AP® Exam Practice Questions for Chapter 1

What You Need to Know...

- The AP® Exam, especially the free-response section, stresses the major applications of calculus, rather than the foundational limit concept.
- You are not required to use the formal epsilon-delta definition of a limit on the AP® Exam.
- Algebraic limit evaluation methods (Section 1.3) are not explicitly tested on the free-response section of the AP® Exam. They are, however, helpful on some multiple-choice questions.
- You should be able to apply the Intermediate Value Theorem, whether the function is presented as an equation or by a table.
- You should be able to find both vertical and horizontal asymptotes given a function or a graph.
- The AP® Exam frequently uses limits at infinity as a way of describing horizontal asymptotes.

Practice Questions

Section 1, Part A, Multiple Choice, No Technology

1. What is the limit of $h(x) = 2$ as x approaches π?

(A) 0 (B) 2
(C) π (D) nonexistent

2. What is the limit of

$$g(x) = \frac{|x+4|}{x+4}$$

as x approaches -4?

(A) -4 (B) 0
(C) 1 (D) nonexistent

3. $\lim_{x\to\pi} \frac{\sin x}{x}$ is

(A) 0. (B) 1.
(C) π. (D) nonexistent.

4. $\lim_{x\to -2} \frac{3x^2 + 5x + 7}{x - 4}$ is

(A) $-\frac{5}{2}$. (B) $-\frac{3}{2}$.
(C) -1. (D) 3.

5. If $\lim_{x\to 5} f(x) = 10$ and $\lim_{x\to 5} g(x) = 1$, what is the limit of $\lim_{x\to 5} [5f(x) - g(x)]$?

(A) 9 (B) 15
(C) 45 (D) 49

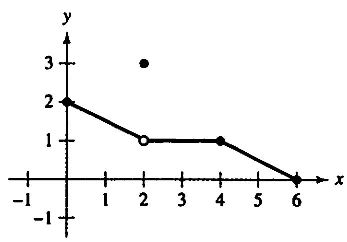
6. $\lim_{x\to\infty} \frac{4x^3 + 3x^2 - x + 5}{6x^4 + x^3 - x^2 + 4x - 1}$ is

(A) -5. (B) 0.
(C) $\frac{2}{3}$. (D) nonexistent.

7. What are the horizontal asymptotes of the graph of $y = \frac{3 + 4^x}{1 - 4^x}$?

(A) $y = -1$ only (B) $y = 3$ only
(C) $y = -1$ and $y = 3$ (D) $y = 0$ and $y = 3$

8.



The graph of function g is shown above. Which of the following is true?

I. $\lim_{x\to 2} g(x) = 1$
II. $\lim_{x\to 2} g(x) = g(2)$
III. g is continuous at $x = 3$.

(A) I only (B) I and III only
(C) III only (D) I, II, and III

Section 1, Part B, Multiple Choice, Technology Permitted

9. $\lim_{x\to 1} \frac{x-1}{\sqrt{x} - 1}$ is

(A) 0. (B) 1.
(C) 2. (D) nonexistent.

10. $\lim_{x\to\infty} \frac{x^3}{\sqrt{x^2 + 4}}$ is

(A) $-\infty$. (B) 0.
(C) 4. (D) ∞.

Section 2, Part A, Free Response, Technology Permitted

11. The function f is defined as $f(x) = \dfrac{10}{1 + \frac{1}{4}e^{-x}}$.

(a) Find $\lim_{x\to 0} f(x)$.

(b) Find $\lim_{x\to 0} (f(x) + 4)$.

(c) State the equation(s) for the horizontal asymptote(s) of the graph of $y = f(x)$. Show the work that leads to your answer.

Section 2, Part B, Free Response, No Technology

12. The function f is defined as $f(x) = \dfrac{x^2 + 5x + 6}{2x^2 + 7x + 3}$.

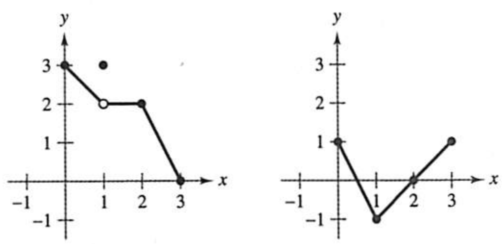
(a) State the value(s) of x for which f is not continuous.

(b) Evaluate $\lim_{x\to -3} f(x)$. Justify your answer.

(c) State the equation(s) for the vertical asymptote(s) of the graph of $y = f(x)$.

(d) State the equation(s) for the horizontal asymptote(s) of the graph of $y = f(x)$. Show the work that leads to your answer.

13.



Graph of f Graph of g

The graphs of functions f and g are shown above. Evaluate each limit using the graphs provided. Show the computations that lead to your answer.

(a) $\lim_{x\to 1} [f(x) + 4]$

(b) $\lim_{x\to 3^-} \dfrac{5}{g(x)}$

(c) $\lim_{x\to 2} [f(x) \cdot g(x)]$

(d) $\lim_{x\to 3^-} \dfrac{f(x)}{g(x) - 1}$ (Assume that f and g are linear on the interval $[2, 3]$.)

14. Let f be a function defined by

$$f(x) = \begin{cases} e^{2x}, & x \le 0 \\ 4 - 3\cos x, & x > 0 \end{cases}.$$

(a) Find $\lim_{x\to -1} f(x)$.

(b) Show that f is continuous at $x = 0$.

(c) Find $\lim_{x\to -\infty} f(x)$.

