Solutions



# 6.4 - Word Problems Extra Practice

1. The half life of Plutonium 239 is 24100 years. If 10 grams of Plutonium 239 were released in a nuclear accident, then how long will it take for the 10 grams to decay to 1 gram?

let y be the quantity of plutonium.

$y = y_0 e^{kt}$

$0.5 = e^{24100k}$

$\ln 0.5 = 24100k$

$k = \frac{1}{24100} \ln 0.5$

$\to y = y_0 e^{\frac{1}{24100}\ln(0.5)t}$

$1 = 10 e^{\frac{1}{24100}\ln(0.5)t}$

$\ln 0.1 = \frac{1}{24100}\ln(0.5)t$

$t = \frac{24100 \ln 0.1}{\ln 0.5} \approx \boxed{80058.467 \text{ years}}$

or

Precalc 12 method

$1 = 10 \cdot \left(\frac{1}{2}\right)^{t/24100}$

$0.1 = 0.5^{t/24100}$

$\ln 0.1 = \frac{t}{24100} \ln 0.5$

$t = \frac{24100 \ln 0.1}{\ln 0.5}$

2. An experimental population of fruit flies increases according to the law of exponential growth. There were 100 flies after the second day of the experiment and 300 flies after the 4th day. Approximately, how many flies were in the original population?

let t be the number of days.

$P = P_0 e^{kt}$

$\hookrightarrow 100 = P_0 e^{2k}$

you could also try to solve for $P_0$ here ...

$\frac{100}{P_0} = e^{2k}$

$\ln\left(\frac{100}{P_0}\right) = 2k$

$k = \frac{1}{2}\ln\left(\frac{100}{P_0}\right)$

$\hookrightarrow 300 = P_0 e^{4k}$

$300 = P_0 e^{2\ln\left(\frac{100}{P_0}\right)}$

$300 = P_0 \cdot \left(\frac{100}{P_0}\right)^2$

$300 = \frac{10000}{P_0}$

$P_0 = \frac{10000}{300}$

$\boxed{P_0 \approx 33 \text{ flies}}$

3. Four months after it stops advertising, a manufacturing company notices that its sales have dropped from 100 000 units per month to 80 000 units per month. The sales follow an exponential pattern of decline. What will the sales be after another 2 months?

let y be the number of units sold per month.

$y = y_0 e^{kt}$

$80000 = 100000 e^{4k}$

$0.8 = e^{4k}$

$\ln 0.8 = 4k$

$k = \frac{1}{4}\ln 0.8$

$y = 100000 e^{\left(\frac{1}{4}\ln 0.8\right) \times 6}$

$\boxed{y \approx 71554 \text{ units per month.}}$

4. Newton's Law of Cooling states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature in the surrounding medium. Let T represent the temperature (in °F) of an object in a room whose temperature is kept at a constant 70°F. The object cools from 120°F to 100°F in 12 minutes. How much longer will it take for the temperature of the object to decrease to 90°F?

• $T' = k(T - 70)$

let $y = T - 70$

$y' = ky$

$\Rightarrow y = y_0 e^{kt}$

i.e $T - 70 = 50e^{kt}$

• when $t = 12$, $T = 100$

$30 = 50 \cdot e^{12k}$

$0.6 = e^{12k}$

$\ln 0.6 = 12k$

$k = \frac{1}{12} \ln 0.6$

• $20 = 50 \times 0.6^{t/12}$

$0.4 = 0.6^{t/12}$

$\ln 0.4 = \frac{t}{12} \ln 0.6$

$t = \frac{12 \ln 0.4}{\ln 0.6} \simeq \boxed{21.5 \text{ min}}$

5. If you invest \$15 000 with a 6.4% interest rate, how much will you have in 5 years, if the interest is compounded:

a) quarterly? $A = 15000\left(1 + \frac{0.064}{4}\right)^{4 \times 5} \simeq \boxed{\$20604.66}$

b) monthly? $A = 15000\left(1 + \frac{0.064}{12}\right)^{12 \times 5} \simeq \boxed{\$20639.36}$

c) continuously? $A = 15000\, e^{0.064 \times 5} \simeq \boxed{\$20656.92}$

6. 10 years ago, you invested \$20 000. You now have \$25 000. What was the interest rate

a) if it was compounded quarterly?

$25000 = 20000\left(1 + \frac{i}{4}\right)^{40}$

$\frac{5}{4} = \left(1 + \frac{i}{4}\right)^{40}$

$\sqrt[40]{\frac{5}{4}} = 1 + \frac{i}{4}$

$\frac{i}{4} = \sqrt[40]{\frac{5}{4}} - 1$

$i = 4\left(\sqrt[40]{\frac{5}{4}} - 1\right)$

$i \simeq 0.0224$

$\Rightarrow \boxed{2.2\%}$

b) if it was compounded continuously?

