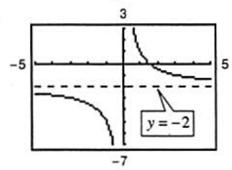
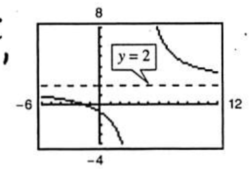
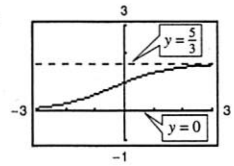
61. (a) -4 (b) 4 (c) Limit does not exist.
63. $x = \pm 3$ **65.** $x = \pm 8$ **67.** $x = \pm 5$ **69.** $-\infty$
71. $\frac{1}{3}$ **73.** $-\infty$ **75.** $\frac{4}{5}$ **77.** $-\infty$
79. (a) \$14,117.65; \$80,000.00; \$720,000.00
(b) ∞; No matter how much the company spends, the company will never be able to remove 100% of the pollutants.
81. 8 **83.** $\frac{2}{3}$ **85.** $-\infty$ **87.** 6
89.



91.



93.



AP® Exam Practice Questions for Chapter 1 *(page 120)*

1. B **2.** D **3.** A **4.** B **5.** D **6.** B
7. C **8.** B **9.** C **10.** D
11. (a) $\lim_{x\to 0} f(x) = 8$ (b) $\lim_{x\to 0} (f(x) + 4) = 12$
(c) $y = 0, y = 10$;

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{10}{1 + \frac{1}{4}e^{-x}} = \lim_{x\to\infty} \frac{10}{1 + \frac{1}{4e^x}} = \frac{10}{1 + \frac{1}{4e^\infty}} = \frac{10}{1+0} = 10$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{10}{1 + \frac{1}{4}e^{-x}} = \frac{10}{1 + \frac{1}{4}e^{\infty}} = \frac{10}{\infty} = 0$$

12. (a) $f(x)$ has discontinuities at $x = -3$ and $x = -\frac{1}{2}$.

(b) $$\lim_{x\to-3} f(x) = \lim_{x\to-3} \frac{x^2 + 5x + 6}{2x^2 + 7x + 3} = \lim_{x\to-3} \frac{(x+2)\cancel{(x+3)}}{(2x+1)\cancel{(x+3)}} = \lim_{x\to-3} \frac{x+2}{2x+1} = \frac{-3+2}{2(-3)+1} = \frac{1}{5}$$

(c) $x = -\frac{1}{2}$

(d) $y = \frac{1}{2}$;

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2 + 5x + 6}{2x^2 + 7x + 3} = \lim_{x\to\infty} \frac{1 + \frac{5}{x} + \frac{6}{x^2}}{2 + \frac{7}{x} + \frac{3}{x^2}} = \frac{1+0+0}{2+0+0} = \frac{1}{2}$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x^2 + 5x + 6}{2x^2 + 7x + 3} = \lim_{x\to-\infty} \frac{1 + \frac{5}{x} + \frac{6}{x^2}}{2 + \frac{7}{x} + \frac{3}{x^2}} = \frac{1+0+0}{2+0+0} = \frac{1}{2}$$

13. (a) 6; $\lim_{x\to1} (f(x) + 4) = \lim_{x\to1} f(x) + \lim_{x\to1} 4 = 2 + 4 = 6$
(b) 5; $\lim_{x\to3^-} \frac{5}{g(x)} = \frac{5}{1} = 5$
(c) 0; $\lim_{x\to2} (f(x) \cdot g(x)) = 2 \cdot 0 = 0$
(d) -2;

$$\lim_{x\to3} \frac{f(x)}{g(x) - 1} = \lim_{x\to3} \frac{-2x + 6}{(x-2) - 1} = \lim_{x\to3} \frac{-2x+6}{x-3} = \lim_{x\to3} \frac{-2\cancel{(x-3)}}{\cancel{x-3}} = -2$$

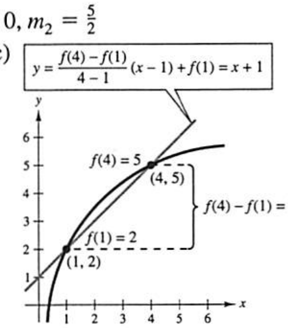
14. (a) $\frac{1}{e^2}$
(b) $f(0)$ is defined as $f(0) = e^{2(0)} = 1$. $\lim_{x\to0^-} f(x) = 1$ and $\lim_{x\to0^+} f(x) = 1$, so $\lim_{x\to0} f(x) = 1$. Also, $\lim_{x\to0} f(x) = f(0) = 1$. So, f is continuous at $x = 0$.
(c) 0
15. (a) 172; Because $T(x)$ is continuous on $[0, 10)$, $\lim_{x\to4} T(x) = T(4) = 172$.
(b) -2 degrees per minute
(c) (6, 8); $T(x)$ is continuous and when $x = 6$, $T(x) > 166.5°$ and when $x = 8$, $T(x) < 166.5°$.
(d) Slope $= -2$
16. (a) Because $s(t)$ is a continuous function and $s(1) = 392$ and $s(2) = 377.3$, there exists a time t, $1 < t < 2$ where $s(t)$ is a value between 377.3 and 392.
(b) 9 seconds
(c) -83.3 m/sec; This is a good measure of velocity because it is the object's average rate of change right before it hits the ground; It can be improved by finding the instantaneous rate of change using calculus.
(d) $$\lim_{x\to3} \frac{s(t) - s(3)}{t - 3} = \lim_{x\to3} \frac{(-4.9t^2 + 396.9) - (-4.9(3)^2 + 396.9)}{t-3} = \lim_{x\to3} \frac{-4.9t^2 + 4.9(9)}{t-3} = \lim_{x\to3} \frac{-4.9(t^2 - 9)}{t-3} = \lim_{x\to3} \frac{-4.9\cancel{(t-3)}(t+3)}{\cancel{t-3}} = \lim_{x\to3} -4.9(t+3) = -4.9(3+3) = -29.4 \text{ m/sec}$$

17. (a) $a = 2, b = 3$ (b) 9 (c) 5

Chapter 2

Section 2.1 *(page 131)*

1. $m_1 = 0, m_2 = \frac{5}{2}$
3. (a)–(c)



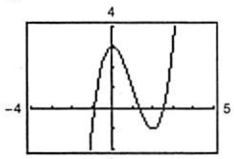
5. $m = -5$ **7.** $m = 8$ **9.** $m = 3$ **11.** $f'(x) = 0$
13. $f'(x) = -5$ **15.** $h'(s) = \frac{2}{3}$ **17.** $f'(x) = 2x + 1$

CHAPTER 2

3.

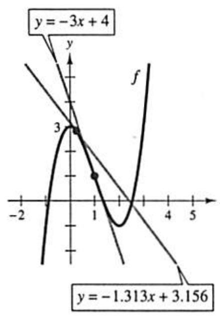
n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	1.6	−0.0292	−0.9996	0.0292	1.5708
2	1.5708	0	−1	0	1.5708

5. -1.587 **7.** 0.682 **9.** 1.250, 5.000 **11.** 0.567
13. 0.900, 1.100, 1.900 **15.** 1.935 **17.** 0.567
19. 4.493 **21.** (a) Proof (b) $\sqrt{5} \approx 2.236$; $\sqrt{7} \approx 2.646$
23. $f'(x_1) = 0$ **25.** 0.74 **27.** 1.12 **29.** Proof
31. (a)



(b) 1.347 (c) 2.532

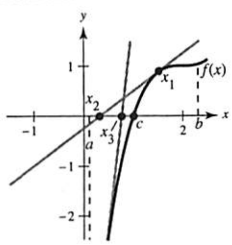
(d)



x-intercept of $y = -3x + 4$ is $\frac{4}{3}$.
x-intercept of $y = -1.313x + 3.156$ is approximately 2.404.

(e) If the initial estimate $x = x_1$ is not sufficiently close to the desired zero of a function, then the x-intercept of the corresponding tangent line to the function may approximate a second zero of the function.

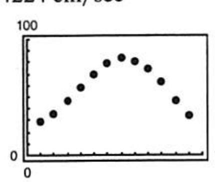
33. Answers will vary. Sample answer: If f is a function continuous on $[a, b]$ and differentiable on (a, b), where $c \in [a, b]$ and $f(c) = 0$, then Newton's Method uses tangent lines to approximate c. First, estimate an initial x_1 close to c. (See graph.) Then determine x_2 using $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$. Calculate a third estimate x_3 using $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$. Continue this process until $|x_n - x_{n+1}|$ is within the desired accuracy, and let x_{n+1} be the final approximation of c.