15. A hot cup of tea is placed on a counter and left to cool. The temperature (in degrees Fahrenheit) of the tea x minutes after the cup is placed on the counter is modeled by a continuous function $T(x)$ for $0 \le x < 10$. Values of $T(x)$ at various times x are shown in the table.

x	0	3	4	6	8	9
$T(x)$	180°	174°	172°	168°	164°	162°

(a) Find $\lim_{x\to 4} T(x)$. Justify your answer.

(b) Use the data to find the average rate of change in the temperature of the tea for $3 \le x \le 8$. Include units in your final answer.

(c) Use the data to identify the shortest interval during which there must exist a time x for which the temperature of the tea is 166.5°. Justify your answer.

(d) Use the data to find the best estimate of the slope of the line tangent to the graph of T at $x = 8$.

16. The position function $s(t) = -4.9t^2 + 396.9$ gives the height (in meters) of an object that has fallen from a height of 396.9 meters after t seconds.

(a) Explain why there must exist at a time t, $1 < t < 2$, at which the height of the object must be 382 meters above the ground.

(b) After how many seconds does the object hit the ground?

(c) Find the average rate of change in s over the interval $[8, 9]$. Include units of measure. Explain why this is a good estimate of the velocity at which the object hits the ground. How can this estimate be improved?

(d) Find

$$\lim_{t\to 3} \frac{s(t) - s(3)}{t - 3}.$$

Show the work that leads to your answer. Include units.

17. Let a and b represent real numbers. Define

$$f(x) = \begin{cases} ax^2 + x - b, & \text{if } x \le 2 \\ ax + b, & \text{if } 2 < x < 5. \\ 2ax - 7, & \text{if } x \ge 5 \end{cases}$$

(a) Find the values of a and b such that f is continuous on the entire real number line.

(b) Evaluate $\lim_{x\to 3} f(x)$.

(c) Let $g(x) = \dfrac{f(x)}{x - 1}$. Evaluate $\lim_{x\to 1} g(x)$.

AP® Exam Practice Questions for Chapter 2

What You Need to Know . . .

- The definition of the derivative is primarily tested on the multiple-choice section of the AP® Exam.
- The AP® Exam requires that you have proficiency with using a function's equation, table of values, or graph when finding the average velocity or average rate of change.
- Extremely complex examples of the Product Rule, Quotient Rule, and Chain Rule do not typically appear on the AP® Exam. The more difficult problems in this chapter will help you master and remember the concepts.
- Although you should know the derivatives of the six trigonometric functions, the derivatives of the sine, cosine, and tangent functions are the most commonly tested.
- Related rate problems make frequent appearances on the AP® Exam because they represent a powerful application of implicit derivatives.

Practice Questions

Section 1, Part A, Multiple Choice, No Technology

1. What is an equation of the tangent line to the graph of $f(x) = 4e^x - x + 6$ at $(0, 10)$?

(A) $y = 4x + 10$

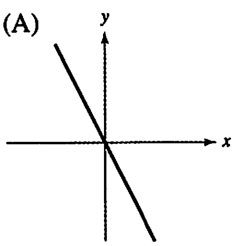
(B) $y = 4x - 10$

(C) $y = 10x - 4$

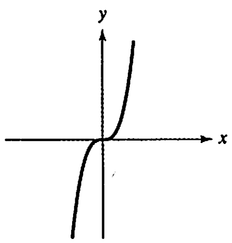
(D) $y = 3x + 10$

2. Which graph shows a function whose derivative is always negative?

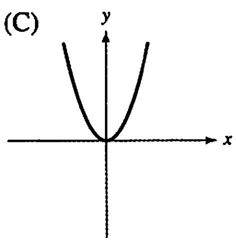
(A)



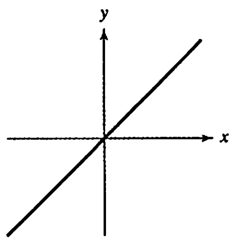
(B)



(C)



(D)



3. If $y = \dfrac{6x^4 - 3x^5 + 5x^3}{x^3}$, then $\dfrac{d^2y}{dx^2} =$

(A) $6 - 6x$.

(B) 6.

(C) $6x$.

(D) -6.

4. If $h(x) = |2x - 5|$, which of the following is true?

(A) h is continuous but is not differentiable at $x = \frac{5}{2}$.

(B) h is not continuous but is differentiable at $x = \frac{5}{2}$.

(C) h is continuous and differentiable at $x = \frac{5}{2}$.

(D) h is neither continuous nor differentiable at $x = \frac{5}{2}$.

5. If $f(x) = \dfrac{\sin x}{x^2}$, then $f'(x) =$

(A) $\dfrac{\cos x}{2x}$.

(B) $\dfrac{x\cos x - 2\sin x}{x^2}$.

(C) $\dfrac{x\cos x - 2\sin x}{x^3}$.

(D) $\dfrac{\cos x - 2\sin x}{x^2}$.

6. If $y = \sqrt[4]{8x + 3}$, then $y' =$

(A) $\dfrac{2}{(8x + 3)^{3/4}}$.

(B) $\dfrac{1}{4(8x + 3)^{3/4}}$.

(C) $\frac{1}{4}(8x + 3)^{3/4}$.

(D) $\dfrac{8}{(8x + 3)^{3/4}}$.

7. If $y = 6\cos 2x$, then $y^{(6)} =$

(A) $384\cos 2x$.

(B) $-384\cos 2x$.

(C) $384\sin 2x$.

(D) $-384\sin 2x$.

8. The table shows the position $s(t)$ of a particle that moves along a straight line at several times t, where t is measured in seconds and s is measured in meters.

t	2.0	2.7	3.2	3.8
$s(t)$	5.2	7.8	10.6	12.2

Which of the following best estimates the velocity of the particle at $t = 3$?

