2.2 – day 2 – Extra Practice – solutions (EVEN)

AP Calc

2 – a) $\left.\begin{array}{l}\lim\limits_{x\to 1^+} g(x) = 0 \\ \lim\limits_{x\to 1^-} g(x) = 1\end{array}\right\} \lim\limits_{x\to 1} g(x)$ DNE

b) $\lim\limits_{x\to 2} g(x) = 1$ c) $\lim\limits_{x\to 3} g(x) = 0$

4 – $\lim\limits_{x\to 1^+} g(x) = 0$ 6 – $\lim\limits_{x\to 3^-} g(x) = 0$

8 – $\lim\limits_{x\to 2} 3(1-x)(2-x) = 0$

10 – $\lim\limits_{t\to -4} \dfrac{t^2}{4-t} = \dfrac{16}{8} = 2$

12 – $\lim\limits_{x\to -1} \dfrac{x^2-1}{x+1} = \lim\limits_{x\to -1} \dfrac{(x+1)(x-1)}{x+1} = \lim\limits_{x\to -1} (x-1) = -2$

14 – $\lim\limits_{x\to -2} \dfrac{x^2+2x}{x^2-4} = \lim\limits_{x\to -2} \dfrac{x(x+2)}{(x+2)(x-2)} = \lim\limits_{x\to -2} \dfrac{x}{x-2} = \dfrac{1}{2}$

16 – $\lim\limits_{h\to 0} \dfrac{3h+4h^2}{h^2-h^3} = \lim\limits_{h\to 0} \dfrac{h(3+4h)}{h^2(1-h)} = \lim\limits_{h\to 0} \dfrac{3+4h}{h(1-h)}$ DNE ($\pm\infty$ depending on side

18 – $\lim\limits_{h\to 0} \dfrac{\sqrt{4+h}-2}{h} = \lim\limits_{h\to 0} \dfrac{4+h-4}{h(\sqrt{4+h}+2)} = \lim\limits_{h\to 0} \dfrac{1}{\sqrt{4+h}+2} = \dfrac{1}{4}$

20 – $\lim\limits_{x\to -2} |x-2| = 4$ 22 – $\lim\limits_{x\to 0} \dfrac{|x-2|}{x-2} = -1$

24 – $\lim\limits_{x\to 2} \dfrac{\sqrt{4-4x+x^2}}{x-2} = \lim\limits_{x\to 2} \dfrac{\sqrt{(x-2)^2}}{x-2} = \lim\limits_{x\to 2} \dfrac{|x-2|}{x-2}$ DNE ($^{+1}_{-1}$ depending on side)

26 – $\lim\limits_{x\to 1} \dfrac{x^2-1}{\sqrt{x+3}-2} = \lim\limits_{x\to 1} \dfrac{(x+1)(x-1)(\sqrt{x+3}+2)}{x+3-4} = \lim\limits_{x\to 1} (x+1)(\sqrt{x+3}+2) = 8$

28 – $\lim\limits_{s\to 0} \dfrac{(s+1)^2-(s-1)^2}{s} = \lim\limits_{s\to 0} \dfrac{s^2+2s+1-(s^2-2s+1)}{s} = \lim\limits_{s\to 0} \dfrac{4s}{s} = 4$

30 – $\lim\limits_{x\to -1} \dfrac{x^3+1}{x+1} = \lim\limits_{x\to -1} \dfrac{(x+1)(x^2-x+1)}{x+1} = \lim\limits_{x\to -1} (x^2-x+1) = 3$

32 – $\lim\limits_{x\to 8} \dfrac{x^{2/3}-4}{x^{1/3}-2} = \lim\limits_{x\to 8} \dfrac{(x^{1/3}-2)(x^{1/3}+2)}{x^{1/3}-2} = \lim\limits_{x\to 8} (x^{1/3}+2) = 4$

34 – $\lim\limits_{x\to 2} \left(\dfrac{1}{x-2} - \dfrac{1}{x^2-4}\right) = \lim\limits_{x\to 2} \left(\dfrac{x+2-1}{(x+2)(x-2)}\right) =$ DNE ($^{+\infty}_{-\infty}$ depending on side)

