AP Calculus Summative Review Name Solutions

Sections 2-1 through 3-9

NO CALCULATOR

Please use a separate sheet of paper to complete the following problems. Be sure to show all of your work.

1. a. Find $\lim\limits_{x\to 5}\dfrac{x^2-3x-10}{x^2-10x+25}$ $= \lim\limits_{x\to 5}\dfrac{(x-5)(x+2)}{(x-5)^2} = \lim\limits_{x\to 5}\dfrac{x+2}{x-5}$ DNE $\left[\begin{array}{l}\pm\infty \\ \text{depending} \\ \text{on side}\end{array}\right.$

   b. Find $\lim\limits_{h\to 0}\dfrac{\sqrt{2x+2h}-\sqrt{2x}}{h}$ $= \lim\limits_{h\to 0}\dfrac{2x+2h-2x}{h(\sqrt{2x+2h}+\sqrt{2x})} = \lim\limits_{h\to 0}\dfrac{2}{\sqrt{2x+2h}+\sqrt{2x}} = \dfrac{1}{\sqrt{2x}}$

2. If there are two functions f(x) and g(x) with both $\lim\limits_{x\to\infty} f(x)=\infty$ and $\lim\limits_{x\to\infty} g(x)=\infty$, give an example for f (x) and g(x) for which $\lim\limits_{x\to\infty}\dfrac{f(x)}{g(x)}=\dfrac{4}{3}$.

   $f(x)=4x$ and $g(x)=3x$ for example

3. If f(x) = $x^3 - 3x + 4$, explain why there exists an x-value, c, on the interval (–3 , 0) such that f(c) = 0. Be complete and specific and name any theorem that you use.

   f is continuous on $[-3,0]$. $f(-3)=-14<0$ and $f(0)=4>0$
   IVT applies and guarantees the existence of $c\in(-3,0)$ such that $f(c)=0$.

4. a. Let $f(x) = tan^3 6x$, find $f'(x)$

   $f'(x) = 3\tan^2(6x)\cdot\sec^2(6x)\cdot 6 = 18\tan^2(6x)\cdot\sec^2(6x)$

   b. Let $f(x) = x^3 \cdot e^{\cos(2x)}$, find $f'(x)$

   $f'(x) = 3x^2 e^{\cos(2x)} + e^{\cos(2x)}\cdot(-\sin(2x))\cdot x^3 = x^2 e^{\cos(2x)}(3 - x\sin(2x))$

   c. Let $f(x) = \log_2 x^3$, find $f'(x)$

   $f'(x) = \dfrac{1}{\ln 2}\cdot\dfrac{1}{x^3}\cdot 3x^2 = \dfrac{3}{x\ln 2}$

   d. Find $\dfrac{dy}{dx}$ if $y = cos^{-1}(\sqrt{x})$

   $\dfrac{dy}{dx} = -\dfrac{1}{\sqrt{1-x}}\cdot\dfrac{1}{2\sqrt{x}} = -\dfrac{1}{2\sqrt{x}\sqrt{1-x}}$

5. Use logarithmic differentiation to find $\frac{dy}{dx}$ when $y = x^{\sqrt{x}}$

$\ln y = \sqrt{x}\ln x$

$\frac{y'}{y} = \frac{1}{2\sqrt{x}} \cdot \ln x + \sqrt{x} \cdot \frac{1}{x}$

$\frac{y'}{y} = \frac{\ln x + 2}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{\ln x + 2}{2\sqrt{x}} \cdot x^{\sqrt{x}}$

6. Let $m(x) = \dfrac{g(x)}{1+f(x)}$. Using the numerical values in the table, find the value of each of the derivatives below. Justify each answer

x	f (x)	g (x)	f ′(x)	g ′(x)
0	1	1	6	1/3
1	3	0	-1/3	-8/3

Find m′(0) $= \dfrac{\frac{2}{3} - 6}{4} = -\dfrac{4}{3}$

$m'(x) = \dfrac{g'(x)(1+f(x)) - f'(x).g(x)}{(1+f(x))^2}$

$m'(0) = \dfrac{\frac{1}{3}(1+1) - 6.1}{(1+1)^2}$

7. Use the one of the correct versions of the definition of the derivative to find derivative of the function $y = \sqrt{3x-2}$. THEN find an equation of the tangent line at x = 9 .