(A) 3.7 m/sec (B) 3.9 m/sec
(C) 5.6 m/sec (D) 7.8 m/sec

9. If $2y^3 - 3xy + x^2 = 4$, then $\dfrac{dy}{dx} =$

(A) $-\dfrac{2x}{6y^2 - 3}$. (B) $\dfrac{2x - 3y}{3x - 6y^2}$.
(C) $\dfrac{2x - 3}{6y^2}$. (D) $-\dfrac{2x}{6y^2 - 3x}$.

10. The volume of a cylinder with radius r and height h is given by $V = \pi r^2 h$. The radius of the cylinder is increasing at a rate of 1/3 centimeter per second and the height of the cylinder is increasing at a rate of 1/2 centimeter per second. At what rate, in cubic centimeters per second, is the volume of the cylinder increasing when its height is 9 centimeters and the radius is 4 centimeters?

(A) $\dfrac{4\pi}{3}$ (B) $\dfrac{8\pi}{3}$ (C) 6π (D) 32π

Section 1, Part B, Multiple Choice, Technology Permitted

11. Two roads intersect at right angles. You are standing 25 meters north of the intersection on one of the roads. You are watching a car traveling west at 30 meters per second. At how many meters per second is the car traveling away from you 3 seconds after it passes through the intersection?

(A) 23.047 (B) 28.906
(C) 29.032 (D) 30

12. The position $s(t)$ of a particle moving along the x-axis at time t is given by $s(t) = -t^3 + 2t^2 + \frac{3}{2}$, where s is measured in meters and t is measured in seconds. At what time is the particle's instantaneous velocity equal to its average velocity on the interval $[0, 4]$?

(A) 1.097 seconds (B) 2 seconds
(C) 2.333 seconds (D) 2.431 seconds

Section 2, Part A, Free Response, Technology Permitted

13. The function f is defined as $f(x) = 3e^{2x^2}$.

(a) Find $f'(x)$.

(b) For what value of x is the slope of the tangent line to the graph of f equal to 2?

(c) For what value(s) of x does the tangent line to the graph of f intersect the x-axis at the point $\left(\frac{1}{2}, 0\right)$?

Section 2, Part B, Free Response, No Technology

14. Evaluate each limit analytically.

(a) $\displaystyle\lim_{h\to 0} \frac{\sin(x + h) - \sin x}{h}$ (b) $\displaystyle\lim_{h\to 0} \frac{\sqrt[3]{x + h} - \sqrt[3]{x}}{h}$

(c) $\displaystyle\lim_{h\to 0} \frac{\sqrt{16 + h} - 4}{h}$ (d) $\displaystyle\lim_{h\to 0} \frac{\frac{1}{5 + h} - \frac{1}{5}}{h}$

15. Given:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	−3	1	5	−2
5	4	7	−1	2

(a) If $h(x) = \dfrac{f(x)}{g(x)}$, find $h'(2)$.

(b) If $j(x) = f(g(x))$, find $j'(2)$.

(c) If $k(x) = \sqrt{f(x)}$, find $k'(5)$.

16. The figure below shows the graph of the velocity, in feet per second, for a particle moving along the line $x = 4$.

(a) During which time interval(s) is the particle:

(i) moving upward?

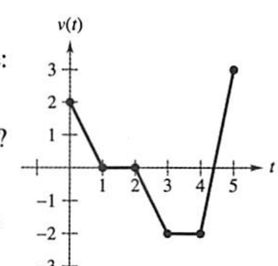
(ii) moving downward?

(iii) at rest?

(b) What is the acceleration of the particle at

(i) $t = 0.75$ and

(ii) $t = 4.2$? Be sure to include units.



17. Given: $g(x) = f(x) \cdot \tan x + kx$, where k is a real number. f is differentiable for all x; $f(\pi/4) = 4$; $f'(\pi/4) = -2$.

(a) For what values of x, if any, in the interval $0 < x < 2\pi$ will the derivative of g fail to exist? Justify your answer.

(b) If $g'\left(\dfrac{\pi}{4}\right) = 6$, find the value of k.

AP® Exam Practice Questions for Chapter 3

What You Need to Know . . .

- On some free-response questions, there may be more than one way of applying derivatives and theorems to justify your answer.
- Be prepared to apply the Mean Value Theorem. It may be referred to directly, or it may be necessary to use the theorem to justify your answer.
- Questions that involve position, velocity, and acceleration functions are very common on the AP® Exam.
- Be prepared to apply the Second Derivative Test to justify whether a point is a local maximum, a local minimum, or a point of inflection. For a point of inflection, make sure to also check for a sign change.
- Tangent line approximations, and whether such an approximation overestimates or underestimates a function value, is commonly tested on the free-response section.

Practice Questions

Section 1, Part A, Multiple Choice, No Technology

1. What are the critical numbers of

$f(x) = 4x^3 + 6x^2 - 72x - 9$?

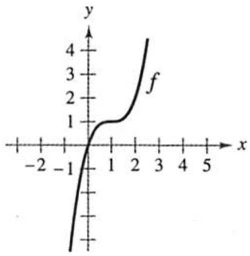
(A) $x = -2$ and $x = 3$

(B) $x = -3$ and $x = 2$

(C) $x = -2$

(D) $x = -3$

2. The graph of the function f is shown. Which of the following is true?



I. $f'(x) > 0$ on the entire real number line.

II. $f''(x) < 0$ on the interval $(-\infty, 1)$.

III. $f''(x) > 0$ on the interval $(1, \infty)$.

(A) I only (B) II and III only

(C) I and III only (D) I, II and III

3. The position of an object along a vertical line is given by $s(t) = -t^3 + 3t^2 + 9t + 5$, where s is measured in feet and t is measured in seconds. The maximum velocity of the object in the time interval $[0, 4]$ is

(A) 9 feet per second.