35. 4.486 hours **37.** True **39.** 0.217 **41.** A

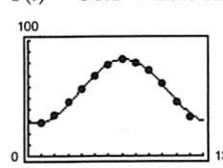
Review Exercises for Chapter 2 *(page 204)*

1. $f'(x) = 0$ **3.** $f'(x) = 2x - 4$ **5.** 5
7. f is differentiable for all $x \neq 3$.
9. f is differentiable on the interval $(1, \infty)$.
11. 0 **13.** $3x^2 - 22x$ **15.** $\frac{3}{\sqrt{x}} + \frac{1}{\sqrt[3]{x^2}}$ **17.** $-\frac{4}{3t^3}$
19. $4 - 5\cos\theta$ **21.** $-3\sin t - 4e^t$ **23.** -1 **25.** 0
27. (a) 50 (vibrations/sec)/lb (b) 33.33 (vibrations/sec)/lb
29. (a) $s(t) = -16t^2 - 30t + 600$; $v(t) = -32t - 30$
(b) -94 ft/sec
(c) $v'(1) = -62$ ft/sec; $v'(3) = -126$ ft/sec
(d) About 5.258 sec
(e) About -198.256 ft/sec
31. $4(5x^3 - 15x^2 - 11x - 8)$ **33.** $\frac{1}{2\sqrt{x}}\sin x + \sqrt{x}\cos x$
35. $-\frac{x^2+1}{(x^2-1)^2}$ **37.** $\frac{4x^3\cos x + x^4\sin x}{\cos^2 x}$
39. $3x^2\sec x\tan x + 6x\sec x$ **41.** $4xe^x + 4e^x + \csc^2 x$
43. $y = 4x + 10$ **45.** $y = -8x + 1$ **47.** $y = \frac{1}{2}x + 3$
49. $-48t$ **51.** $\frac{225}{4}\sqrt{x}$ **53.** $6\sec^2\theta\tan\theta$
55. (a) Proof (b) $-7920\sqrt{3}$ mi/degree
57. $28(7x + 3)^3$ **59.** $-\frac{2x}{(x^2+4)^2}$ **61.** $-45\sin(9x + 1)$
63. $\sin^2 x$ **65.** $(6x + 1)^4(36x + 1)$ **67.** $\frac{3}{(x^2+1)^{3/2}}$
69. $\frac{1}{4}te^{t/4}(t + 8)$ **71.** $\frac{e^{2x} - e^{-2x}}{\sqrt{e^{2x} + e^{-2x}}}$ **73.** $\frac{x(2-x)}{e^x}$
75. $\frac{1}{2x}$ **77.** $\frac{1 + 2\ln x}{2\sqrt{\ln x}}$ **79.** $\frac{x}{(a+bx)^2}$ **81.** $\frac{1}{x(a+bx)}$
83. $-\frac{3x^2}{2\sqrt{1-x^3}}$; -2 **85.** $-\frac{8x}{(x^2+1)^2}$; 2
87. $-(\csc 2x)\cot 2x$; 0 **89.** $384(8x + 5)$ **91.** $2\csc^2 x\cot x$
93. (a) -18.667°F/h (b) -7.284°F/h
(c) -3.240°F/h (d) -0.747°F/h
As the time increases, the rate of change of the temperature decreases.
95. 0.04224 cm/sec²
97. (a)



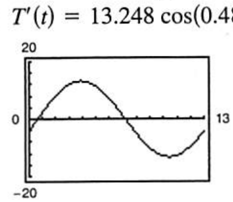
$T(t) = 56.1 + 27.6\sin(0.48t - 1.86)$

(b)



The model is a good fit.

(c) $T'(t) = 13.248\cos(0.48t - 1.86)$



(d) The temperature changes most rapidly around spring (March–May) and fall (Oct.–Nov.). The temperature changes most slowly around winter (Dec.–Feb.) and summer (June–Aug.). Yes. Explanations will vary.

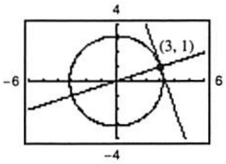
99. $-\frac{x}{y}$ **101.** $\frac{y(y^2 - 3x^2)}{x(x^2 - 3y^2)}$ **103.** $\frac{y\sin x + \sin y}{\cos x - x\cos y}$

105. $y = -6x + 28$ **107.** $y = -\frac{1}{2}x + 5$

109. Tangent line:

$y = -3x + 10$

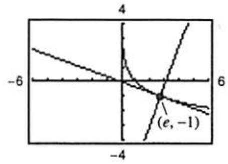
Normal line:
$y = \frac{1}{3}x$



111. Tangent line:

$y = -\frac{1}{e}x$

Normal line:
$y = ex - e^2 - 1$



113. $\dfrac{x^3 + 8x^2 + 4}{(x + 4)^2\sqrt{x^2 + 1}}$ **115.** $\dfrac{1}{3(\sqrt[3]{-3})^2} \approx 0.160$ **117.** $\dfrac{3}{4}$

119. $(1 - x^2)^{-3/2}$ **121.** $\dfrac{x}{|x|\sqrt{x^2 - 1}} + \operatorname{arcsec} x$

123. (a) $2\sqrt{2}$ units/sec (b) 4 units/sec (c) 8 units/sec

125. 450π km/h **127.** $\dfrac{d\theta}{dt} = \dfrac{1}{25}\cos^2\theta, -\dfrac{\pi}{4} \le \theta \le \dfrac{\pi}{4}$

129. $-0.347, -1.532, 1.879$ **131.** 1.202

133. 0.264, 1, 1.737

AP® Exam Practice Questions for Chapter 2 *(page 208)*

1. D **2.** A **3.** D **4.** A **5.** C **6.** A **7.** B
8. C **9.** B **10.** D **11.** B **12.** D

13. (a) $12xe^{2x^2}$ (b) 0.158 (c) $\dfrac{1 \pm \sqrt{5}}{4}$

14. (a) $\cos x$ (b) $\dfrac{1}{3x^{2/3}}$ (c) $\dfrac{1}{8}$ (d) $-\dfrac{1}{25}$

15. (a) $-\frac{1}{25}$ (b) -14 (c) $\frac{7}{4}$

16. (a) (i) (0, 1), (4.4, 5)
(ii) (2, 4.4)
(iii) (1, 2)
(b) (i) -2 ft/sec^2
(ii) 5 ft/sec^2

17. (a) $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$; $\tan x$ and $\sec^2 x$ are undefined at these values.
(b) $k = 0$

Chapter 3

Section 3.1 *(page 217)*

1. $f'(0) = 0$ **3.** $f'(2) = 0$ **5.** $f'(-2)$ is undefined.

7. 2, absolute maximum (and relative maximum)

9. 1, absolute maximum (and relative maximum)
2, absolute minimum (and relative minimum)
3, absolute maximum (and relative maximum)

11. $x = \frac{3}{4}$ **13.** $t = \frac{8}{3}$

15. $x = \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$ **17.** $t = \dfrac{1}{2}$ **19.** $x = 0$

21. Minimum: (2, 1); Maximum: $(-1, 4)$

23. Minimum: $(-3, -13)$; Maximum: (0, 5)

25. Minimum: $\left(-1, -\frac{5}{2}\right)$; Maximum: (2, 2)

27. Minimum: (0, 0); Maximum: $(-1, 5)$

29. Minimum: $(1, -1)$; Maximum: $\left(0, -\frac{1}{2}\right)$

31. Minimum: $(-1, -1)$; Maximum: (3, 3)

33. Minimum value is -2 for $-2 \le x < -1$; Maximum: (2, 2)

35. Minimum: $\left(\dfrac{3\pi}{2}, -1\right)$; Maximum: $\left(\dfrac{5\pi}{6}, \dfrac{1}{2}\right)$

37. Minimum: $(\pi, -3)$; Maxima: (0, 3) and $(2\pi, 3)$

39. Minimum: (0, 0); Maximum: $(-2, \arctan 4)$

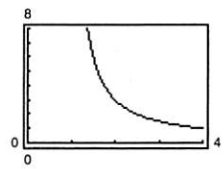
41. Minimum: $(2, 5e^2 - e^4)$; Maximum: $\left(\ln\frac{5}{2}, \frac{25}{4}\right)$

43. Minima: (0, 0) and $(\pi, 0)$; Maximum: $\left(\dfrac{3\pi}{4}, \dfrac{\sqrt{2}}{2}e^{3\pi/4}\right)$

45. $x = 0$ is a critical number but not a relative extremum; $x = -\frac{3}{2}$ and $x = 3$

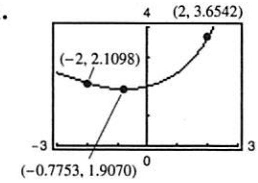
47. (a) Minimum: $(0, -3)$
Maximum: (2, 1)
(b) Minimum: $(0, -3)$
(c) Maximum: (2, 1)
(d) No extrema

49.



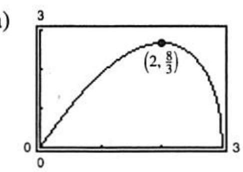
Minimum: (4, 1)

51.



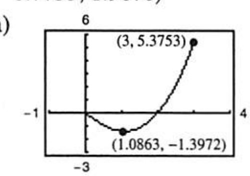
Minimum:
$(-0.7753, 1.9070)$

53. (a)



(b) Minima: (0, 0) and (3, 0)
Maximum: $\left(2, \frac{8}{3}\right)$

55. (a)



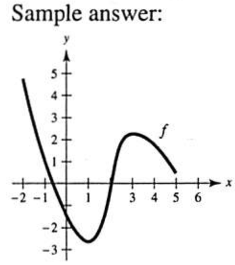
(b) Minimum
$(1.0863, -1.3972)$

57. Maximum: $\left|f''\left(\sqrt[3]{-10 + \sqrt{108}}\right)\right| = f''\left(\sqrt{3} - 1\right) \approx 1.47$

59. Maximum: $|f''(0)| = 1$

61. f is continuous on $\left[0, \dfrac{\pi}{4}\right]$ but not on $[0, \pi]$.

