AP CALCULUS PROBLEM SET #11 INTEGRATION III FTC ANSWER KEY

1. a) Inflection points at $x=-2$ and $x=0$

   b) $f(-4)=5+\int_0^{-4} g(x)dx = 2\pi - 3$

   $f(4)=5+\int_0^{4}\left(5e^{-x/3}-3\right)dx = 8-15e^{-4/3}$

   c) $g(3)=5+\int_0^{3} g'(x)dx = \frac{13}{2}+\pi$

   $f(-2)=5+\int_0^{-2} g'(x)dx = 5-\pi$

2. a) $g(3)=5+\int_0^{3} g'(x)dx = \frac{13}{2}+\pi$

   $f(-2)=5+\int_0^{-2} g'(x)dx = 5-\pi$

   b) Inflection points at $x=0$, 2 and 3

   c) $h'(x)=g'(x)-x$ when $x=\sqrt{2}$ and $x=3$

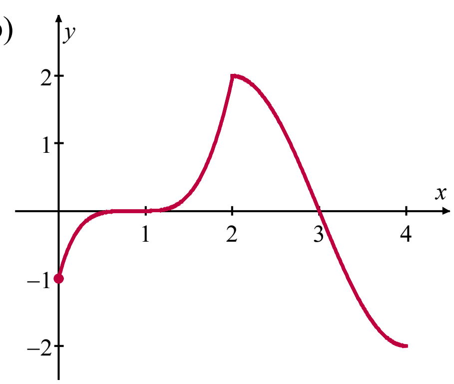
   h has a relative max at $x=\sqrt{2}$,

   and neither a maximum nor minimum

   at $x=3$

3. a) Relative maximum at $x=2$

   b)



   c) $g'(x)=f(x)=0$ at $x=1, 3$

   g has rel. min. at $x=1$,

   rel. max. at $x=3$

   d) g has an inflection point at $x=2$.

4. a) $g(0)=4.5$, $g'(0)=1$

   b) g has rel. max. at $x=3$

   c) g has abs. min. at $x=-4$, $g=-1$

   d) $x=-3, 1, 2$

5. a) f is increasing on $[-3,-2]$

   b) $x=0$ and $x=2$

   c) $y=-2x+3$

   d) $f(-3)=9/2$, $f(4)=-5+2\pi$

6. a) $6\pi^2-\left[2\sin\left(\frac{x}{2}\right)\right]_{x=-2\pi}^{x=4\pi}=6\pi^2$

   b) $f'(x)=g'(x)+\frac{1}{2}\sin\left(\frac{x}{2}\right)$

   $=\begin{cases}1+\frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } -2\pi<x<0\\ -\frac{1}{2}+\frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } 0<x<4\pi\end{cases}$

   $f'(x)$ does not exist at $x = 0$.

   For $-2\pi<x<0$, $f'(x)\neq 0$.

   For $0<x<4\pi$, $f'(x)=0$ when $x=\pi$.

   f has critical points at $x=0$ and $x=\pi$.

   c) $h'(x)=g(3x)\cdot 3$

   $h'\left(-\frac{\pi}{3}\right)=3g(-\pi) = 3\pi$

7. a) $g(4)=3$, $g'(4)=0$, $g''(4)=-2$

   b) g has rel. min. at $x=-1$

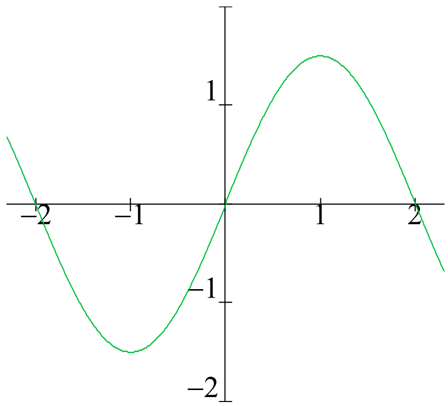
   c) at $x=108$, $y-44=2(x-108)$

8. a) $g(-1) = -1.5,\ g'(-1) = 0,\ g''(-1) = 3$

b) g is increasing on $-1 < x < 1$ since $g'(x) = f(x) > 0$

c) g is concave down on $0 < x < 2$ since $g''(x) = f'(x) < 0$

d)



9. a) *i)* $5x^3 + 40 = \int_c^x f(t)dt$

$$15x^2 = f(x)\ \therefore\ f(t) = 15t^2$$

ii) $5x^3 + 40 = \int_c^x 15t^2 dt = 5t^3\Big]_c^x = 5x^3 - 5c^3$

$$c = -2$$

b) $F(x) = \int_x^3 \sqrt{1+t^{16}}\,dt = -\int_3^x \sqrt{1+t^{16}}\,dt$

$$F'(x) = -\sqrt{1+x^{16}}$$

10. a) $0 \le \left(\frac{x}{2} + 3\right) \le 5$

$$-6 \le x \le 4$$

b) $h'(x) = f\left(\frac{x}{2} + 3\right)\left(\frac{1}{2}\right)$

$$h'(2) = f(4)\left(\frac{1}{2}\right) = -\frac{3}{2}$$

c) $h(-6) = \int_0^0 f(t)dt = 0$

$$h(4) = \int_0^5 f(t)dt < 0$$

Since area below x axis is greater than the area above, min @ $x = 4$.

11. a) f has rel. min. at $x = 1$

b) f' is differentiable and therefore continuous on $-1 \le x \le 1$

c) $h'(3) = \frac{1}{f(3)} f'(3) = \left(\frac{1}{7}\right)\left(\frac{1}{2}\right) = \frac{1}{14}$

d) $\int_{-2}^{3} f'(g(x))g'(x)dx$

$$= \left[f(g(x))\right]_{x=-2}^{x=3} = \left[f(g(3))\right] - \left[f(g(-2))\right]$$

$$= f(1) - f(-1) = 2 - 8$$

$$= -6$$