(B) 12 feet per second.

(C) 16 feet per second.

(D) 32 feet per second.

4. The function g is continuous and differentiable on the interval $[2, 6]$. The table shows selected values of g on $[2, 6]$. Which of the following statements must be true?

x	2	3	4	5	6
$g(x)$	7	4	1	4	7

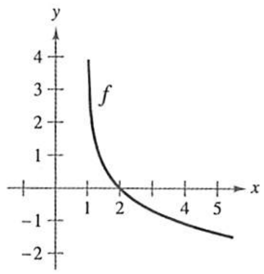
(A) The minimum value of g on $[2, 6]$ is 1.

(B) The maximum value of g on $[2, 6]$ is 7.

(C) There exists a number c, with $2 < c < 6$, for which $g'(c) = 0$.

(D) $g'(x) < 0$ for $2 < x < 4$

5. Consider the graph of $y = f(x)$ shown below. If f is a function such that f' and f'' are defined in a region around $x = 2$, then which of the following must be true?



(A) $f''(2) < f(2)$ (B) $f''(2) < f'(2)$

(C) $f(2) = f'(2)$ (D) $f''(2) > f(2)$

6. If $y = \arctan 4x$, then $dy =$

(A) $\dfrac{4}{1 + 16x^2}\,dx.$ (B) $\dfrac{4x}{1 + 16x^2}\,dx.$

(C) $-\dfrac{4x}{1 + 16x^2}\,dx.$ (D) $-\dfrac{4}{1 + 16x^2}\,dx.$

Section 1, Part B, Multiple Choice, Technology Permitted

7. If the Mean Value Theorem is applied to the function $f(x) = \ln(x - 3)$ on the interval $[4, 8]$, then the number c that must exist in $(4, 8)$ is

(A) 5.485.

(B) 5.885.

(C) 6.

(D) 6.368.

Section 2, Part A, Free Response, Technology Permitted

8. The table below shows the behavior of a function f that is continuous on the entire real number line. For the function, $f(2) = 4$, and $\lim_{x \to \infty} f(x) = 0$.

	$x < 4$	$x = 4$	$x > 4$
$f'(x)$	positive	does not exist	negative
$f''(x)$	negative	does not exist	positive

(a) For what values of x is f increasing?

(b) Does f have a relative maximum at $x = 4$? Explain.

(c) If possible, name the x-coordinate of the point of inflection on the graph of f. Justify your answer.

(d) Does the Mean Value Theorem apply over the interval $[3, 5]$? Justify your answer.

(e) Sketch a possible graph of f.

9. Consider the function $f(x) = \dfrac{x^3}{2} - \sin x + 1$.

(a) Approximate the relative extrema of f.

(b) Find the tangent line approximation of f at $\dfrac{\pi}{2}$.

(c) Use your tangent line approximation to approximate the value of $f(1.5)$. Is your approximation an underestimate or an overestimate of the actual value of $f(1.5)$? Justify your answer.

Section 2, Part B, Free Response, No Technology

10. Consider the function

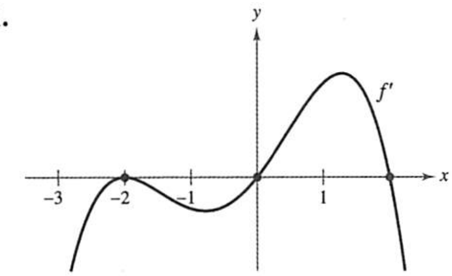
$$f(x) = 2x + \cos 2x$$

on the interval $[0, \pi]$.

(a) Find the maximum value of f. Justify your answer.

(b) Explain how the conditions of the Mean Value Theorem are satisfied by f for $0 \le x \le \pi$. Find the value of x whose existence is guaranteed by the Mean Value Theorem.

11.



The figure above shows the graph of f', the derivative of f. The function f is a twice differentiable function on $x \in (-\infty, \infty)$, $f''(-0.8) = 0$, and $f''(1.3) = 0$.

(a) For what values of x is f increasing?

(b) For what values of x is the graph of f concave downward? Justify your answer.

(c) Is

$$\frac{f(-0.5) - f(0)}{-0.5 - 0}$$

positive or negative? Justify your answer.

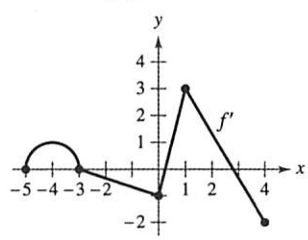
12. Consider the function $f(x) = \dfrac{1 - 4x^2}{x}$.

(a) For what values of x is f decreasing?

(b) For what values of x is the graph of f concave downward? Justify your answer.

(c) Does the graph of f have any points of inflection? Justify your answer.

13.



The figure above shows the graph of f', the derivative of f, on the interval $[-5, 4]$. The function f is differentiable on the interval and $f''(-4) = 0$.

(a) Find $f'(-1)$ and $f''(-1)$.

(b) At which x-values does f have a relative extrema on the interval $(-5, 0)$? Justify your answer.

(c) Find all intervals on which the graph of f is concave downward. Explain your reasoning.

(d) Find the x-coordinate of each of the points of inflection of the graph of f. Justify your answer.