63. Answers will vary.
Sample answer:



65. (a) Yes (b) No

67. Maximum: $P(12) = 72$; No, P is decreasing for $I < 12$.

69. $\theta = \operatorname{arcsec}\sqrt{3} \approx 0.9553$ rad **71.** True

73. Proof **75.** D

77. (a) $y = -\dfrac{1}{2}x + \left(\dfrac{\pi}{6} + \dfrac{\sqrt{3}}{4}\right)$

(b) $x = \dfrac{\pi}{4}$, relative maximum; $x = \dfrac{3\pi}{4}$, relative minimum;

$x = \dfrac{5\pi}{4}$, relative maximum; $x = \dfrac{7\pi}{4}$, relative minimum

39. $f(x) = \sqrt{x}, dy = \dfrac{1}{2\sqrt{x}}\,dx$

$f(99.4) \approx \sqrt{100} + \dfrac{1}{2\sqrt{100}}(-0.6) = 9.97$

Calculator: 9.97

41. $f(x) = \sqrt[4]{x}, dy = \dfrac{1}{4x^{3/4}}\,dx$

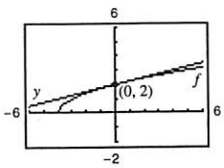
$f(624) \approx \sqrt[4]{625} + \dfrac{1}{4(625)^{3/4}}(-1) = 4.998$

Calculator: 4.998

43. $y - f(0) = f'(0)(x - 0)$

$y - 2 = \dfrac{1}{4}x$

$y = 2 + \dfrac{x}{4}$



45. (a) $f(x) = \sqrt{x}; dy = \dfrac{1}{2\sqrt{x}}\,dx$

$f(4.02) \approx \sqrt{4} + \dfrac{1}{2\sqrt{4}}(0.02) = 2 + \dfrac{1}{4}(0.02)$

(b) $f(x) = \tan x; dy = \sec^2 x\,dx$

$f(0.05) \approx \tan 0 + (\sec^2 0)(0.05) = 0 + 1(0.05)$

47. True

49. False; Let $f(x) = \sqrt{x}$, $x = 1$, and $\Delta x = dx = 3$.

51. A

53. (a) $P' = \frac{1}{4}e^{-x/400}(400 - x)$ (b) 400 units

(c) \$503; 5.8%

Review Exercises for Chapter 3 *(page 274)*

1. Maximum: $(0, 0)$

Minimum: $\left(-\frac{5}{2}, -\frac{25}{4}\right)$

3. Maximum: $(4, 0)$

Minimum: $(0, -2)$

5. Maximum: $\left(3, \frac{2}{3}\right)$

Minimum: $\left(-3, -\frac{2}{3}\right)$

7. Maximum: $(2\pi, 17.57)$

Minimum: $(2.73, 0.88)$

9. $f(0) \neq f(4)$

11. Not continuous on $[-2, 2]$

13. $f'\left(\frac{2744}{729}\right) = \frac{3}{7}$

15. f is not differentiable at $x = 5$.

17. $f'(0) = 1$

19. No; The function has a discontinuity at $x = 0$, which is in the interval $[-2, 1]$.

21. Increasing on $\left(-\frac{3}{2}, \infty\right)$; Decreasing on $\left(-\infty, -\frac{3}{2}\right)$

23. Increasing on $(-\infty, 1)$ and $\left(\frac{7}{3}, \infty\right)$; Decreasing on $\left(1, \frac{7}{3}\right)$

25. Increasing on $(1, \infty)$; Decreasing on $(0, 1)$

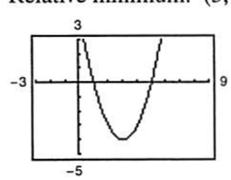
27. Increasing on $\left(-\infty, 2 - \dfrac{1}{\ln 2}\right)$; Decreasing on $\left(2 - \dfrac{1}{\ln 2}, \infty\right)$

29. (a) Critical number: $x = 3$

(b) Increasing on $(3, \infty)$; Decreasing on $(-\infty, 3)$

(c) Relative minimum: $(3, -4)$

(d)

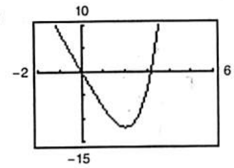


31. (a) Critical number: $t = 2$

(b) Increasing on $(2, \infty)$; Decreasing on $(-\infty, 2)$

(c) Relative minimum: $(2, -12)$

(d)



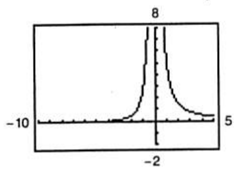
33. (a) Critical number: $x = -8$; Discontinuity: $x = 0$

(b) Increasing on $(-8, 0)$

Decreasing on $(-\infty, -8)$ and $(0, \infty)$

(c) Relative minimum: $\left(-8, -\frac{1}{16}\right)$

(d)



35. (a) Critical numbers: $x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

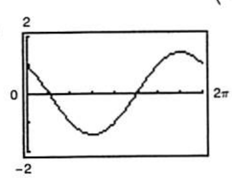
(b) Increasing on $\left(\dfrac{3\pi}{4}, \dfrac{7\pi}{4}\right)$

Decreasing on $\left(0, \dfrac{3\pi}{4}\right)$ and $\left(\dfrac{7\pi}{4}, 2\pi\right)$

(c) Relative minimum: $\left(\dfrac{3\pi}{4}, -\sqrt{2}\right)$

Relative maximum: $\left(\dfrac{7\pi}{4}, \sqrt{2}\right)$

(d)



37. $(3, -54)$; Concave upward: $(3, \infty)$;

Concave downward: $(-\infty, 3)$

39. No points of inflection; Concave upward: $(-5, \infty)$

41. $\left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right), \left(\dfrac{3\pi}{2}, \dfrac{3\pi}{2}\right)$; Concave upward: $\left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$;

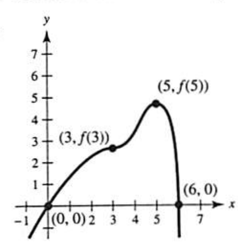
Concave downward: $\left(0, \dfrac{\pi}{2}\right), \left(\dfrac{3\pi}{2}, 2\pi\right)$

43. Relative minimum: $(-9, 0)$

45. Relative maxima: $\left(\pm\dfrac{\sqrt{2}}{2}, \dfrac{1}{2}\right)$; Relative minimum: $(0, 0)$

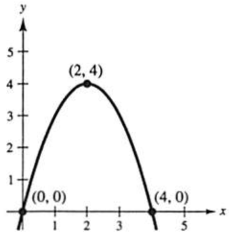
47. Relative maximum: $(-3, -12)$; Relative minimum: $(3, 12)$

49.

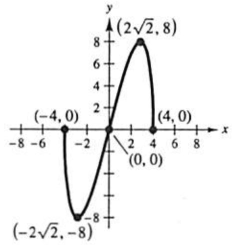


51. $x = \sqrt{\dfrac{2Qs}{r}}$

53.



55.



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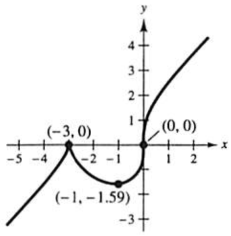
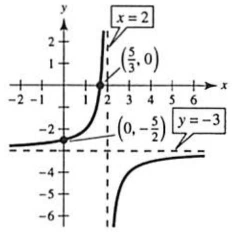
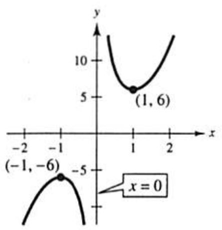


59.



61.



63. $x = 50$ ft, $y = \frac{200}{3}$ ft **65.** (0, 0), (5, 0), (0, 10)

67. 14.05 ft **69.** $\dfrac{32\pi r^3}{81}$ units^3

71. $\Delta y = 0.03005$; $dy = 0.03$

73. $(1 + x \sin x - \cos x)\,dx$ **75.** $\Delta p = -\frac{1}{4}$; $dp = -\frac{1}{4}$

AP® Exam Practice Questions for Chapter 3 *(page 276)*

1. B **2.** B **3.** B **4.** C **5.** D **6.** A **7.** A

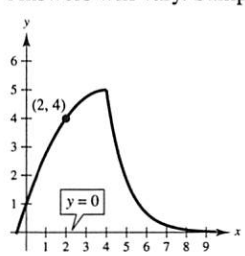
8. (a) $(-\infty, 4)$

(b) Yes; It goes from increasing to decreasing and $f(x)$ is continuous.

(c) $x = 4$; At this value, $f''(x)$ changes sign.

(d) No; $f(x)$ is not differentiable on (3, 5).

(e) Answers will vary. Sample answer:



9. (a) $x \approx \pm 0.7108$ (b) $y = \dfrac{3\pi^2}{8}x - \dfrac{\pi^3}{8}$

(c) $f(1.5) \approx 1.6759$; This is an underestimate because $f'\left(\frac{\pi}{2}\right) > f'(1.5)$.

10. (a) $2\pi + 1$; Critical number: $x = \dfrac{\pi}{4}$,

$$f(\pi) = 2\pi + 1 > f\left(\frac{\pi}{4}\right) > f(0)$$

(b) $f(x)$ is continuous on $[0, \pi]$ and differentiable on $(0, \pi)$; $x = \dfrac{\pi}{2}$

11. (a) (0, 2)

(b) $(-2, -0.8)$, $(1.3, \infty)$; $f(x)$ is concave downward when $f'(x)$ is decreasing.

(c) Negative; $f'(x)$ is negative so the slope of $f(x)$ is negative there.

12. (a) $(-\infty, 0)$, $(0, \infty)$

(b) $f(x)$ is never concave downward because $f''(x) > 0$ for all real x.

(c) $f(x)$ has no points of inflection because it does not change concavity.

13. (a) $f'(-1) = -\dfrac{2}{3}$; $f''(-1) = -\dfrac{1}{3}$

(b) $x = -3$; $f'(x)$ changes from positive to negative, so f has a relative maximum at $x = -3$.

(c) $(-4, 0)$ and $(1, 4)$; f is concave downward when f' is decreasing.

(d) $x = -4$, $x = 0$, and $x = 1$; $f''(0)$ and $f''(1)$ are undefined, and $f''(-4) = 0$.

(e) Decreasing;

$$g(x) = f(x) + \sin^2 x$$
$$g'(x) = f'(x) + 2 \sin x \cos x$$
$$g'\left(-\frac{\pi}{4}\right) = f'\left(-\frac{\pi}{4}\right) + 2 \sin\left(-\frac{\pi}{4}\right)\cos\left(-\frac{\pi}{4}\right)$$
$$= f'\left(-\frac{\pi}{4}\right) + 2\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$= f'\left(-\frac{\pi}{4}\right) - 1$$

From the graph, $f'\left(-\frac{\pi}{4}\right)$ is negative. So, $g'\left(-\frac{\pi}{4}\right)$ is negative, which means that g is decreasing at $x = -\dfrac{\pi}{4}$.

