cited for speeding. Why?

5. Suppose $f(x)$ is a differentiable function on the interval [–7, 1] such that $f(-7) = 4$ and $f(1) = -1$.

a) *Explain* why f must have at least one value in the interval (–7, 1), where the function equals 2.

b) *Explain* why there must be at least one point in the interval (–7, 1) whose derivative is $-\frac{5}{8}$.

6. Make a sign chart for the following functions:

a) $f(x) = (x-3)^2(x+4)(7-x)$

b) $g(x) = \dfrac{5(2x-7)}{(x+1)(3x-5)}$

7. Summarize how we will use calculus to determine whether a function is increasing or decreasing.

8. For each function, find the critical numbers of f and where the function is increasing or decreasing.

a) $h(x) = \dfrac{2}{x}$

b) $f(x) = x^3 - 6x^2 + 15$

c) $k(x) = \dfrac{-x}{x^2+4}$

9. [Calculator] The Profit P in dollars made by a fast food restaurant selling x hamburgers is given by

$$P = 2.44x - \frac{x^2}{20000} - 5000, \quad 0 \le x \le 35000.$$

a) Find the intervals on which P is increasing or decreasing.

b) Find the maximum profit.

10. Find *all* possible functions f with the given derivative.

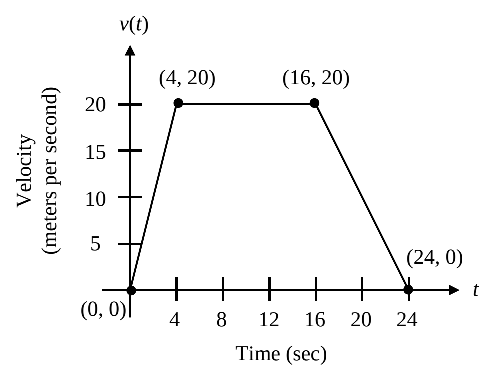
a) $f'(x) = x$ b) $f'(x) = 3x^2 - 2x + 1$ c) $f'(x) = e^x$

11. If you know that the acceleration of gravity is $-32\,{}^{ft}\!/_{s^2}$, for an falling object, we could write the acceleration of the object at time *t* as $a(t) = -32$.

a) Find a function for the velocity of the object at time *t*. What does the constant equal (in words)?

b) Find a function for the position of the object at time *t*. What does the constant equal (in words)?

12. A car is traveling on straight road. For $0 \le t \le 24$ seconds, the car's velocity *v*(*t*), in meters per second, is modeled by the piecewise-linear function defined by the graph below.



a) For each o $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

b) Le *a*(*t*) be the car's acceleration at time *t*, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for *a*(*t*).

c) Find the verage rate of change of *v* over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of *c*, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

13. 216-m^2 pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

AP Calculus
4.3 Worksheet

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1. Complete each statement with the correct word.

   a) When f' is ______________________, the graph of f is increasing,.

   b) When f' is ______________________, the graph of f is decreasing,.

   c) When f'' is ______________________, the graph of f is concave upward.

   d) When f'' is ______________________, the graph of f is concave downward.

   e) When f' is ______________________, the graph of f is concave upward.

   f) When f' is ______________________, the graph of f is concave downward.

2. Use the function $y = 3x - x^3 + 5$. [No calculator allowed]

   a) Where is the function increasing? Justify your response.

   b) Where is the function decreasing? Justify your response.

   c) Where is the function concave up? Justify your response.

   d) Where is the function concave down? Justify your response.

   e) Where are the point(s) of inflection? Justify your response.

   f) Find ALL extrema and justify your response.

   g) Create a *sketch* of the function using the information you have found from *a – f*.

3. Find all local extrema and justify your response for each function:

a) $y = -2x^3 + 6x^2 - 3$

b) $y = xe^{1/x}$

4. Determine the intervals on which the graph of each function is concave up or concave down and determine all points of inflection. Justify your responses.

a) $y = \frac{1}{20}x^5 + \frac{1}{4}x^4 - \frac{3}{2}x^3 - \frac{27}{2}x^2 + x - 4$

b) $y = 2x^{1/5} + 3$

5. If f is continuous on [0, 3] and satisfies the following:

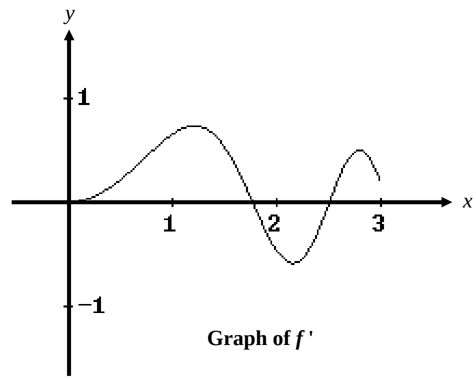
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3
$f(x)$	0	+	2	+	0	–	–2
$f'(x)$	3	+	0	–	DNE	–	–3
$f''(x)$	0	–	–1	–	DNE	–	0

a) Find the absolute extrema of f and where they occur. Justify your response.

b) Find any points of inflection. Justify your response.

c) Sketch a possible graph of f.