(e) If $g(x) = f(x) + \sin^2 x$, is g increasing or decreasing at $x = -\pi/4$? Justify your answer.

AP® Exam Practice Questions for Chapter 4

What You Need to Know . . .

- Approximating the area under a curve using rectangles and basic geometry is often tested on the AP® Exam.
- Be prepared to approximate definite integrals using left, right, or midpoint Riemann sums, or trapezoidal sums.
- An alternative form of the Fundamental Theorem of Calculus, $f(b) = f(a) + \int_a^b f'(x)\,dx$, is also emphasized on the AP® Exam.
- Some questions where technology is permitted not only encourage but also require the use of a graphing utility in evaluating definite integrals. This is the case for functions with no elementary antiderivative.

Practice Questions

Section 1, Part A, Multiple Choice, No Technology

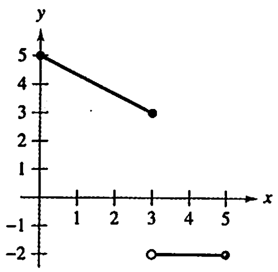
1. The table shows selected values for a continuous function g that is increasing over the interval $[0, 4]$. Which of the following could be the value of $\int_0^4 g(x)\,dx$?

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$g(x)$	0	3	7	12	18	25	33	42	52

(A) 70 (B) 80
(C) 96 (D) 100

2. The graph of f is shown for $0 \le x \le 5$. What is the value of $\int_0^5 f(x)\,dx$?

(A) -1
(B) 7
(C) 8
(D) 16



3. $\displaystyle\int \frac{4}{(x-5)^2+9}\,dx =$

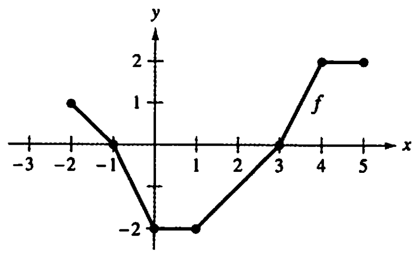
(A) $\frac{4}{3}\tan^{-1}\frac{x-5}{3} + C$ (B) $4\tan^{-1}\frac{x-5}{3} + C$

(C) $\tan^{-1}\frac{x-5}{3} + C$ (D) $\frac{1}{3}\tan^{-1}\frac{x-5}{3} + C$

4. The velocity of a particle is given by $v(t) = 4t^3 - 4t$ for the times $0 \le t \le 2$ in seconds. What is the average velocity of the particle over that interval?

(A) 4 (B) 5
(C) 10 (D) 24

5.



The graph of a piecewise linear function f is shown above. If g is the function defined by

$$g(x) = \int_4^x f(t)\,dt$$

find $g(-1)$.

(A) -6 (B) -4
(C) 4 (D) 6

Section 1, Part B, Multiple Choice, Technology Permitted

6. If $0 \le b \le \pi$, and the area under the curve $y = \sin x$ from $x = b$ to $x = \pi$ is 0.4, what is the value of b?

(A) 0.927 (B) 1.159
(C) 1.982 (D) 2.214

7. Let $f(x)$ be a continuous function such that $f(1) = 2$ and $f'(x) = \sqrt{x^3 + 6}$. What is the value of $f(5)$?

(A) 11.446 (B) 13.446
(C) 24.672 (D) 26.672

8. What is the average value of the function $y = x + \sin x$ on the interval

$$\left[0, \frac{3\pi}{2}\right]?$$

(A) 2.144 (B) 2.356 (C) 2.568 (D) 2.781

Section 2, Part A, Free Response, Technology Permitted

9. As a pot of coffee cools down, the temperature of the coffee is modeled by a differentiable function C, for $0 \le t \le 12$, where time t is measured in minutes and the temperature $C(t)$ is measured in degrees Celsius. Selected values of t are shown in the table.

t (minutes)	0	3	5	7	8	12
$C(t)$ (degrees Celsius)	65	57	50	46	44	40

(a) Evaluate $\int_0^{12} C'(t)\,dt$. Explain the meaning of your answer in the context of the problem. Indicate units of measure.

(b) Explain the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem. Use a trapezoidal sum with 5 subintervals indicated by the table to approximate $\frac{1}{12}\int_0^{12} C(t)\,dt$. Indicate units of measure.

(c) Use the data in the table to approximate the rate at which the temperature is changing at time $t = 4$. Show the work that leads to your answer.

(d) For $12 \le t \le 15$, the rate of cooling is modeled by

$$C'(t) = -2\cos(0.5t).$$

Based on the model, what is the temperature of the coffee when $t = 15$? Assume $C(t)$ is continuous at $t = 12$.

10. On a typical day, the snow on a mountain melts at a rate modeled by the function

$$M(t) = \frac{\pi}{6}\sin\frac{\pi t}{12}.$$

A snow maker adds snow at a rate modeled by the function

$$S(t) = 0.006t^2 - 0.12t + 0.87.$$

Both M and S have units in inches per hour and t is measured in hours for $0 \le t \le 6$. At $t = 0$, the mountain has 40 inches of snow.

(a) How much snow will melt during the 6 hour period? Indicate units of measure.

(b) Write an expression for $I(t)$, the total number of inches of snow at any time t.

(c) Find the rate of change of the total amount of snow when $t = 3$.

(d) For $0 \le t \le 6$, at what time t is the amount of snow a maximum? What is the maximum value? Justify your answers.

Section 2, Part B, Free Response, No Technology

11. For $0 \le t \le 9$, a particle moves along the x-axis. The velocity of the particle is given by $v(t) = \sin(\pi t/4)$. The particle is at position $x = -4$ when $t = 0$.