36 – when x is around 0, $3x-1<0$. Therefore $|3x-1| = -3x+1$
$3x+1>0$. Therefore $|3x+1| = 3x+1$

$$\lim_{x\to 0} \frac{|3x-1|-|3x+1|}{x} = \lim_{x\to 0} \frac{-3x+1-(3x+1)}{x} = \lim_{x\to 0}\frac{-6x}{x} = -6$$

38 – $$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^3-x^3}{h} = \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h} = \frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(3x^2+3xh+h^2) = 3x^2$$

40 – $$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \frac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h} = \frac{\frac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2}}{h}$$

$$= \frac{1}{h}\cdot\frac{-2xh-h^2}{x^2(x+h)^2} = \frac{h(-2x-h)}{h(x^2)(x+h)^2} = -\frac{2x+h}{x^2(x+h)^2}$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = -\frac{2x}{x^4} = -\frac{2}{x^3}$$

42 – $$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h} = \frac{1}{h}\cdot\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}} = \frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

$$= -\frac{1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)} \xrightarrow[h\to 0]{} \frac{-1}{2x\sqrt{x}}$$

44 – $\lim_{x\to \pi/4} \cos x = \frac{\sqrt{2}}{2}$ 46 – $\lim_{x\to \frac{2\pi}{3}} \sin x = \frac{\sqrt{3}}{2}$

48 – We notice that it seems like $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$

50 – $y=\sqrt{2-x}$ is defined on $(-\infty; 2]$ so $\lim_{x\to 2^+}\sqrt{2-x}$ DNE

52 – $\lim_{x\to -2^+}\sqrt{2-x} = 2$

54 – $x^3-x = x(x^2-1) = x(x+1)(x-1)$ and we have:

x	$-\infty$		-1		0		1		$+\infty$
x^3-x		$-$	0	$+$	0	$-$	0	$+$	

Therefore: $y=\sqrt{x^3-x}$ is defined on $[-1,0]\cup[1,+\infty)$.

$\therefore \lim_{x\to 0^-}\sqrt{x^3-x} = 0$

56 – $x^2-x^4 = x^2(1-x^2) = x^2(1+x)(1-x)$ and we have:

x	$-\infty$		-1		0		1		$+\infty$
x^2-x^4		$-$	0	$+$	0	$+$	0	$-$	

Therefore $y=\sqrt{x^2-x^4}$ is defined on $[-1,1]$

$\therefore \lim_{x\to 0^+}\sqrt{x^2-x^4} = 0$

58 - If $x > a$, then $x - a > 0$ Therefore $|x-a| = x - a$

$$\lim_{x \to a^+} \frac{|x-a|}{x^2 - a^2} = \lim_{x \to a^+} \frac{x-a}{(x-a)(x+a)} = \lim_{x \to a^+} \frac{1}{x+a} = \frac{1}{2a}$$

60 - $\lim_{x \to 2^+} \frac{x^2 - 4}{x+2} = 0$ (direct substitution)

62 - If $x \longrightarrow -1^+$, then $f(x) = x^2 + 1$ $\therefore \lim_{x \to -1^+} f(x) = 2$

64 - If $x \longrightarrow 0^-$, then $f(x) = x^2 + 1$ $\therefore \lim_{x \to 0^-} f(x) = 1$

66 - a) 2 b) -8 c) -8 d) -2

68 - Since $\lim_{x \to 0} x^2 = 0$, then $\lim_{x \to 0} f(x) = 0$

Since $\lim_{x \to 0} x = 0$, then $\lim_{x \to 0} \frac{f(x)}{x} = 0$

70 - $\lim_{x \to 0} \frac{\sin(2\pi x)}{\sin(3\pi x)} = \lim_{x \to 0} \frac{\sin(2\pi x)}{2\pi x} \cdot \frac{3\pi x}{\sin(3\pi x)} \cdot \frac{2}{3} = \frac{2}{3}$

72 - $\frac{x - \sqrt{x}}{\sqrt{\sin x}} = \frac{\sqrt{x}(\sqrt{x} - 1)}{\sqrt{\sin x}} = \sqrt{\frac{x}{\sin x}}\,(\sqrt{x} - 1) \xrightarrow[x \to 0^+]{} -1$

74 - $\lim_{x \to 0} \sqrt{5 - 2x^2} = \sqrt{5}$ and $\lim_{x \to 0} \sqrt{5 - x^2} = \sqrt{5}$ $\therefore \lim_{x \to 0} f(x) = \sqrt{5}$