Additional Prerequisites for Calculus

1. Even and Odd functions:

Recognizing the behavior of functions is not limited to their domain and range. Many functions have the symmetric property of being odd or even. You need to be able to recognize the graph of a function as odd or even, AND you need to understand how to show/verify/prove that a function is even or odd algebraically.

Graphical Recognition of Even and Odd Functions

An **EVEN** function is symmetrical about the y – axis. *Example*: $y = \cos x$

An **ODD** function is symmetrical about the origin. *Example*: $y = \sin x$

Algebraic Properties of Even and Odd Functions

An **EVEN** function has the property that $f(-x) = f(x)$.
That is, if you plug in "$-x$" into the function and simplify, you will obtain the original function.

An **ODD** function has the property that $f(-x) = -f(x)$.
That is, if you plug in "$-x$" into the function and simplify, you will obtain the opposite of the original function.

Example 3: Prove whether the following functions are even, odd, or neither.

a) $g(x) = x^3 - x$

$g(-x) = (-x)^3 - (-x)$

$= -x^3 + x$

$= -g(x)$

$\Rightarrow$ odd

b) $h(x) = 1 + \cos x$

$h(-x) = 1 + \cos(-x)$

$= 1 + \cos x$ (because $y = \cos x$ is even)

$= h(x)$

$\Rightarrow$ even

c) $f(x) = 5x^2 - 3$

$f(-x) = 5(-x)^2 - 3$

$= 5x^2 - 3$

$= f(x)$

$\Rightarrow$ even

d) $k(x) = (x+1)(3x-3) = 3x^2 - 3$

$k(-x) = 3(-x)^2 - 3$

$= 3x^2 - 3$

$= k(x)$

$\Rightarrow$ even

⚠ most functions are neither!

2. Piecewise functions:

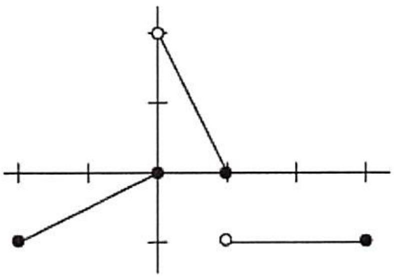
Some functions are broken into pieces and behave differently depending on the restricted domain of each piece. Such functions are called piecewise functions. An example of a function that can be written as a piecewise function is the absolute value function $f(x)=|x|$. Be sure to use correct domain restrictions.

Example 4: Sketch $f(x)=|x|$, and write an equation for the two "pieces" using a domain appropriate to each piece.

$$f(x) = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

Example 5: Write a piecewise function for the graph at the right.

$$y = \begin{cases} \frac{1}{2}x \text{ if } -2 \leq x \leq 0 \\ -2x+2 \text{ if } 0 < x \leq 1 \\ -1 \text{ if } 1 < x \leq 3 \end{cases}$$



Hwk: worksheet + p 19 # 21 - 34

3. One to one functions:

Definition: *One – to – One Function* A function $f(x)$ is **one – to – one** on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$. A function that is one – to – one has an inverse.

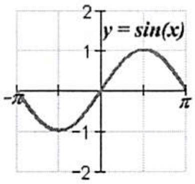
The definition above can be seen graphically with the use of a horizontal line test. If there are two x – values for any given y – value of function, then the function does NOT have an inverse.

Note: A function that is always increasing* is one to one.
A function that is always decreasing* is one to one.

Hwk: worksheet + p 44 # 1 – 6,13 – 24,52,53
extend: p 45 # 59,60,62.