(a) For $0 \le t \le 9$, when is the particle moving to the right? Justify your answer.

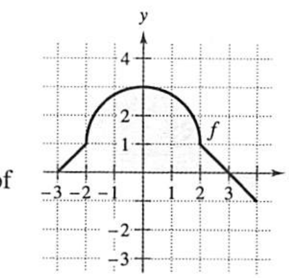
(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to $t = 9$.

(c) Find the acceleration of the particle at time $t = 3$. Is the particle speeding up, slowing down, or neither at $t = 3$? Justify your answer.

(d) Find the position of the particle at time $t = 3$.

12. Let

$$F(x) = \int_3^x f(t)\,dt.$$



The graph of f on the interval $[-3, 4]$ consists of two line segments and a semicircle, as shown in the graph.

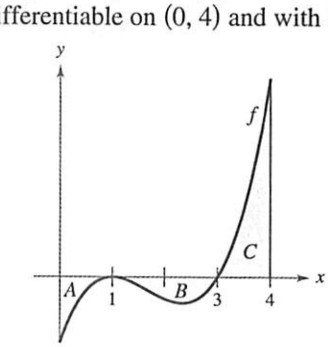
(a) Find $F(0)$, $F'(0)$, and $F(4)$.

(b) Find all relative minimum values of $F(x)$ on the interval $[-3, 4]$. Justify your answer.

(c) Find the x-coordinates of the inflection points of $F(x)$ on the interval $[-3, 4]$. Justify your answer.

(d) Write the equation of the line tangent to the point where $x = 2$.

13. The graph of a continuous function f is shown. The three regions between the graph of f and the x-axis are marked A, B, and C, and have unsigned areas 5.5, 6, and 15.5, respectively. Let $F(x)$ be an antiderivative of f that is differentiable on $(0, 4)$ and with $F(1) = 9$.



(a) Find $F(0)$ and $F(4)$.

(b) What is the minimum number of times F equals 5 on the interval $[0, 4]$? Show the work that leads to your answer.

(c) Find all intervals where F is increasing. Justify your answer.

AP® Exam Practice Questions for Chapter 5

What You Need to Know . . .

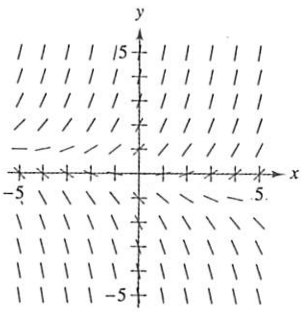
- You may have to sketch a slope field for a given differential equation at a specified number of points.
- You may have to sketch a solution curve on a given portion of a slope field. When this occurs, make sure that your curve follows the slope field appropriately, and passes through the indicated point.
- Be prepared to solve differential equations completely using separation of variables with a given initial condition. The final answer should be of the form $y = f(x)$.
- On the AP® Calculus BC Exam, be prepared to do at least two iterations of Euler's Method, showing all of your work without using a calculator.

Practice Questions

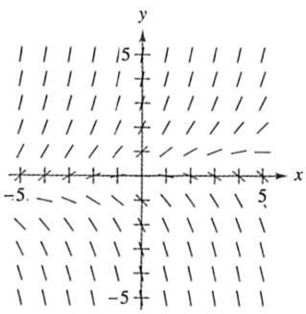
Section 1, Part A, Multiple Choice, No Technology

1. Which of the following is a slope field for $\frac{dy}{dx} = y - \frac{x}{5}$?

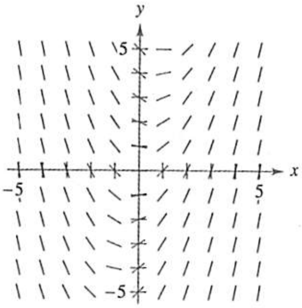
(A)



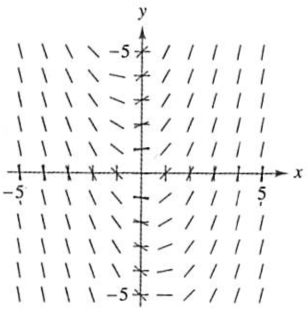
(B)



(C)



(D)



2. A population P grows according to the equation $dP/dt = kP$, where k is a constant and t is measured in years. If the population triples every 15 years, what is the value of k?

(A) $\ln \frac{1}{5}$ (B) $\frac{1}{15} \ln 3$

(C) $\ln 3$ (D) 5

3. Let $y = f(x)$ be a solution of the differential equation $y' = ky$, where k is a constant. If $f(0) = 8$ and $f(6) = 2$, which of the following is an expression for $f(x)$?

(A) $8e^{(x/6)\ln(1/4)}$ (B) $-e^{(x/6)\ln 7} + 9$

(C) $-x + 8$ (D) $x^2 + 8$

4. If $\frac{dy}{dx} = 2xy^2$ and $y(-1) = 2$, find $y(2)$.

(A) $-4e^3$ (B) $-\frac{3}{2}$

(C) $-\frac{2}{5}$ (D) $-\frac{1}{4}$

5. Which of the following is the solution of the differential equation

$$\frac{dy}{dx} = \frac{3y}{x}$$

with the initial condition $y(1) = -1$?

(A) $y = x^3$ (B) $y = -x^3$

(C) $y = x^3 - 2$ (D) $y = -x^3 - 2$

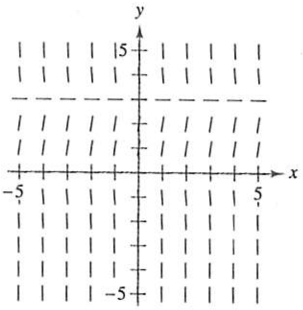
6. Which of the following differential equations produces the slope field shown below?