7. [Calculator Required] Let f be a function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown below.

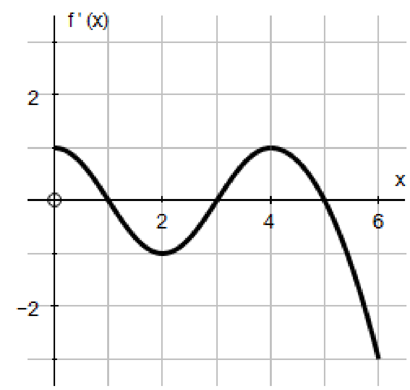


Graph of f'

a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

b) Write an equation for the line tangent to the graph of f at the point (2, 5.623).

8. Use the graph of $f'(x)$ defined on [0, 6] provided below to estimate the following:



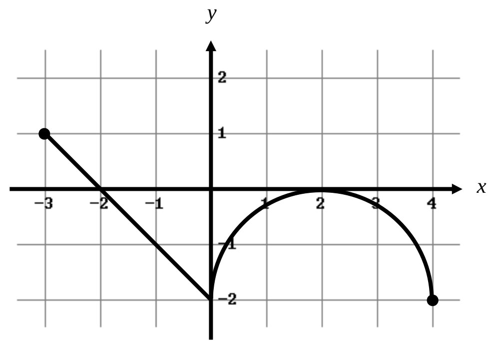
a) When is f increasing? When is f decreasing? Justify your response.

b) Determine the x-coordinates of all local extrema. Justify your response.

c) Determine when f is concave up and concave down. Justify your response.

d) Determine whether f has any points of inflection. Justify your response.

9. [No Calculator Allowed] Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown below.



Graph of f'

a) On what intervals, if any, is f increasing? Decreasing? Justify your answer.

b) Find all values of x for which f assumes a relative maximum. Justify your answer.

c) Where is the graph of f concave up? concave down? Justify your answers.

d) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

e) Find an equation for the line tangent to the graph of f at the point (0, 3).

f) Sketch a possible graph of f.

10. If g is a differentiable function such that $g(x) < 0$ for all real numbers x, and if $f'(x) = (x^2 - 9)g(x)$, which of the following is true?

A) f has a relative maximum at $x = -3$ and a relative minimum at $x = 3$.

B) f has a relative minimum at $x = -3$ and a relative maximum at $x = 3$.

C) f has relative minima at $x = -3$ and at $x = 3$.

D) f has relative maxima at $x = -3$ and at $x = 3$.

E) It cannot be determined if f has any relative extrema.

AP Calculus
4.4 Worksheet

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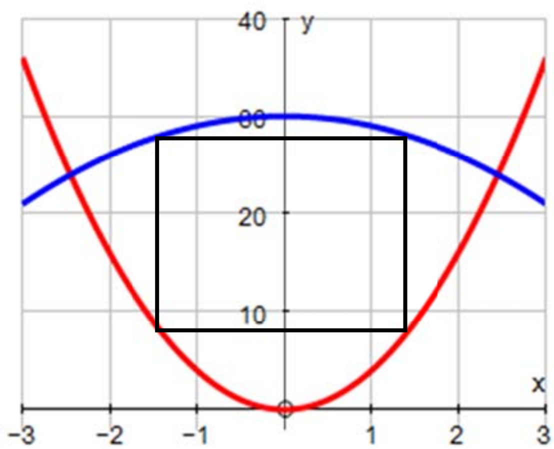
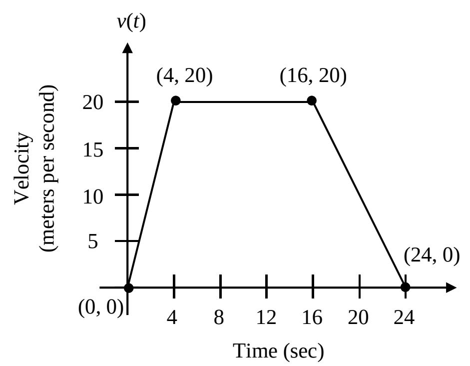
Textbook p 226 odd numbers

11. Suppose that at any time t (sec) the current I (amp) in an alternating current circuit is $I = 2\cos t + 2\sin t$. What is the peak (largest magnitude) current for this circuit? Justify your response.

12. A rectangle is inscribed between the parabolas $y = 4x^2$ and $y = 30 - x^2$ as shown in the picture. What is the maximum area of such a rectangle? Justify your response.



12. A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph below.



a) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

b) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

c) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

13. A 216-m^2 pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

AP Calculus
4.5 Worksheet

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1. Consider the function $y = \sin x$.

a) Find the equation of the tangent line when $x = 0$.

b) Graph both equations on your calculator in a standard viewing window.
When would the tangent line be a good approximation for the curve? (Try zooming in at the origin)

c) Use the tangent line to approximate sin (0.2).
[Why is using the tangent line from part *a* a good approximation?]

2. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is
[Why does the question ask you to use the tangent line at $x = 0$?]

