6. [Calculator] During the zombie invasion of a small town the number of infected people is proportional to the difference between the town's population and the number of zombies currently roaming around. There are 8 zombies roaming around when they are first discovered (call this time $t = 0$ hours). If $Z(t)$ represents the number of zombies roaming the town at time t, then

$$\frac{dZ}{dt} = 0.05(1100 - Z)$$

a) Find a tangent line to the graph of Z when $t = 0$.

$$\left.\frac{dz}{dt}\right|_{t=0} = 0.05(1100 - Z_0) = 0.05(1100 - 8) = 54.6$$

$$y - 8 = 54.6(t - 0)$$

$$\boxed{y = 54.6t + 8}$$

b) Find $\frac{d^2Z}{dt^2}$ in terms of Z.

$$\frac{d^2z}{dt^2} = -0.05\frac{dz}{dt}$$

$$= -0.05 \times 0.05(1100 - z)$$

$$\boxed{\frac{d^2z}{dt^2} = -0.0025(1100 - z)}$$

c) Use your tangent line from part *a* to estimate the number of zombies roaming the town 12 hours after they are first discovered ($t = 12$). Is this an over approximation or an under approximation? Explain.

when $t = 12$, $Z \simeq y|_{t=12} = 54.6 \times 12 + 8 \simeq 663$ zombies.

$Z'' < 0 \Rightarrow$ concave down $\Rightarrow$ overapproximation

d) Use separation of variables to find the particular solution for $Z(t)$ if $Z(0) = 8$.

$$\frac{z'}{1100 - z} = 0.05$$

$$-\ln|1100 - z| = 0.05t + C, \quad C \in \mathbb{R}$$

when $t = 0$, $z = 8 \Rightarrow C = -\ln 1092$

$$\ln|1100 - z| = -0.05t - C$$

$$|1100 - z| = e^{-0.05t - C}$$

because $1100 - z > 0$

$$1100 - z = + e^{-0.05t - C}$$

$$z = -e^{-0.05t - C} + 1100$$

$$\boxed{z = -1092e^{-0.05t} + 1100}$$