(A) $\frac{dy}{dx} = 10y\left(1 - \frac{y}{3}\right)$

(B) $\frac{dy}{dx} = \frac{y}{2}\left(1 - \frac{y}{3}\right)$

(C) $\frac{dy}{dx} = y\left(1 - \frac{y}{3}\right)$

(D) $\frac{dy}{dx} = 5y\left(1 - \frac{y}{6}\right)$



Section 1, Part B, Multiple Choice, Technology Permitted

7. Consider the differential equation

$$y' = 0.5(y - 1)(t + 1)$$

with an initial value of $y(0) = -3$. Using Euler's Method with a step of $h = \frac{1}{3}$, what is the approximate value of $y(1)$?

(A) -6.288 (B) -6.125

(C) -4.753 (D) -4.703

Section 2, Part A, Free Response, Technology Permitted

8. At any time $t \geq 0$, in hours, the rate of growth of a population of bacteria is given by $dy/dt = 0.5y$. Initially, there are 200 bacteria.

(a) Solve for y, the number of bacteria present, at any time $t \geq 0$.

(b) Write and evaluate an expression to find the average number of bacteria in the population for $0 \leq t \leq 10$.

(c) Write an expression that gives the average rate of bacteria growth over the first 10 hours of growth. Indicate units of measure.

Section 2, Part B, Free Response, No Technology

9. Let $y = f(x)$ be a particular solution of the differential equation

$$\frac{dy}{dx} = \frac{1}{xy}$$

with $f(1) = 2$.

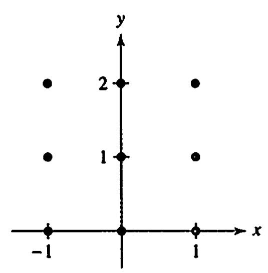
(a) Find $\frac{d^2y}{dx^2}$ at the point $(1, 2)$.

(b) Write an equation for the line tangent to the graph of f at $(1, 2)$ and use it to approximate $f(1.1)$. Is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the solution of the given differential equation that satisfies the initial condition $f(1) = 2$.

10. Consider the differential equation $\frac{dy}{dx} = x^2(1 - y)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



(b) While the slope field in part (a) is drawn only at nine points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution in the form of $y = f(x)$ to the given differential equation with the initial condition $f(0) = 2$.

11. Consider the differential equation $y' = (2x)/y$ with a particular solution in the form of $y = f(x)$ that satisfies the initial condition $f(1) = 2$.

(a) Use Euler's Method, starting at $x = 1$ with two steps of equal size, to approximate $y(1.4)$. Show the work that leads to your answer.

(b) Find the particular solution of the given differential equation that passes through $(1, 2)$ and state its domain.

12. Consider the differential equation $dy/dx = xy$.

(a) Let $y = f(x)$ be the function that satisfies the differential equation with initial condition $f(1) = 1$. Use Euler's Method, starting at $x = 1$ with a step size of 0.1, to approximate $f(1.2)$. Show the work that leads to your answer.

(b) Find d^2y/dx^2. Determine whether the approximation found in part (a) is less than or greater than $f(1.2)$. Justify your answer.

(c) Find the particular solution of the given differential equation that passes through $(1, 1)$.

13. At any time $t \geq 0$, the rate of the spread of an epidemic is modeled by a function y that satisfies the differential equation

$$\frac{dy}{dt} = \frac{1}{10}y\left(1 - \frac{y}{1000}\right).$$

In an isolated town of 1000 inhabitants, 100 people have a disease at the beginning of the week.

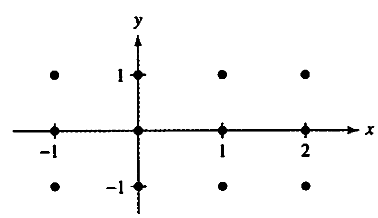
(a) Is the disease spreading faster when 100 people have the disease or when 200 people have the disease? Explain your reasoning.

(b) Write a model for the population $y = f(t)$ at any time $t \geq 0$.

(c) What is $\lim_{t \to \infty} y(t)$?

14. Consider the differential equation $\frac{dy}{dx} = \frac{x}{y^2}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(c) Find the particular solution of the given differential equation that satisfies the initial condition $y(0) = 2$.

AP® Exam Practice Questions for Chapter 6

What You Need to Know . . .

- The shell method is not required on the AP® Exam, but some free-response questions may be solvable by the shell method in addition to the disk method.
- The shell method is particularly advantageous when it is difficult to express one variable in terms of the other and when more than one integral is required to find a volume.
- On the AP® Calculus BC Exam, you may need to use the concept of arc length to find the perimeter of a given region.
- When limits of integration are irrational numbers, you can assign variables to represent each limit of integration. You can then write the correct integral expression using the variables, instead of writing out each integral in its entirety. This will help you save time.
- Some questions may just ask you to write, and not necessarily evaluate, the correct integral you can use to solve the problem.

