

Match each derivative

- 1)  $f(x) = \tan x$  d)
- 2)  $f(x) = \sec x$  c)
- 3)  $f(x) = \csc x$  b)
- 4)  $f(x) = \sin x$  f)
- 5)  $f(x) = \cos x$  a)
- 6)  $f(x) = \cot x$  e)

- a)  $f'(x) = -\sin x$
- b)  $f'(x) = -\csc x \cot x$
- c)  $f'(x) = \sec x \tan x$
- d)  $f'(x) = \sec^2 x$
- e)  $f'(x) = -\csc^2 x$
- f)  $f'(x) = \cos x$

7)  $f(x) = 3x^2 - 7x + 8$

a)  $f'(x) = 6x - 7$

b)  $f'(2) = 5$

c) Equation of the tangent line at  $x = 2$

$y - 6 = 5(x - 2)$

point (2, 6)

d) Equation of the normal line at  $x = 2$

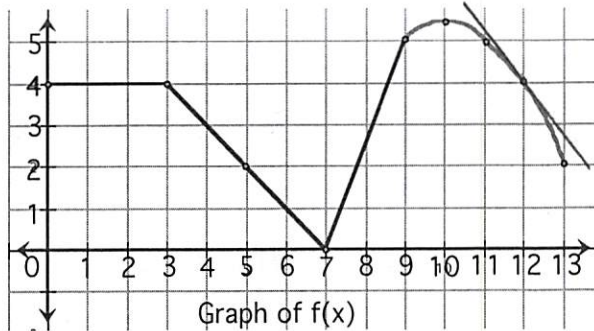
$y - 6 = -\frac{1}{5}(x - 2)$

Use picture at right for #8

8 a)  $f'(2) = 0$     b)  $f'(3) = \text{DNE}$     c)  $f'(5) = -1$

d)  $f'(8) = \frac{5}{2}$     e)  $f'(9) = \text{DNE}$     f)  $f'(10) = 0$

g)  $f'(12) = -\frac{4}{3}$     h) Equation of tangent line at 12  
 $y - 4 = -\frac{4}{3}(x - 12)$



9)  $\lim_{h \rightarrow 0} \frac{\tan(x+h) - \tan x}{h} = \sec^2 x$

10)  $\lim_{h \rightarrow 0} \frac{3(x+h)^7 - 3x^7}{h} = 21x^6$

11)  $\lim_{h \rightarrow 0} \frac{(1+h)^7 - 1}{h} = 7$

12)  $\lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{6}+h\right) - \frac{\sqrt{3}}{2}}{h} = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

Find equation of the tangent line and normal line to the given equation at the given point.

3.7 13)  $2xy^2 + x = 12$ ; (4, -1)

$2y^2 + 4xy \cdot \frac{dy}{dx} + 1 = 0$      $\frac{dy}{dx} \Big|_{x=4, y=-1} = \frac{-3}{-16}$

$\frac{dy}{dx} = \frac{-1 - 2y^2}{4xy}$

tangent line

$y + 1 = \frac{3}{16}(x - 4)$

normal line

$y + 1 = -\frac{16}{3}(x - 4)$

3.7 14) Given:  $x^2 - 6x + y^2 + 4y - 12 = 0$  and  $\frac{dy}{dx} = \frac{-2x + 6}{2y + 4}$

a) Find the horizontal tangent(s).

$\frac{dy}{dx} = 0 \Leftrightarrow x = 3$

if  $x = 3$ :  $9 - 18 + y^2 + 4y - 12 = 0$

$y^2 + 4y - 21 = 0$

$(y+7)(y-3) = 0$

$y = -7$      $y = 3$

Points: (3, 3)  
& (3, -7)

b) Find the vertical tangent(s).