Chapter 4

Section 4.1 *(page 287)*

1. Proof **3.** $y = 3t^3 + C$ **5.** $y = \dfrac{2}{5}x^{5/2} + C$

	Original Integral	*Rewrite*	*Integrate*	*Simplify*
7.	$\int \sqrt[3]{x}\,dx$	$\int x^{1/3}\,dx$	$\dfrac{x^{4/3}}{4/3} + C$	$\dfrac{3}{4}x^{4/3} + C$
9.	$\int \dfrac{1}{x\sqrt{x}}\,dx$	$\int x^{-3/2}\,dx$	$\dfrac{x^{-1/2}}{-1/2} + C$	$-\dfrac{2}{\sqrt{x}} + C$

11. $\frac{3}{4}x^4 - 2x^3 + 2x + C$ **13.** $\frac{2}{5}x^{5/2} + x^2 + x + C$

15. $\dfrac{3}{5}x^{5/3} + C$ **17.** $-\dfrac{1}{4x^4} + C$ **19.** $\dfrac{2}{3}x^{3/2} + 12x^{1/2} + C$

21. $x^3 + \dfrac{1}{2}x^2 - 2x + C$ **23.** $5 \sin x - 4 \cos x + C$

25. $-2 \cos x - 5e^x + C$ **27.** $\tan y + C$

29. $x^2 - \dfrac{4^x}{\ln 4} + C$ **31.** $\dfrac{1}{2}x^2 - 5 \ln|x| + C$

Review Exercises for Chapter 4 *(page 362)*

1. $\frac{4}{3}x^3 + \frac{1}{2}x^2 + 3x + C$ **3.** $\frac{x^2}{2} - \frac{4}{x^2} + C$

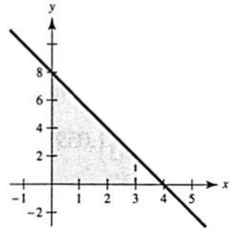
5. $5x - e^x + C$ **7.** $f(x) = -3x^2 + 1$

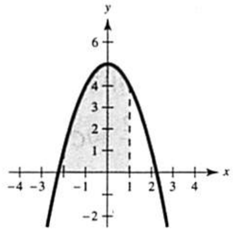
9. $f(x) = 4x^3 - 5x - 3$

11. (a) 3 sec; 144 ft (b) $\frac{3}{2}$ sec (c) 108 ft

13. 60 **15.** $\sum_{n=1}^{10} \frac{1}{3n}$ **17.** 420 **19.** 3310

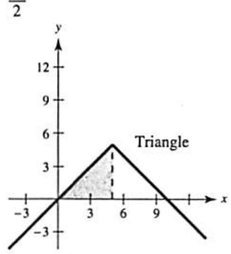
21. $A = 15$ **23.** $A = 12$





25. $\frac{27}{2}$

27.



$A = \frac{25}{2}$

29. (a) 17 (b) 7 (c) 9 (d) 84

31. (a) $x = -5$ (b) -56

33. Trapezoidal Rule: 0.285; Graphing utility: 0.284

35. Trapezoidal Rule: 3.432; Graphing utility: 3.406

37. 56 **39.** $\frac{422}{5}$ **41.** $1 + e^2$ **43.** 30

45. $2 \ln 3 \approx 2.1972$ **47.** Average value $= \frac{2}{5}$; $x = \frac{25}{4}$

49. $x^2\sqrt{1 + x^3}$ **51.** $x^2 + 3x + 2$ **53.** $\frac{2}{3}\sqrt{x^3 + 3} + C$

55. $\frac{1}{30}(3x^2 - 1)^5 + C$ **57.** $\frac{1}{4}\sin^4 x + C$

59. $-2\sqrt{1 - \sin\theta} + C$ **61.** $\frac{1}{2 \ln 5} 5^{(x+1)^2} + C$

63. $\frac{1}{3\pi}(1 + \sec \pi x)^3 + C$ **65.** $\frac{455}{2}$ **67.** 2 **69.** $\frac{28\pi}{15}$

71. 2 **73.** $\frac{1}{7}\ln|7x - 2| + C$ **75.** $-\ln|1 + \cos x| + C$

77. $3 + \ln 2$ **79.** $\ln(2 + \sqrt{3})$ **81.** $\frac{1}{2}\arctan e^{2x} + C$

83. $\frac{1}{2}\arcsin x^2 + C$ **85.** $\frac{1}{4}\left(\arctan \frac{x}{2}\right)^2 + C$

AP® Exam Practice Questions for Chapter 4 *(page 364)*

1. B **2.** C **3.** A **4.** A **5.** C

6. D **7.** D **8.** C

9. (a) $-25°$; This represents the total temperature lost from $t = 0$ to $t = 12$.

(b) The average temperature for $0 \le t \le 12$; 49.9°C

(c) -3.5°C/min (d) 35.1°C

10. (a) 2 in. (b) $I(t) = 0.006t^2 - 0.12t + 40.87 - \frac{\pi}{6}\sin\frac{\pi t}{12}$

(c) -0.1809 in./h

(d) $t = 0$; $I(t)$ is decreasing over $[0, 6]$.

11. (a) The particle is moving to the right on the t-intervals $(0, 4)$ and $(8, 9)$ because $v(t) > 0$ on these intervals.

(b) $\int_0^9 \sin\frac{\pi t}{4}\,dt$

(c) -0.555; slowing down because $-0.555 < 0$

(d) -1.83

12. (a) $F(0) \approx -5.64$; $F'(0) = 3$; $F(4) = -\frac{1}{2}$

(b) None; On $[-3, 4]$, $f(x)$ does not change from negative to positive at any point.

(c) $x = 0$; $F'(x)$ changes sign at $x = 0$. (d) $y = x - \frac{5}{2}$

13. (a) $F(0) = 14.5$; $F(4) = 18.5$

(b) 2 times; $F(0) > 5$, $F(x)$ is decreasing on $(0, 3)$, $F(3) < 5$, $F(x)$ is increasing on $(3, 4)$, $F(4) > 5$

(c) $(3, 4)$; $f(x) > 0$ at $(3, 4)$

Chapter 5

Section 5.1 *(page 373)*

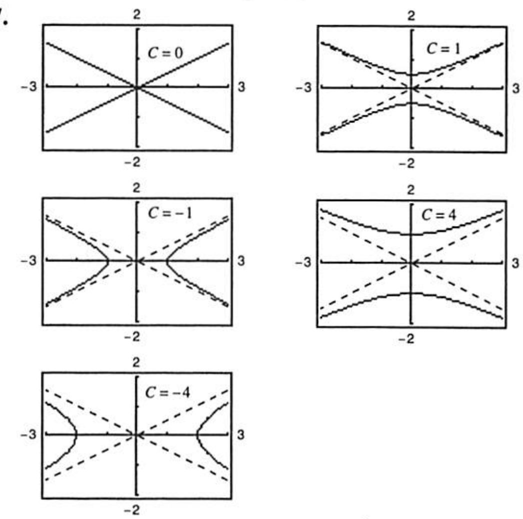
1. Proof **3.** Proof **5.** Proof **7.** Proof **9.** Proof

11. Proof **13.** Not a solution **15.** Solution

17. Solution **19.** Not a solution **21.** Solution

23. Not a solution **25.** $y = 3e^{-x/2}$

27.



29. $y = 3e^{-6x}$ **31.** $y = 2\sin 3x - \frac{1}{3}\cos 3x$

33. $y = -2x + \frac{1}{2}x^3$ **35.** $y = 4x^3 + C$

37. $y = \frac{1}{2}\ln(1 + x^2) + C$ **39.** $y = x - \ln x^2 + C$

41. $y = -\frac{1}{2}\cos 2x + C$

43. $y = \frac{2}{5}(x - 6)^{5/2} + 4(x - 6)^{3/2} + C$ **45.** $y = \frac{1}{2}e^{x^2} + C$

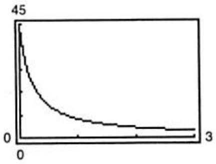
47.

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
$\frac{dy}{dx}$	-4	Undef.	0	1	$\frac{4}{3}$	2

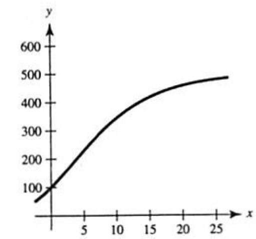
49.

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
$\frac{dy}{dx}$	$-2\sqrt{2}$	-2	0	0	$-2\sqrt{2}$	-8

51. $y = \dfrac{360}{8 + 41t}$



53. $y = 500e^{-1.6094e^{-0.1451t}}$

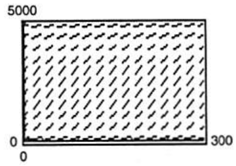


55. 34 beavers **57.** (a) $Q = 25e^{-(1/20)t}$ (b) $t \approx 10.2$ min

59. (a) $y = Ce^{kt}$ (b) About 6.2 h **61.** About 3.15 h

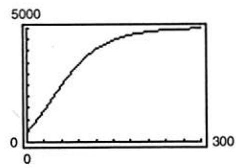
63. $A = \dfrac{P}{r}(e^{rt} - 1)$ **65.** $23,981,015.77

67. (a)



(b) As $t \to \infty$, $y \to L$.
(c) $y = 5000e^{-2.303e^{-0.02t}}$

(d)



The graph is concave upward on (0, 41.7) and concave downward on $(41.7, \infty)$.