$25000 = 20000\, e^{10r}$

$1.25 = e^{10r}$

$10r = \ln 1.25$

$r = \frac{1}{10} \ln 1.25$

$\boxed{2.2\%}$

7. After 1000 years, 4.43 grams of Carbon 14 are remaining from a 5 grams sample. What is the half-life of Carbon 14?

- $4.43 = 5e^{1000k}$

$\ln 0.886 = 1000k$

$k = \frac{1}{1000}\ln 0.886$

- $0.5 = e^{\frac{1}{1000}(\ln 0.886)t}$

OT

$0.5 = 0.886^{t/1000}$

$\ln 0.5 = \frac{t}{1000}\ln 0.886$

$t = \frac{1000 \ln 0.5}{\ln 0.886}$

$\boxed{t \simeq 5727 \text{ years}}$

Precalc 12 method

$4.43 = 5\left(\frac{1}{2}\right)^{1000/n}$

$0.886 = 0.5^{\frac{1000}{n}}$

$\ln 0.886 = \frac{1000}{n}\ln 0.5$

$n = \frac{1000 \ln 0.5}{\ln 0.886}$ $n \simeq 5727$ years.

8. Newton's Law of Cooling states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature in the surrounding medium. When an object is removed from a furnace and placed in an environment with a constant temperature of 80°F, its core temperature is 1500°F. One hour after it is removed, the core temperature is 1120°F. Find the core temperature 5 hours after the object is removed from the furnace.

$T' = k(T - 80)$

let $y = T - 80$

$y' = ky$

$\Rightarrow y = y_0 e^{kt}$

$T - 80 = 1420e^{kt}$

- $1040 = 1420e^{k}$

$\frac{1040}{1420} = e^k$

$\ln\left(\frac{1040}{1420}\right) = k$

- $T = 1420\left(\frac{1040}{1420}\right)^5 + 80$

$\boxed{T \simeq 379.236°F}$

9. Newton's Law of Cooling states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature in the surrounding medium. A container of hot liquid is placed in a freezer that is kept at a constant temperature of 20°F. The initial temperature of the liquid is 160°F. After 5 minutes, the liquid's temperature is 60°F. How much longer will it take for its temperature to decrease to 30°F?

$T' = k(T - 20)$

let $y = T - 20$

$y' = ky$

$\Rightarrow y = y_0 e^{kt}$

$T - 20 = 140e^{kt}$

$T = 140e^{kt} + 20$

- $60 = 140e^{5k} + 20$

$\frac{40}{140} = e^{5k}$

$\ln\left(\frac{4}{14}\right) = 5k$

$k = \frac{1}{5}\ln\left(\frac{2}{7}\right)$

$T = 140\left(\frac{2}{7}\right)^{t/5} + 20$

- $30 = 140\left(\frac{2}{7}\right)^{t/5} + 20$

$\frac{10}{140} = \left(\frac{2}{7}\right)^{t/5}$

$\ln\left(\frac{1}{14}\right) = \frac{t}{5}\ln\left(\frac{2}{7}\right)$

$t = \frac{5\ln(1/14)}{\ln(2/7)}$

$\boxed{t \simeq 10.533 \text{ min}}$

10. The rate of change of the number of coyotes $N(t)$ in a population is directly proportional to $650 - N(t)$, where $t$ is the time in years. When $t = 0$, the population is 300, and when $t = 2$, the population has increased to 500. Find the population when $t = 3$.

$N'(t) = k(650 - N(t))$

or $N'(t) = -k(N(t) - 650)$

let $y(t) = N(t) - 650$

$y' = -ky$

$\Rightarrow y = y_0 e^{-kt}$

• $N(t) - 650 = -350\, e^{-kt}$

$500 - 650 = -350\, e^{-2k}$

$\frac{150}{350} = e^{-2k}$

$\ln\left(\frac{150}{350}\right) = -2k$

$k = -\frac{1}{2}\ln\left(\frac{150}{350}\right)$

• $N(3) = -350\left(\frac{150}{350}\right)^{3/2} + 650$

$\boxed{N(3) \simeq 551.802}$

11. A Calf that weighs 60 pounds at birth gains weight at the rate: $\frac{dw}{dt} = 0.8(1200 - w)$. The animal is sold when its weight reaches 800 pounds. Find the time of sale for this calf.

$W' = 0.8(1200 - W)$

$W' = -0.8(W - 1200)$

let $y = W - 1200$

$y' = -0.8y$

$\Rightarrow y = y_0 e^{-0.8t}$

$W - 1200 = -1140\, e^{-0.8t}$

• $W = -1140\, e^{-0.8t} + 1200$

$800 = -1140 e^{-0.8t} + 1200$

$\frac{-400}{-1140} = e^{-0.8t}$

$\ln\left(\frac{20}{57}\right) = -0.8t$

$t = -\frac{1}{0.8}\ln\left(\frac{20}{57}\right)$

$\boxed{t \simeq 1.309 \text{ years}}$

12. A wet towel hung from a clothesline to dry loses moisture through evaporation at a rate proportional to its moisture content. After 1 hour, the towel has lost 40% of its original moisture content. After how long will it have lost 80%?

let M be the moisture content of the towel

$M' = kM$

$\Rightarrow M = M_0 e^{kt}$

• $60 = 100 e^{k}$

$\ln(0.6) = k$

• $20 = 100\,(0.6)^t$

$\ln(0.2) = t \ln 0.6$

$t = \frac{\ln 0.2}{\ln 0.6}$

$\boxed{t \simeq 3.151 \text{ hours}}$