4. Trigonometric inverse functions:

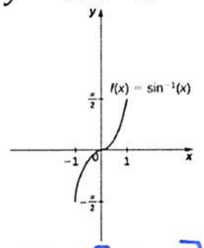
$y = sinx$



$D_R = [-\pi/2, \pi/2]$

$R = [-1, 1]$

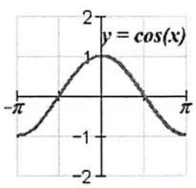
$y = sin^{-1}x$



$D = [-1, 1]$

$R = [-\pi/2, \pi/2]$

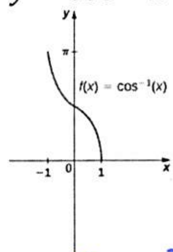
$y = cosx$



$D_R = [0, \pi]$

$R = [-1, 1]$

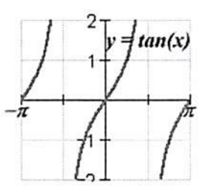
$y = cos^{-1}x$



$D = [-1, 1]$

$R = [0, \pi]$

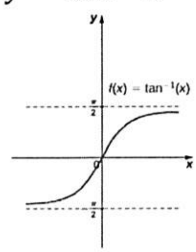
$y = tanx$



$D = (-\frac{\pi}{2}, \frac{\pi}{2})$

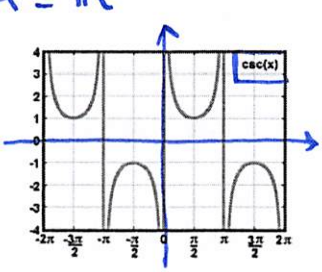
$R = \mathbb{R}$

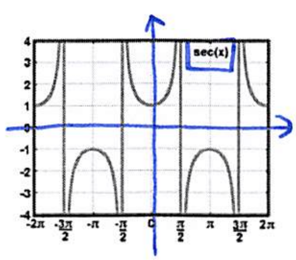
$y = tan^{-1}x$

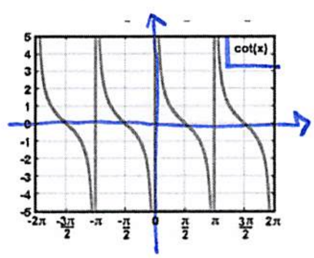


$D = \mathbb{R}$

$R = (-\frac{\pi}{2}, \frac{\pi}{2})$





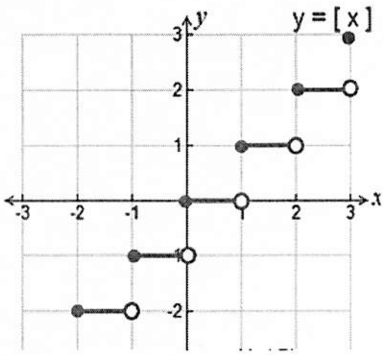


5. The Greatest Integer Function:

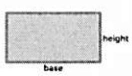
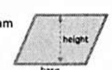
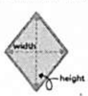
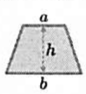
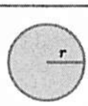
The Greatest Integer Function is denoted by $y = [x]$ or $y = Int(x)$.

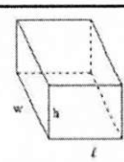
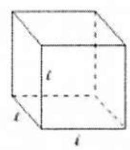
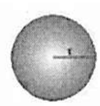
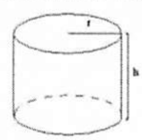
For all real numbers, x, the **greatest integer function** returns the largest integer less than or equal to x. In essence, it rounds down a real number to the nearest integer.

For example: [1] = 1 [1.5] = 1 [3.7] = 3 [4.3] = 4
Beware! [-2] = -2 [-1.6] = -2 [-2.1] = -3 [-5.5] = -6



6. Usual shapes areas and volumes:

Shape		Formula
Triangle		$Area = \frac{1}{2} \times base \times height$
Rectangle		$Area = base \times height$
Parallelogram		$Area = base \times height$
Rhombus		$Area = \frac{1}{2} \times diagonal \times diagonal$
Trapezium		$Area = \frac{1}{2}(a + b)h$
Circle		$Area = \pi r^2$

3-D Shape	Surface Area	Volume	Sketch
Rectangular Prism	$2(lw + lh + wh)$	lwh	
Cube	$6l^2$	l^3	
Sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$	
Cylinder	$2\pi r^2 + 2\pi rh$	$\pi r^2 h$	
Cone	$\pi r\sqrt{r^2 + h^2}$	$\frac{1}{3}\pi r^2 h$