69. Separable; $\dfrac{1}{y}\,dy = -\dfrac{1 + x}{x}\,dx$ **71.** Not separable

73. (a) $v = 20(1 - e^{-1.386t})$
(b) $s \approx 20t + 14.43(e^{-1.386t} - 1)$

75. False; $y' = \dfrac{x}{y}$ is separable, but $y = 0$ is not a solution.

77. A **79.** (a) $y = -4e^{-1/x + 1/3} + 4$ (b) $-4e^{1/3} + 4$

Section 5.4 *(page 400)*

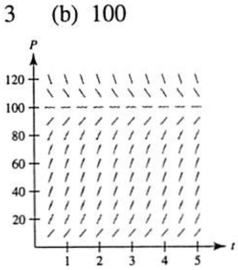
1. d **2.** a **3.** b **4.** c

5. $y(0) = 4$ **7.** $y(0) = \frac{12}{7}$

9. (a) 0.75 (b) 2100 (c) 70 (d) 4.49 yr
(e) $\dfrac{dP}{dt} = 0.75P\left[1 - \dfrac{P}{2100}\right]$

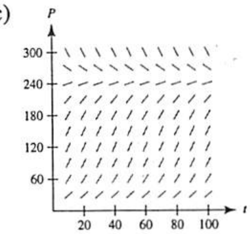
11. (a) 0.8 (b) 6000 (c) 1.2 (d) 10.65 yr
(e) $\dfrac{dP}{dt} = 0.8P\left[1 - \dfrac{P}{6000}\right]$

13. (a) 3 (b) 100
(c)



(d) 50

15. (a) 0.1 (b) 250
(c)



(d) 125

17. $y = \dfrac{36}{1 + 8e^{-t}}$; 34.16; 36.00

19. $y = \dfrac{120}{1 + 14e^{-0.8t}}$; 95.51; 120.0

21. L represents the value that y approaches as t approaches infinity. L is the carrying capacity.

23. Yes; It can be written as $\dfrac{dy}{ky\left(1 - \dfrac{y}{L}\right)} = dt.$

25. (a) $P = \dfrac{200}{1 + 7e^{-0.2640t}}$ (b) 70 panthers (c) 7.37 yr
(d) $\dfrac{dP}{dt} = 0.2640P\left(1 - \dfrac{P}{200}\right)$; 69.25 panthers (e) 100 yr

27. Proof

29. False; $\dfrac{dy}{dt} < 0$ and the population decreases to approach L.

31. (a) 5 (b) $y = 10$ (c) $y = \dfrac{5}{1 + \frac{2}{3}e^{-2t}}$

Review Exercises for Chapter 5 *(page 402)*

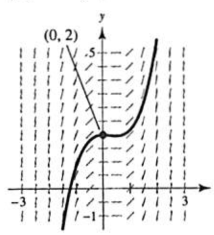
1. Solution **3.** $y = \frac{4}{3}x^3 + 7x + C$

5. $y = \frac{1}{2}\sin 2x + C$ **7.** $y = -e^{2-x} + C$

9.

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
$\dfrac{dy}{dx}$	-10	-4	-4	0	2	8

11. (a) and (b)



13.

n	0	1	2	3	4	5
x_n	0	0.05	0.1	0.15	0.2	0.25
y_n	4	3.8	3.6125	3.4369	3.2726	3.1190

n	6	7	8	9	10
x_n	0.3	0.35	0.4	0.45	0.5
y_n	2.9756	2.8418	2.7172	2.6038	2.4986

15. $y = -\frac{5}{3}x^3 + x^2 + C$ **17.** $y = -3 - \dfrac{1}{x + C}$

19. $y = \dfrac{Ce^x}{(2 + x)^2}$ **21.** $\dfrac{dy}{dt} = \dfrac{k}{t^3}$; $y = -\dfrac{k}{2t^2} + C$

23. $y \approx \frac{3}{4}e^{[(1/5)\ln(20/3)]t} \approx \frac{3}{4}e^{0.3794t}$

25. $y = \frac{9}{20}e^{[(1/2)\ln(10/3)]t} \approx \frac{9}{20}e^{0.6020t}$

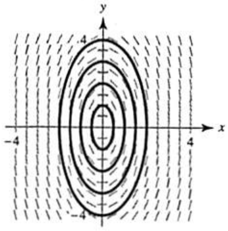
27. About 7.79 in. **29.** About 37.5 yr

31. (a) $S \approx 30e^{-1.7918/t}$ (b) 20,965 units

33. $y^2 = 5x^2 + C$ **35.** $y = Ce^{8x^2}$

37. $y^4 = 6x^2 - 8$ **39.** $y^4 = 2x^4 + 1$

41. Graphs will vary.



$4x^2 + y^2 = C$

43. (a) 0.55 (b) 5250 (c) 150 (d) 6.41 yr

(e) $\dfrac{dP}{dt} = 0.55P\left(1 - \dfrac{P}{5250}\right)$

45. $y = \dfrac{80}{1 + 9e^{-t}}$

47. (a) $P = \dfrac{20{,}400}{1 + 16e^{-0.553t}}$

(b) 17,118 trout

(c) 4.94 yr

49. $\dfrac{dS}{dt} = k(L - S)$; $S = L(1 - e^{-kt})$

51. $\dfrac{dP}{dn} = kP(L - P)$; $P = \dfrac{CL}{e^{-Lkn} + C}$

AP® Exam Practice Questions for Chapter 5 *(page 404)*

1. B **2.** B **3.** A **4.** C **5.** B **6.** A **7.** A

8. (a) $y = 200e^{0.5t}$ (b) $\dfrac{1}{10}\displaystyle\int_0^{10} 200e^{0.5t}\,dt = 40(e^5 - 1)$

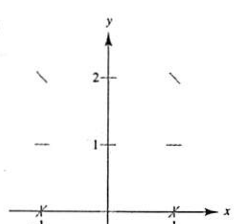
(c) $\dfrac{1}{20}\displaystyle\int_0^{10} 200e^{0.5t}\,dt$ bacteria per hour

9. (a) $-\frac{5}{8}$

(b) $y = \frac{1}{2}x + \frac{3}{2}$; $f(1.1) \approx 2.05$; $2.05 > f(1.1)$ because $f''(1.1) < 0$.

(c) $f(x) = \sqrt{2\ln|x| + 4}$

10. (a)



(b) $\dfrac{dy}{dx} > 0$ when $y < 1$

(c) $f(x) = 1 + e^{(-1/3)x^3}$

11. (a) $y_1 = 2.2$, $y_2 \approx 2.418$; $f(1.4) \approx 2.418$

(b) $f(x) = \sqrt{2x^2 + 2}$; Domain: $(-\infty, \infty)$

12. (a) $y_1 = 1.1$, $y_2 = 1.221$; $f(1.2) \approx 1.221$

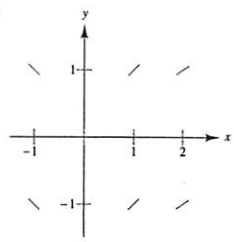
(b) $\dfrac{d^2y}{dx^2} = x^2y + y$; On the interval $[1, 1.2]$, x is positive. Because $\dfrac{dy}{dx} = xy$, $\dfrac{dy}{dx}$ and y have the same sign on $[1, 1.2]$. Because $y = f(1) = 1$, $\dfrac{dy}{dx}$ and y are both positive on $[1, 1.2]$. Therefore, $\dfrac{d^2y}{dx^2} > 0$ and the approximation found in part (a) is less than $f(1.2)$.

(c) $f(x) = e^{0.5x^2 - 0.5}$

13. (a) It is spreading faster when 200 people have the disease; Because when $y = 200$, $\dfrac{dy}{dx} > \dfrac{dy}{dx}$, when $y = 100$.

(b) $f(t) = \dfrac{1000}{1 + 9e^{-0.1t}}$ (c) 1000

14. (a)



(b) $\dfrac{d^2y}{dx^2} = \dfrac{y^3 - 2x^2}{y^5}$

(c) $y = \sqrt[3]{\frac{3}{2}x^2 + 8}$

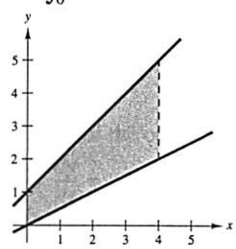
Chapter 6

Section 6.1 *(page 414)*

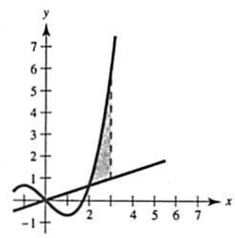
1. $-\displaystyle\int_0^6 (x^2 - 6x)\,dx$ **3.** $\displaystyle\int_0^3 (-2x^2 + 6x)\,dx$

5. $-6\displaystyle\int_0^1 (x^3 - x)\,dx$

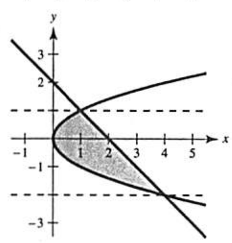
7.



9.



11.



13. Should have subtracted g from f;

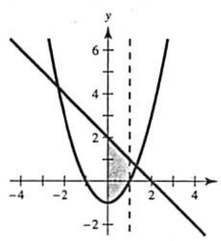
$A = \displaystyle\int_0^2 (2x^3 + 2x^2 - 4x)\,dx$

15. d

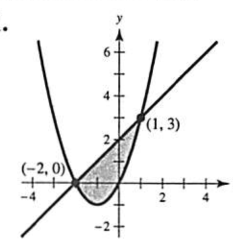
17. (a) $\frac{125}{6}$ (b) $\frac{125}{6}$

(c) Integrating with respect to y; Answers will vary.