$\frac{dy}{dx}$  undefined if  $y = -2$

$x^2 - 6x + 4 - 8 - 12 = 0$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = 8$      $x = -2$

Points: (8, -2)

& (-2, -2)

3.7 (15) Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ .  $x^2 - 9y^2 = 7$

$$2x - 9 \times 2y \times \frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = \frac{2x}{18y}}$$

$$2 - 18 \left(\frac{dy}{dx}\right)^2 - 18y \cdot \frac{d^2y}{dx^2} = 0$$

$$2 - 18 \left(\frac{4x^2}{18^2 y^2}\right) - 18y \frac{d^2y}{dx^2} = 0$$

$$2 - \frac{4x^2}{18y^2} = 18y \frac{d^2y}{dx^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{1}{9y} - \frac{4x^2}{324y^3}}$$

$$f(3) = 3$$

$$f'(3) = 7$$

$$f(9) = -2$$

$$f'(9) = 3$$

$$g(3) = -1$$

$$g'(3) = 4$$

$$g(9) = 0$$

$$g'(9) = 6$$

16) Use information above to find  $h'(x)$  and  $h'(3)$ .

a)  $h(x) = f(x) \cdot g(x)$

$$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\begin{aligned} h'(3) &= 7 \cdot (-1) + 3 \cdot 4 \\ &= -7 + 12 \\ &= 5 \end{aligned}$$

b)  $h(x) = g(f(x))$

$$h'(x) = g'(f(x)) \times f'(x)$$

$$\begin{aligned} h'(3) &= g'(f(3)) \times f'(3) \\ &= g'(3) \times f'(3) \\ &= 4 \times 7 \\ &= 28 \end{aligned}$$

c)  $h(x) = (f(3x))^3$

$$\begin{aligned} h'(x) &= 3(f(3x))^2 \times f'(3x) \times 3 \\ &= 9f'(3x) \times (f(3x))^2 \end{aligned}$$

$$\begin{aligned} h'(3) &= 9f'(9) \times (f(9))^2 \\ &= 9 \times 3 \times (-2)^2 \\ &= 108 \end{aligned}$$

Find derivatives for each.

17)  $f(x) = \frac{2}{x}$

$$f'(x) = -\frac{2}{x^2}$$

18)  $f(x) = \cos(7x^3)$

$$\begin{aligned} f'(x) &= -\sin(7x^3) \times 21x^2 \\ &= -21x^2 \sin(7x^3) \end{aligned}$$

19)  $f(x) = x^{7/9}$

$$\begin{aligned} f'(x) &= \frac{7}{9} x^{-2/9} \\ &= \frac{7}{9x^{2/9}} \end{aligned}$$

20)  $f(x) = \tan(\sec x)$

$$f'(x) = \sec^2(\sec x) \cdot \sec x \tan x$$

21)  $f(x) = \cot 17x$

$$\begin{aligned} f'(x) &= -\csc^2(17x) \times 17 \\ &= -17 \csc^2(17x) \end{aligned}$$

22)  $f(x) = (\sin 5x)^4$

$$\begin{aligned} f'(x) &= 4(\sin 5x)^3 \cdot \cos(5x) \cdot 5 \\ &= 20 \sin^3 5x \cdot \cos 5x \end{aligned}$$

23)  $f(x) = x^2 \sqrt{x^2 - 9}$

$$f'(x) = 2x \sqrt{x^2 - 9} + x^2 \cdot \frac{2x}{2\sqrt{x^2 - 9}}$$

$$= 2x \sqrt{x^2 - 9} + \frac{x^3}{\sqrt{x^2 - 9}}$$

$$= \frac{2x(x^2 - 9) + x^3}{\sqrt{x^2 - 9}}$$

$$= \frac{3x(x^2 - 6)}{\sqrt{x^2 - 9}}$$

24)  $f(x) = 3x^3(5x^2 + 7)^5$

$$\begin{aligned} f'(x) &= 9x^2(5x^2 + 7)^5 + 15x^3(5x^2 + 7)^4(10x) \\ &= 3x^2(5x^2 + 7)^4(3(5x^2 + 7) + 5x(10x)) \\ &= 3x^2(5x^2 + 7)^4(65x^2 + 21) \end{aligned}$$

25)  $f(x) = \frac{x^2 + 5}{x^3 - 9}$

$$\begin{aligned} f'(x) &= \frac{2x(x^3 - 9) - 3x^2(x^2 + 5)}{(x^3 - 9)^2} \\ &= \frac{2x^4 - 18x - 3x^4 - 15x^2}{(x^3 - 9)^2} \end{aligned}$$

$$= \frac{-x^4 - 15x^2 - 18x}{(x^3 - 9)^2}$$

$$= \frac{-x(x^3 + 15x + 18)}{(x^3 - 9)^2}$$