A 2.00

B 2.03

C 2.06

D 2.12

E 2.24

3. Use linearization to approximate $f(5.02)$ if $f(x) = \dfrac{1}{\sqrt{4+x}}$. Find the error for your approximation.

4. Suppose you were asked to determine the value of $(2.003)^4$.

Use linearization to approximate the value of $(\quad 2.003)^4$.

5. [With calculator] Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate the values of $f(x)$. Which of the following is the greatest value for which the error resulting from this tangent line approximation is less than 0.5?

A 2.4

B 2.5

C 2.6

D 2.7

E 2.8

AP Calculus
4.6 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. The radius r and area A of a circle are related by the equation: $A = \pi r^2$
Write an equation that relates $\frac{dA}{dt}$ and $\frac{dr}{dt}$.

2. A spherical container is deflated such that its volume is decreasing at a constant rate of 3141cm^3/min.
[The Surface area of a sphere is $S = 4\pi r^2$ The volume of a sphere is $V = \frac{4}{3}\pi r^3$]

a) At what rate is the radius changing when the radius is 5 cm? Indicate units of measure.

b) At that same moment, how fast is the Surface Area changing? Indicate units of measure.

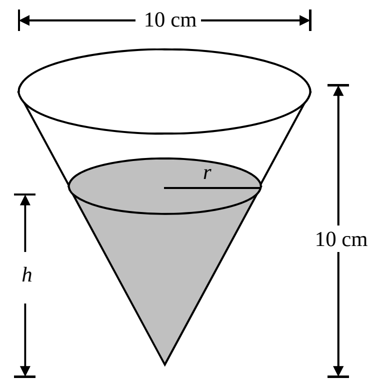
3. A 14 ft ladder is leaning against a wall. The top of the ladder is slipping down the wall at a rate of 2 ft/s.

a) How fast will the end of the ladder be moving away from the wall when the top is 6 ft above the ground? Indicate units of measure.

b) At the same moment, how fast is the angle between the ground and the ladder changing?

4. A pebble is dropped into a still pool and sends out a circular ripple whose radius increases at a constant rate of 4 ft/s. How fast is the area of the region enclosed by the ripple increasing at the end of 8 s? Indicate units of measure.

5. A container has the shape of an open right circular cone, as shown in the figure to the right. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth *h* is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

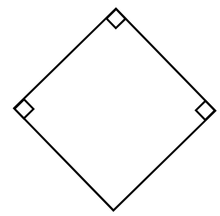


(The volume of a cone of height *h* and radius *r* is given by $V = \frac{1}{3}\pi r^2 h$.)

a) Find the volume *V* of water in the container when *h* = 5 cm. Indicate units of measure.

b) Find the rate of change of the volume of water in the container, with respect to time, when *h* = 5 cm. Indicate units of measure.

6. A baseball diamond has the shape of a square with sides 90 feet long. Tweety is just flying around the bases, running from 2^{nd} base (top of the diamond) to third base (left side of diamond) at a speed of 28 feet per second. When Tweety is 30 feet from third base, at what rate is Tweety's distance from home plate (bottom of diamond) changing? Indicate units of measure.



7. The radius r, height h, and volume V of a right circular cylinder are related by the equation $V = \pi r^2 h$.

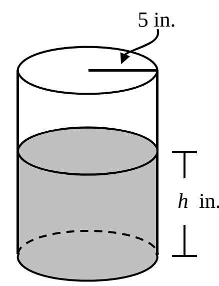
a) How is $\frac{dV}{dt}$ related to $\frac{dh}{dt}$ <u>if r is constant</u>?

b) How is $\frac{dV}{dt}$ related to $\frac{dr}{dt}$ <u>if h is constant</u>?

c) How is $\frac{dV}{dt}$ related to $\frac{dr}{dt}$ and $\frac{dh}{dt}$ if neither r nor h is constant?

8. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure to the right. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

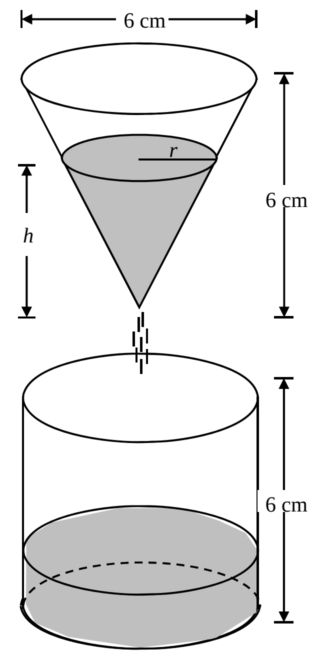


9. Sand pours out of a chute into a conical pile whose height is always one half its diameter. If the height increases at a constant rate of 4 ft/min, at what rate is sand pouring from the chute when the pile is 15 ft high? Indicate units of measure.

10. A camera man is standing 1000 feet from the launch of a rocket. As the rocket launches, the camera man must change the angle of elevation of his camera to keep the rocket in the camera's view. How fast is the angle of elevation changing when the rocket is 1 mile (5280 feet) in the air if the rocket is moving at 2300 feet per second?

11. Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of 10 in^3/min.

a) How fast is the level in the pot rising when the coffee in the cone is 5 in. deep?



b) How fast is the level in the cone falling at that moment?