19. $\frac{13}{6}$



21. $\frac{9}{2}$

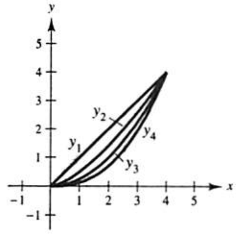


45. A rectifiable curve is a curve with a finite arc length.

47. The integral formula for the area of a surface of revolution is derived from the formula for the lateral surface area of the frustum of a right circular cone. The formula is $S = 2\pi rL$, where $r = \frac{1}{2}(r_1 + r_2)$, which is the average radius of the frustum, and L is the length of a line segment on the frustum. The representative element is

$$2\pi f(d_i)\sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\,\Delta x_i.$$

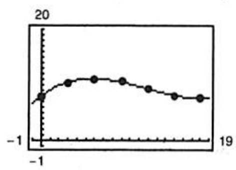
49. (a)



(b) y_1, y_2, y_3, y_4
(c) $s_1 \approx 5.657$
$s_2 \approx 5.759$
$s_3 \approx 5.916$
$s_4 \approx 6.063$

51. 20π **53.** $6\pi(3 - \sqrt{5}) \approx 14.40$

55. (a) Answers will vary. Sample answer: 5207.62 in.^{3}
(b) Answers will vary. Sample answer: 1168.64 in.^{2}
(c) $r = 0.0040y^3 - 0.142y^2 + 1.23y + 7.9$



(d) 5279.64 in.^{3}; 1179.5 in.^{2}

57. (a) $\pi\left(1 - \frac{1}{b}\right)$ (b) $2\pi\int_1^b \frac{\sqrt{x^4+1}}{x^3}\,dx$

(c) $\lim_{b\to\infty} V = \lim_{b\to\infty}\left[\pi\left(1 - \frac{1}{b}\right)\right] = \pi$

(d) Because $\frac{\sqrt{x^4+1}}{x^3} > \frac{\sqrt{x^4}}{x^3} = \frac{1}{x} > 0$ on $[1, b]$, you have

$\int_1^b \frac{\sqrt{x^4+1}}{x^3}\,dx > \int_1^b \frac{1}{x}\,dx = \Big[\ln x\Big]_1^b = \ln b$ and

$\lim_{b\to\infty} \ln b \to \infty$. So, $\lim_{b\to\infty} 2\pi\int_1^b \frac{\sqrt{x^4+1}}{x^3}\,dx = \infty.$

59. Fleeing object: $\frac{2}{3}$ unit

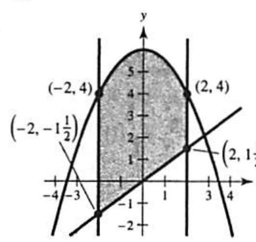
Pursuer: $\frac{1}{2}\int_0^1 \frac{x+1}{\sqrt{x}}\,dx$

61. Proof **63.** B **65.** C

Review Exercises for Chapter 6 *(page 448)*

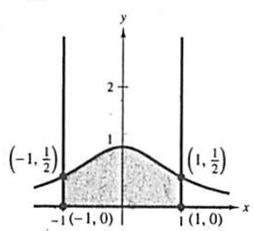
1. $\int_0^2 (\sin \pi x - x^3 + 4x)\,dx$ **3.** $2\int_0^2 (-2x^3 + 8x)\,dx$

5.



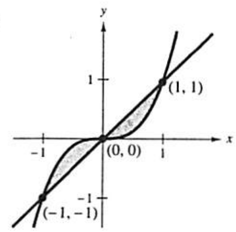
$\frac{64}{3}$

7.



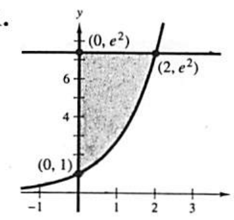
$\frac{\pi}{2}$

9.



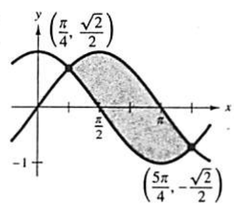
$\frac{1}{2}$

11.



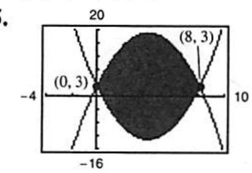
$e^2 + 1$

13.



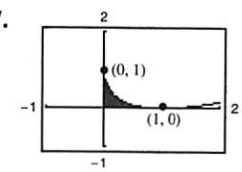
$2\sqrt{2} \approx 2.828$

15.



$\frac{512}{3} \approx 170.667$

17.



$\frac{1}{6} \approx 0.1667$

19. 9920 ft^2 **21.** $\frac{27}{4}$

23. (a) 9π (b) 18π (c) 9π (d) 36π

25. $\frac{\pi^2}{4}$ **27.** $2\pi \ln\frac{5}{2} \approx 5.757$ **29.** $\frac{\pi}{3}$ **31.** 1.958 ft

33. (a) and (b) 13 **35.** $\frac{8}{15}(1 + 6\sqrt{3}) \approx 6.076$

37. $100(e^{1/2} - e^{-1/2}) \approx 104.22$ ft

39. $3 \arcsin\frac{2}{3} \approx 2.1892$ **41.** 15π

43. $2\pi\int_0^8 (4 - x^{2/3})^{3/2}\sqrt{\frac{4}{x^{2/3}}}\,dx$

AP® Exam Practice Questions for Chapter 6 *(page 450)*

1. A **2.** C **3.** A **4.** B **5.** D **6.** B
7. A **8.** B **9.** D

10. (a) $\int_1^{10}\left(\frac{y-1}{6}\right)^{2/3} dy = 2\int_0^{1+6k^{3/2}}\left(\frac{y-1}{6}\right)^{2/3} dy$
(b) 6.1032 (c) 15.9210

11. (a) 1.4706 (b) 18.7834
(c) $\pi\int_{-2.888703}^{0.58307388}\left[\left(\frac{y+3}{2}\right)^2 - (e^y)^2\right] dy$

12. (a) 1.3767 (b) 11.5014
(c) $\pi\int_0^{0.5248886}\left[(\sqrt{x}+1)^2 - (-\sqrt{x}+1)^2\right] dx$
$+ \pi\int_{0.5248886}^{1.4902161}\left[(\sqrt{x}+1)^2 - (x^2)^2\right] dx$

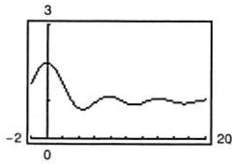
85. $v = 32t + v_0$ **87.** Proof **89.** $c = \frac{2}{3}$ **91.** $c = \frac{\pi}{4}$

93. False; L'Hôpital's Rule does not apply because $\lim_{x \to 0} (x^2 + x + 1) \neq 0$.

95. True **97.** $\frac{3}{4}$ **99.** $\frac{4}{3}$ **101.** $a = 1, b = \pm 2$

103. Proof **105.** Proof **107.** Proof

109. (a)

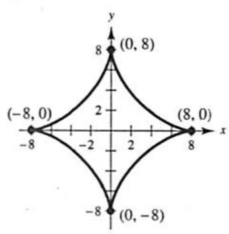


(b) $\lim_{x \to \infty} h(x) = 1$
(c) No

111. B **113.** C

Section 7.8 *(page 522)*

1. Improper; infinite discontinuity at $x = \frac{3}{5}$
3. Not improper; continuous on $[0, 1]$
5. Not improper; continuous on $[0, 2]$
7. Improper; infinite limits of integration
9. Infinite discontinuity at $x = 0$; 4
11. Infinite discontinuity at $x = 1$; diverges
13. Infinite discontinuity at $x = 0$; diverges
15. Infinite limit of integration; converges to 1
17. $\frac{1}{2}$ **19.** Diverges **21.** Diverges **23.** 2
25. $\frac{1}{2(\ln 4)^2}$ **27.** π **29.** $\frac{\pi}{4}$ **31.** Diverges
33. Diverges **35.** 0 **37.** $-\frac{1}{4}$ **39.** Diverges
41. $\frac{\pi}{3}$ **43.** $\ln 3$ **45.** $\frac{\pi}{6}$ **47.** $\frac{2\pi\sqrt{6}}{3}$ **49.** $p > 1$
51. Proof **53.** Diverges **55.** Converges
57. Converges **59.** Diverges **61.** Converges
63. When the limit of the integral exists, the improper integral converges. When the limit does not exist, the improper integral diverges.
65. $\frac{7}{8}$ **67.** π **69.** (a) 1 (b) $\frac{\pi}{2}$ (c) 2π
71.

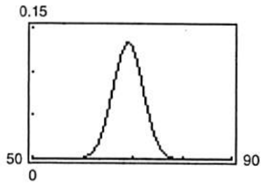


Perimeter = 48

73. (a) Proof (b) $P = 43.53\%$ (c) $E(x) = 7$
75. (a) \$757,992.41 (b) \$837,995.15 (c) \$1,066,666.67
77. $P = \frac{2\pi NI\left(\sqrt{r^2 + c^2} - c\right)}{kr\sqrt{r^2 + c^2}}$
79. False; Let $f(x) = \frac{1}{x + 1}$. **81.** True
83. (a) and (b) Proofs
(c) The definition of the improper integral $\int_{-\infty}^{\infty}$ is not $\lim_{a \to \infty} \int_{-a}^{a}$ but rather that if you rewrite the integral that diverges, you can find that the integral converges.
85. $\frac{1}{s}, s > 0$ **87.** $\frac{2}{s^3}, s > 0$ **89.** $\frac{s}{s^2 + a^2}, s > 0$

91. $\int_0^1 2 \sin u^2 \, du$; 0.6278 **93.** Proof

95. (a)



(b) About 0.1587
(c) 0.1587; same by symmetry

97. (a) $\Gamma(1) = 1, \Gamma(2) = 1, \Gamma(3) = 2$ (b) Proof
(c) $\Gamma(n) = (n - 1)!$
99. $c = 1$; $\ln 2$
101. $8\pi\left[\frac{(\ln 2)^2}{3} - \frac{2 \ln 2}{9} + \frac{2}{27}\right] \approx 2.0155$ **103.** B

