AP CALCULUS PROBLEM SET 4 ANSWER KEY

1. a) $x < 0$ or $x > 1$

   b) $f'(-1) = -\frac{1}{2}$

   c) $f^{-1}(x) = \frac{e^x}{e^x - 1}$

2. a) $f(g(x)) = (1 - \ln x)^2 = h(x)$

   b) $h'(x) = -\frac{2}{x}(1 - \ln x)$

   c) $h''(x) = \frac{-2(\ln x - 2)}{x^2}$

3. a) even

   b) $\left(-\infty, -\frac{1}{\sqrt{2}}\right]$ or $\left[\frac{1}{\sqrt{2}}, \infty\right)$

   c) $[1, \infty)$

   d) $f'(x) = \frac{(2x\ln 5)5^{\sqrt{2x^2-1}}}{\sqrt{2x^2 - 1}}$

4. a) $f(x)$ is defined for all $x \neq 0$

   b) $x = \pm e$

   c) $y = 2x - 4$

5 a) $m = \frac{1}{e}$

   b) Show that $\frac{1}{e}(x) - \ln x,\ x > 0$ is always positive

   $f(x) = \frac{1}{e}(x) - \ln x = \frac{x}{e} - \ln x$

   $f'(x) = \frac{1}{e} - \frac{1}{x} = 0$ when $x = e$

   Sign Analysis shows absolute minimum at $x = e$,

   $f(e) = 0,\ \therefore \frac{x}{e} - \ln x \geq 0$

   c) from b), $\frac{x}{e} \geq \ln x$

   $x \geq e\ln x$

   $x \geq \ln x^e$

   $e^x \geq x^e$

6. a) $\lim_{x\to 0^-}(1 - 2\sin x) = 1$

   $\lim_{x\to 0^+} e^{-4x} = 1$

   $f(0) = 1$

   $\therefore \lim_{x\to 0} f(x) = f(0)$

   b) $f'(x) = \begin{cases} -2\cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$

   $f'(x) = -3$ for $x = -\frac{1}{4}\ln\left(\frac{3}{4}\right)$

   c) Average value requires integration (later)

   $$\frac{1}{1-(-1)}\left(\int_{-1}^{0}(1 - 2\sin x)dx + \int_{0}^{1} e^{-4x}dx\right)$$

   $$= \frac{1}{2}\left[\left(x + 2\cos x\right)_{-1}^{0} + \left(\frac{e^{-4x}}{-4}\right)_{0}^{1}\right]$$

   $$= \frac{1}{2}\left[(0 + 2\cos 0) - (-1 + 2\cos(-1)) + \left(\frac{e^{-4}}{-4}\right) - \left(\frac{e^{0}}{-4}\right)\right]$$

   $$= \frac{1}{2}\left[2 + 1 - 2\cos(-1) - \left(\frac{e^{-4} - 1}{4}\right)\right]$$

   $$= \frac{12 - 8\cos(-1) - e^{-4} + 1}{8}$$

   $$= \frac{13}{8} - \cos(-1) - \frac{1}{8}e^{-4}$$