

AP CALCULUS PROBLEM SET #13 DIFFERENTIAL EQUATIONS ANSWER KEY

1. a) $N = 10000e^{kt}$

b) $k = \frac{\ln 10}{5},$

$$N(20) = 10000e^{\frac{20 \ln 5}{10}} = 250000$$

c) $t_1 = \frac{10 \ln 2}{\ln 5} \approx 4.307$ minutes

2. a) $y = -3(x - 1)$

$$f(1.2) \approx -0.6$$

b) $y = -\ln(-x^3 + 3x^2 - 1)$

3. a) $P(t) = 800 - 300e^{kt}$

b) $k = \frac{\ln\left(\frac{1}{3}\right)}{2}$

c) $\lim_{t \rightarrow \infty} P(t) = 800$

4. a) $v(t) = -34e^{-2t} - 16$

b) $\lim_{t \rightarrow \infty} v(t) = -16$ ft/s

c) $t \approx 1.070$ s

5. a) $y = \frac{1}{2} \ln |2x^3 + e|$

b) $x > \left(-\frac{e}{2}\right)^{\frac{1}{3}}, y \in R$

6. a) $W = 44t + 1400 \quad W(0.25) \approx 1411$ tons

b) $\frac{d^2W}{dt^2} = \frac{1}{25} > 0$

The approximation 1411 is less than $W(0.25)$

c) $W(t) = 1100e^{\frac{t}{25}} + 300$

7. a) $y = 10^6 e^{\left(\frac{\ln 0.5}{6}\right)t} = 10^6 e^{\left(\frac{-\ln 2}{6}\right)t} = 10^6 (2)^{-\frac{t}{6}}$

b) decreasing at $10^5 \ln 2$ gallons/year

c) $\frac{6 \ln 20}{\ln 2} \approx 25.932$ years after starting

8. a) $y = 8x - 6$

b) $f(1.1) \approx 2.8$

$$\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2) > 0$$

The approximation 2.8 is less than $f(1.1)$

c) $f(x) = \frac{2}{\sqrt{5-4x^2}}, \quad \frac{-\sqrt{5}}{2} < x < \frac{\sqrt{5}}{2}$

9. a) $v(t) = -3t^2 + 300$

b) 2000 m

10. a) $r = \sqrt[4]{t+1}$

b) $t = 80$