Practice Questions

Section 1, Part A, Multiple Choice, No Technology

1. What is the area of the region bounded by the y-axis, the line $y = e$, and the graph of the function $y = e^{3x}$?

(A) $\frac{1}{3}$ (B) $e^{3e} - \frac{1}{3}$

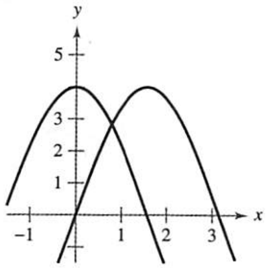
(C) $1 - \frac{2}{3}e$ (D) $3 - \frac{8}{3}e$

2. What is the area enclosed by the curves $y = x^3 - 7x^2 + 12x + 4$ and $y = 2x + 4$?

(A) $\frac{125}{12}$ (B) $\frac{63}{4}$

(C) $\frac{253}{12}$ (D) $\frac{445}{12}$

In Exercises 3 and 4, use the figure shown below. Let R be the region bounded by the graphs of $y = 4\cos x$, $y = 4\sin x$, and the y-axis.



3. Which expression represents the area of R?

(A) $4(\sqrt{2} - 1)$

(B) $\sqrt{2} - 1$

(C) $4(\sqrt{3} - 1)$

(D) $2\sqrt{2} - 1$

4. The horizontal line $y = 2$ splits the region R into two parts. What is the area of the part of R that is below this horizontal line?

(A) $\frac{2\pi}{3} - 2$ (B) $\frac{\pi}{3} + 2\sqrt{3} - 4$

(C) $\frac{2\pi}{3} + 2$ (D) $\frac{\pi}{3} + 2\sqrt{3}$

5. What is the area of the region bounded by the curves $x = y^2 - 4y$ and $y = -x + 4$?

(A) $\frac{95}{6}$ (B) 18

(C) $\frac{56}{3}$ (D) $\frac{125}{6}$

6. Which of the following integrals gives the length of the graph of $y = \ln(\sec x)$ from $x = 0$ to $x = \frac{\pi}{4}$?

(A) $\int_0^{\pi/4} \sec^2 x\,dx$ (B) $\int_0^{\pi/4} \sec x\,dx$

(C) $\int_0^{\pi/4} \sec x \tan x\,dx$ (D) $\int_0^{\pi/4} \sqrt{1 + \cos^2 x}\,dx$

7. Which of the following integrals gives the length of the graph of $y = 4e^{0.5x}$ between 1 and 4?

(A) $\int_1^4 \sqrt{1 + 4e^x}\,dx$ (B) $\int_1^4 \sqrt{1 + 16e^x}\,dx$

(C) $\int_1^4 \sqrt{1 + 2e^{0.5x}}\,dx$ (D) $\int_1^4 \sqrt{x + 16e^x}\,dx$

8. What is the arc length of the graph of $y = \frac{2}{3}x^{3/2}$ from $x = 3$ to $x = 8$?

(A) $\frac{32}{3}$ (B) $\frac{38}{3}$

(C) $\frac{40}{3}$ (D) 28

Section 1, Part B, Multiple Choice, Technology Permitted

9. The base of a solid is the region in the first quadrant bounded above by the line $y = 2$, below by $y = \sin^{-1} x$, and to the right by the line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 1.429

(B) 2

(C) 2.184

(D) 4

Section 2, Part A, Free Response, Technology Permitted

10. Consider the region bounded by the y-axis,

$$y = 10, \text{ and } y = 1 + 6x^{3/2}.$$

(a) Write, but do not evaluate, an integral equation that will find the value of k so that $x = k$ divides the region into two parts of equal area.

(b) Find the length of the curve $y = 1 + 6x^{3/2}$ on the interval $[0, 1]$.

(c) The region is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are rectangles with a height of 3 times that of its width. Find the volume of this solid.

11. Let R be the region bounded by the graphs of

$$y = \ln x \text{ and } y = 2x - 3.$$

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an expression involving one or more integrals that can be used to find the volume of the solid generated when R is revolved about the y-axis.

12. Let R be the region bounded by the graphs of

$$y = x^2 - 1 \text{ and } x = y^2.$$

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = 2$.

(c) Write, but do not evaluate, an expression involving one or more integrals to find the volume of the solid generated when R is rotated about the horizontal line $y = -1$.

Section 2, Part B, Free Response, No Technology

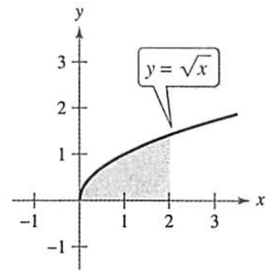
13. A region in the xy-plane is bounded by

$$y = 2x + 2, \quad x = \frac{y^2}{2} + 2, \quad y = -2, \quad \text{and} \quad y = 2.$$

(a) Sketch the bounded region. Label each boundary curve and shade the bounded region.

(b) Find the area of the bounded region. Show the work that leads to your answer.

14. The region shown below is bounded by $f(x) = \sqrt{x}$, $y = 0$, $x = 0$, and $x = 2$.



(a) Find the volume of the solid formed by rotating the region about the x-axis.

(b) Find the volume of the solid formed by rotating the region about the y-axis.

(c) Write an expression that gives the volume of the solid formed by rotating the region about the line $y = -2$.

(d) The region shown is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an equilateral triangle. Find the volume of this solid.

15. Consider the region R bounded by the graphs of $y = x^3$, $y = 8$, and the y-axis. The region S is bounded by $y = x^3$, $x = 2$, and the x-axis.

(a) Find the area of R.

(b) Find the volume of the solid formed by rotating R about the y-axis.

(c) The region S is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a semicircle with diameters extending from $y = x^3$ to the x-axis. Find the volume of this solid.

16. Consider the region T bounded by the graphs of $y = x^2$, $y = -2x$, and $x = 2$.

(a) Find the area of T.

(b) Find the volume of the solid formed by rotating T about the horizontal line $y = -4$.

(c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of T.