$f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} \dfrac{\sqrt{3(x+h)-2} - \sqrt{3x-2}}{h} = \lim\limits_{h\to 0} \dfrac{3(x+h)-2-3x+2}{h(\sqrt{3(x+h)-2} + \sqrt{3x-2})}$

$= \lim\limits_{h\to 0} \dfrac{3}{\sqrt{3(x+h)-2} + \sqrt{3x-2})} = \boxed{\dfrac{3}{2\sqrt{3x-2}}} \Rightarrow f'(9) = \dfrac{3}{10}$

Point $(9, 5)$

$\boxed{y - 5 = \frac{3}{10}(x-9)}$

8. Calculate the equation of the normal line to y = sin(x) + cos(x) at $x = \pi$.

$\frac{dy}{dx} = \cos x - \sin x$ Point $(\pi, -1)$

$\left.\frac{dy}{dx}\right|_{x=\pi} = -1 \Rightarrow m_\perp = +1 \Rightarrow \boxed{y + 1 = x - \pi}$

9. If $x^3 y^3 - 4 = 4$, find the value of $\dfrac{d^2y}{dx^2}$ at the point (–2, –1) . Note: you do not need to simplify your $\dfrac{d^2y}{dx^2}$ algebraically before substituting.

$y = \sqrt[3]{\dfrac{8}{x^3}}$

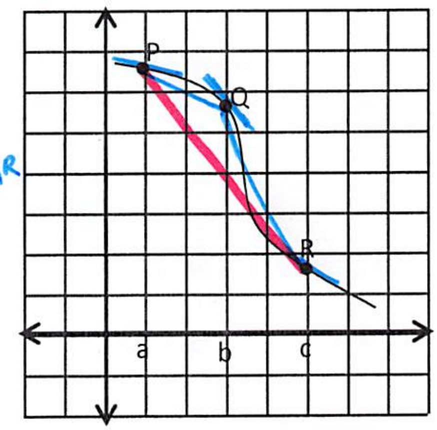
$\frac{dy}{dx} = \frac{1}{3}\left(\frac{8}{x^3}\right)^{-2/3} \cdot \left(\frac{-24}{x^4}\right) = \frac{1}{3}\,\frac{x^2}{4} \cdot \frac{-24}{x^4} = -\frac{2}{x^2}$

$\frac{d^2y}{dx^2} = \frac{4}{x^3}$ $\left.\frac{d^2y}{dx^2}\right|_{x=-2} = \boxed{-\frac{1}{2}}$

10. If the graph of f (x) is shown below, arrange in the following in <u>ascending</u> order (<u>smallest first</u>)

A) $f'(a)$ B) $f'(b)$ C) $f'(c)$ D) Slope of secant line $\overleftrightarrow{PQ}$

E) The slope of secant line $\overleftrightarrow{QR}$ F) $\dfrac{f(a)-f(c)}{a-c}$



$f'(a) < m_{PQ} < f'(c) < f'(b) \simeq \dfrac{f(a)-f(c)}{a-c} < M_{QR}$

$\approx$

11. Let $f(x) = \begin{cases} \sqrt{-x} & \text{if } x<0 \\ 3-x & \text{if } 0 \le x < 3 \\ (3-x)^2 & \text{if } x>3 \end{cases}$

Evaluate each limit, if it exists.

$\lim\limits_{x\to 0^+} f(x) = 3$
$= \lim\limits_{x\to 0} (3-x)$

$\lim\limits_{x\to 0^-} f(x) = 0$
$= \lim\limits_{x\to 0^-} \sqrt{-x}$

11. The graph below contains a function and its derivative. Label which is which. Justify your answer using only the characteristics of the graph, without referring to the possible degree of the function or the derivative. You many add and label any points which might be helpful if necessary.

x	$-\infty$	a		b		c	$+\infty$
$g(x)$	$-$	0	$+$	0	$-$	0	$+$
$f(x)$	↘		↗		↘		↗