Review Exercises for Chapter 7 *(page 526)*

1. $\frac{1}{3}(x^2 - 36)^{3/2} + C$ **3.** $\frac{1}{2}\ln|x^2 - 49| + C$
5. $\frac{1}{2} + \ln 2 \approx 1.1931$ **7.** $100 \arcsin \frac{x}{10} + C$
9. $\frac{1}{9}e^{3x}(3x - 1) + C$
11. $\frac{1}{13}e^{2x}(2 \sin 3x - 3 \cos 3x) + C$
13. $-\frac{1}{2}x^2 \cos 2x + \frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x + C$
15. $\frac{1}{8}\left[(4x^2 - 1) \arcsin 2x + 2x\sqrt{1 - 4x^2}\right] + C$
17. $\frac{1}{3\pi}[\sin(\pi x - 1)][2 + \cos^2(\pi x - 1)] + C$
19. $\frac{2}{3}\left[\tan^3 \frac{x}{2} + 3 \tan \frac{x}{2}\right] + C$ **21.** $\tan \theta + \sec \theta + C$
23. $\frac{3\pi}{16} + \frac{1}{2} \approx 1.0890$ **25.** $\frac{3\sqrt{4 - x^2}}{x} + C$
27. $\frac{1}{3}(x^2 + 4)^{1/2}(x^2 - 8) + C$ **29.** $256 - 62\sqrt{17}$
31. (a)–(c) $\frac{1}{3}\sqrt{4 + x^2}(x^2 - 8) + C$
33. $-5 \ln|x - 4| + 6 \ln|x + 3| + C$
35. $\frac{1}{4}[6 \ln|x - 1| - \ln(x^2 + 1) + 6 \arctan x] + C$
37. $x - \frac{64}{11} \ln|x + 8| + \frac{9}{11} \ln|x - 3| + C$
39. $\frac{1}{25}\left(\frac{4}{4 + 5x} + \ln|4 + 5x|\right) + C$ **41.** $1 - \frac{\sqrt{2}}{2}$
43. $\frac{1}{2} \ln|x^2 + 4x + 8| - \arctan\left(1 + \frac{x}{2}\right) + C$ **45.** Proof
47. $\frac{1}{8}(\sin 2\theta - 2\theta \cos 2\theta) + C$
49. $\frac{4}{3}[x^{3/4} - 3x^{1/4} + 3 \arctan(x^{1/4})] + C$
51. $2\sqrt{1 - \cos x} + C$ **53.** $(\sin x) \ln(\sin x) - \sin x + C$
55. $\frac{5}{2} \ln\left|\frac{x - 5}{x + 5}\right| + C$
57. $x \ln|x^2 + x| - 2x + \ln|x + 1| + C$ **59.** $\frac{1}{5}$
61. $\frac{1}{2}(\ln 4)^2 \approx 0.961$ **63.** π **65.** $\frac{128}{15}$ **67.** 3.82
69. 0 **71.** ∞ **73.** 0 **75.** $1000e^{0.09} \approx 1094.17$
77. $\frac{32}{3}$ **79.** Diverges **81.** 1 **83.** $\frac{\pi}{4}$
85. (a) \$6,321,205.59 (b) \$10,000,000
87. (a) 0.4581 (b) 0.0135

AP® Exam Practice Questions for Chapter 7 *(page 528)*

1. A **2.** B **3.** A **4.** D **5.** A **6.** C **7.** C
8. A **9.** B **10.** C **11.** B

12. (a) π (b) 4.401

(c) $\int_0^{1.6693678} (-2x + 5 - x\sin x)\,dx = 2\int_0^k (-2x + 5 - x\sin x)\,dx$

13. (a) $10 - 15e^{-2/3} \approx 2.2987$ (b) $\int_0^2 \left(te^{-t/3} - \frac{1}{2}\right) dt$

(c) Neither; Because $a(3) = 0$, the particle's speed is not changing at $t = 3$.

14. (a) $y = \dfrac{1}{e}x + 3$

(b) Concave downward on $(1, 5)$ because $f''(x) < 0$ on $(1, 5)$.

(c) $f(x) = -\dfrac{1}{2(\ln x)^2} + \dfrac{9}{2}$

15. (a) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0) = 3$

(b) $f'(x) = \begin{cases} \dfrac{4 - 6x - 4x^2}{(1 + x^2)^2}, & \text{for } x \le 0 \\ 2\tan x \sec^2 x, & \text{for } x > 0 \end{cases}$

(c) $\frac{1}{3}\tan^2 1 + \frac{28}{27} \approx 1.8456$

16. (a) $T = 0.1\ln|R| - 0.1\ln|5 - R| + 0.1\ln\frac{2}{3}$

(b) No points of inflection

17. (a) $\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \dfrac{x}{e^x} = \lim_{x\to\infty} \dfrac{1}{e^x} = 0$

Horizontal asymptote: $y = 0$

(b) Maximum value is $g(1) = e^{-1}$ because $g(x)$ is increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$.

(c) $-2e^{-1} + 1 \approx 0.2642$

18. (a) Converges;

$$\int_0^3 g'(x)\,dx = \lim_{b\to 2}\left[\int_0^b (x-2)^{-2}\,dx + \int_b^3 (x-2)^{-2}\,dx\right]$$
$$= \lim_{b\to 2}\left[-\frac{1}{b-2} - \frac{1}{2} - 1 + \frac{1}{b-2}\right]$$
$$= \lim_{b\to 2}\frac{3}{2}$$
$$= \frac{3}{2}$$

(b) There is a discontinuity at $x = 2$.

(c) $\frac{1}{4}$

19. (a) $\lim_{x\to 0}\dfrac{f(x) + 2}{\tan x} = \lim_{x\to 0}\dfrac{f'(x)}{\sec^2 x} = (-2)^2[4(0) + 1] = 4$

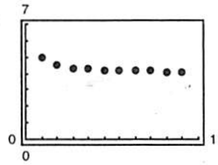
(b) $-\dfrac{1}{2}$ (c) $y = -\left(2x^2 + x + \dfrac{1}{2}\right)^{-1}$

Chapter 8

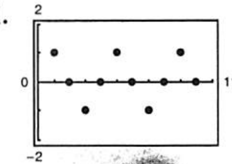
Section 8.1 *(page 540)*

1. 1, 5, 9, 13, 17 **3.** 3, 9, 27, 81, 243 **5.** 1, 0, −1, 0, 1
7. $2, -1, \frac{2}{3}, -\frac{1}{2}, \frac{2}{5}$ **9.** 3, 4, 6, 10, 18
11. c **12.** a **13.** d **14.** b
15. 14, 17; Add 3 to preceding term.
17. 80, 160; Multiply the preceding term by 2.
19. $n + 1$ **21.** $\dfrac{1}{2n(2n+1)}$ **23.** 1 **25.** 2

27. $\lim_{x\to\infty}\dfrac{1}{e^x} = 0$; The sequence converges to 0.

29.


Converges to 4

31.


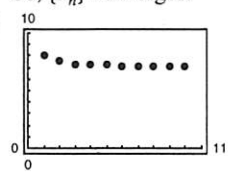
Diverges

33. Converges to 0 **35.** Diverges **37.** Converges to 5
39. Converges to 0 **41.** Diverges **43.** Converges to 0
45. Converges to 1 **47.** Converges to 0
49. Answers will vary. Sample answer: $a_n = -4 + 6n$
51. Answers will vary. Sample answer: $a_n = n^2 - 3$
53. Answers will vary. Sample answer: $a_n = \dfrac{n+1}{n+2}$
55. Answers will vary. Sample answer: $a_n = \dfrac{n+1}{n}$
57. Monotonic, bounded **59.** Not monotonic, bounded
61. Monotonic, bounded **63.** Not monotonic, bounded

65. (a) $\left|7 + \dfrac{1}{n}\right| \ge 7 \implies$ bounded

$a_n > a_{n+1} \implies$ monotonic

So, $\{a_n\}$ converges.

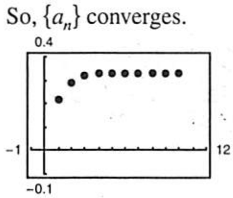
(b)


Limit = 7

67. (a) $\left|\dfrac{1}{3}\left(1 - \dfrac{1}{3^n}\right)\right| < \dfrac{1}{3} \implies$ bounded

$a_n < a_{n+1} \implies$ monotonic

So, $\{a_n\}$ converges.

(b)


Limit = $\frac{1}{3}$

69. (a) No; $\lim_{n\to\infty} A_n$ does not exist.

(b)

n	1	2	3	4
A_n	\$10,045.83	\$10,091.88	\$10,138.13	\$10,184.60

n	5	6	7
A_n	\$10,231.28	\$10,278.17	\$10,325.28

n	8	9	10
A_n	\$10,372.60	\$10,420.14	\$10,467.90

71. $\{a_n\}$ has a limit because it is bounded and monotonic; Because $2 \le a_n \le 4$, $2 \le \lim_{n\to\infty} a_n \le 4$.

73. $10 - \dfrac{1}{n}$

63. (a) Arctangent Formula, Formula 23, $\int \frac{1}{u^2+1}\,du,\ u = e^x$

(b) Log Rule: $\int \frac{1}{u}\,du,\ u = e^x + 1$

(c) Substitution: $u = x^2,\ du = 2x\,dx$
Then use Formula 81.

(d) Integration by parts (e) Cannot be integrated

(f) Formula 16 with $u = e^{2x}$

65. $32\pi^2$ **67.** About 401.4 **69.** C

Section 7.7 *(page 511)*

1.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	1.3177	1.3332	1.3333	1.3333	1.3332	1.3177

$\frac{4}{3}$

3.

x	1	10	10^2	10^3	10^4	10^5
$f(x)$	0.9900	90,483.7	3.7×10^9	4.5×10^{10}	0	0

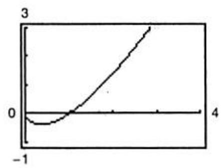
0

5. $\frac{3}{8}$ **7.** $\frac{1}{8}$ **9.** $\frac{5}{3}$ **11.** 4 **13.** 0 **15.** ∞ **17.** $\frac{11}{4}$

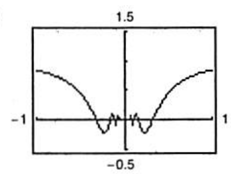
19. $\frac{3}{5}$ **21.** 1 **23.** $\frac{5}{4}$ **25.** ∞ **27.** 0 **29.** 1

31. 0 **33.** 0 **35.** ∞ **37.** $\frac{5}{9}$ **39.** 1 **41.** ∞

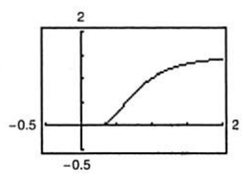
43. (a) Not indeterminate
(b) ∞
(c)



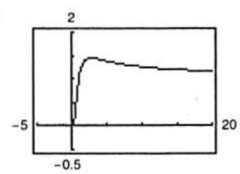
45. (a) $0 \cdot \infty$
(b) 1
(c)



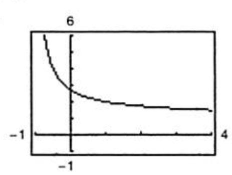
47. (a) Not indeterminate
(b) 0
(c)



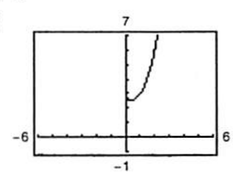
49. (a) ∞^0
(b) 1
(c)



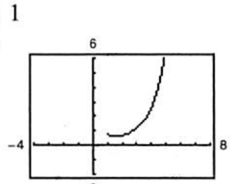
51. (a) 1^∞
(b) e
(c)



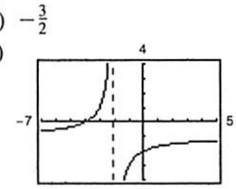
53. (a) 0^0
(b) 3
(c)



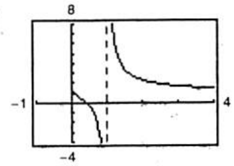
55. (a) 0^0
(b) 1
(c)



57. (a) $\infty - \infty$
(b) $-\frac{3}{2}$
(c)



59. (a) $\infty - \infty$ (b) ∞
(c)



61. $\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, 1^\infty, 0^0, \infty - \infty$

63. Answers will vary. Sample answers:
(a) $f(x) = x^2 - 25,\ g(x) = x - 5$
(b) $f(x) = (x - 5)^2,\ g(x) = x^2 - 25$
(c) $f(x) = x^2 - 25,\ g(x) = (x - 5)^3$

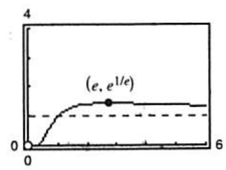
65. (a) Yes: $\frac{0}{0}$ (b) No: $\frac{0}{-1}$ (c) Yes: $\frac{\infty}{\infty}$ (d) Yes: $\frac{0}{0}$
(e) No: $\frac{-1}{0}$ (f) Yes: $\frac{0}{0}$

67.

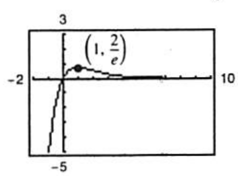
x	10	10^2	10^4	10^6	10^8	10^{10}
$\frac{(\ln x)^4}{x}$	2.811	4.498	0.720	0.036	0.001	0.000

69. 0 **71.** 0 **73.** 0

75. Horizontal asymptote: $y = 1$
Relative maximum: $(e, e^{1/e})$

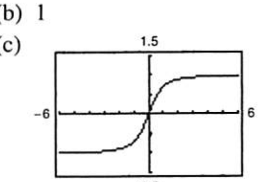


77. Horizontal asymptote: $y = 0$
Relative maximum: $\left(1, \frac{2}{e}\right)$

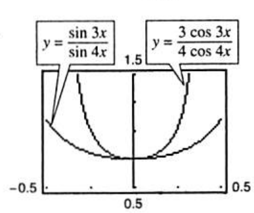


79. Limit is not of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$; 0

81. (a) $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$
Applying L'Hôpital's Rule twice results in the original limit, so L'Hôpital's Rule fails.
(b) 1
(c)



83.



As $x \to 0$, the graphs get closer together (they both approach 0.75). By L'Hôpital's Rule,
$\lim_{x\to 0} \frac{\sin 3x}{\sin 4x} = \lim_{x\to 0} \frac{3\cos 3x}{4\cos 4x} = \frac{3}{4}.$

CHAPTER 7

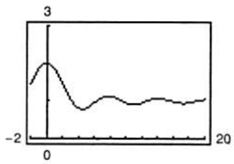
85. $v = 32t + v_0$ **87.** Proof **89.** $c = \frac{2}{3}$ **91.** $c = \frac{\pi}{4}$

93. False; L'Hôpital's Rule does not apply because $\lim_{x\to 0}(x^2 + x + 1) \neq 0$.

95. True **97.** $\frac{3}{4}$ **99.** $\frac{4}{3}$ **101.** $a = 1, b = \pm 2$

103. Proof **105.** Proof **107.** Proof

109. (a)



(b) $\lim_{x\to\infty} h(x) = 1$

(c) No

111. B **113.** C

Section 7.8 *(page 522)*

1. Improper; infinite discontinuity at $x = \frac{3}{5}$

3. Not improper; continuous on $[0, 1]$

5. Not improper; continuous on $[0, 2]$

7. Improper; infinite limits of integration

9. Infinite discontinuity at $x = 0$; 4

11. Infinite discontinuity at $x = 1$; diverges

13. Infinite discontinuity at $x = 0$; diverges

15. Infinite limit of integration; converges to 1

17. $\frac{1}{2}$ **19.** Diverges **21.** Diverges **23.** 2

25. $\frac{1}{2(\ln 4)^2}$ **27.** π **29.** $\frac{\pi}{4}$ **31.** Diverges

33. Diverges **35.** 0 **37.** $-\frac{1}{4}$ **39.** Diverges

41. $\frac{\pi}{3}$ **43.** $\ln 3$ **45.** $\frac{\pi}{6}$ **47.** $\frac{2\pi\sqrt{6}}{3}$ **49.** $p > 1$

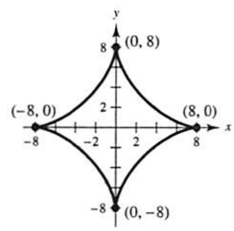
51. Proof **53.** Diverges **55.** Converges

57. Converges **59.** Diverges **61.** Converges

63. When the limit of the integral exists, the improper integral converges. When the limit does not exist, the improper integral diverges.

65. $\frac{7}{8}$ **67.** π **69.** (a) 1 (b) $\frac{\pi}{2}$ (c) 2π

71.



Perimeter = 48

73. (a) Proof (b) $P = 43.53\%$ (c) $E(x) = 7$

75. (a) \$757,992.41 (b) \$837,995.15 (c) \$1,066,666.67

77. $P = \dfrac{2\pi NI\left(\sqrt{r^2 + c^2} - c\right)}{kr\sqrt{r^2 + c^2}}$

79. False; Let $f(x) = \frac{1}{x + 1}$. **81.** True

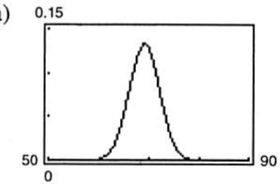
83. (a) and (b) Proofs

(c) The definition of the improper integral $\int_{-\infty}^{\infty}$ is not $\lim_{a\to\infty}\int_{-a}^{a}$ but rather that if you rewrite the integral that diverges, you can find that the integral converges.

85. $\frac{1}{s}, s > 0$ **87.** $\frac{2}{s^3}, s > 0$ **89.** $\frac{s}{s^2 + a^2}, s > 0$

91. $\int_0^1 2 \sin u^2\, du$; 0.6278 **93.** Proof

95. (a)



(b) About 0.1587

(c) 0.1587; same by symmetry

97. (a) $\Gamma(1) = 1, \Gamma(2) = 1, \Gamma(3) = 2$ (b) Proof

(c) $\Gamma(n) = (n - 1)!$

99. $c = 1$; $\ln 2$

101. $8\pi\left[\frac{(\ln 2)^2}{3} - \frac{2\ln 2}{9} + \frac{2}{27}\right] \approx 2.0155$ **103.** B

Review Exercises for Chapter 7 *(page 526)*

1. $\frac{1}{3}(x^2 - 36)^{3/2} + C$ **3.** $\frac{1}{2}\ln|x^2 - 49| + C$

5. $\frac{1}{2} + \ln 2 \approx 1.1931$ **7.** $100 \arcsin \frac{x}{10} + C$

9. $\frac{1}{9}e^{3x}(3x - 1) + C$

11. $\frac{1}{13}e^{2x}(2 \sin 3x - 3 \cos 3x) + C$

13. $-\frac{1}{2}x^2 \cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C$

15. $\frac{1}{8}\left[(4x^2 - 1)\arcsin 2x + 2x\sqrt{1 - 4x^2}\right] + C$

17. $\frac{1}{3\pi}[\sin(\pi x - 1)][2 + \cos^2(\pi x - 1)] + C$

19. $\frac{2}{3}\left[\tan^3\frac{x}{2} + 3\tan\frac{x}{2}\right] + C$ **21.** $\tan\theta + \sec\theta + C$

23. $\frac{3\pi}{16} + \frac{1}{2} \approx 1.0890$ **25.** $\frac{3\sqrt{4 - x^2}}{x} + C$

27. $\frac{1}{3}(x^2 + 4)^{1/2}(x^2 - 8) + C$ **29.** $256 - 62\sqrt{17}$

31. (a)–(c) $\frac{1}{3}\sqrt{4 + x^2}(x^2 - 8) + C$

33. $-5\ln|x - 4| + 6\ln|x + 3| + C$

35. $\frac{1}{4}[6\ln|x - 1| - \ln(x^2 + 1) + 6\arctan x] + C$

37. $x - \frac{64}{11}\ln|x + 8| + \frac{9}{11}\ln|x - 3| + C$

39. $\frac{1}{25}\left(\frac{4}{4 + 5x} + \ln|4 + 5x|\right) + C$ **41.** $1 - \frac{\sqrt{2}}{2}$

43. $\frac{1}{2}\ln|x^2 + 4x + 8| - \arctan\left(1 + \frac{x}{2}\right) + C$ **45.** Proof

47. $\frac{1}{8}(\sin 2\theta - 2\theta\cos 2\theta) + C$

49. $\frac{4}{3}[x^{3/4} - 3x^{1/4} + 3\arctan(x^{1/4})] + C$

51. $2\sqrt{1 - \cos x} + C$ **53.** $(\sin x)\ln(\sin x) - \sin x + C$

55. $\frac{5}{2}\ln\left|\frac{x - 5}{x + 5}\right| + C$

57. $x\ln|x^2 + x| - 2x + \ln|x + 1| + C$ **59.** $\frac{1}{5}$

61. $\frac{1}{2}(\ln 4)^2 \approx 0.961$ **63.** π **65.** $\frac{128}{15}$ **67.** 3.82

69. 0 **71.** ∞ **73.** 0 **75.** $1000e^{0.09} \approx 1094.17$

77. $\frac{32}{3}$ **79.** Diverges **81.** 1 **83.** $\frac{\pi}{4}$

85. (a) \$6,321,205.59 (b) \$10,000,000

87. (a) 0.4581 (b) 0.0135