Applications of Integrals

Brief Review

The application of integrals we will focus on this week is area and volume.

AREA BETWEEN TWO CURVES:

You can do this with an x-axis orientation in which case your limits of integration come from the x-axis and you do TOP-BOTTOM.

y-axis orientation is RIGHT-LEFT and your limits come from y. Your functions must be solve for x in terms of y.

VOLUME BY DISKS:

$$\pi \int_a^b (R(x))^2 dx$$

VOLUME BY WASHERS:

$$\pi \int_a^b (R(x))^2 - (r(x))^2 \, dx$$

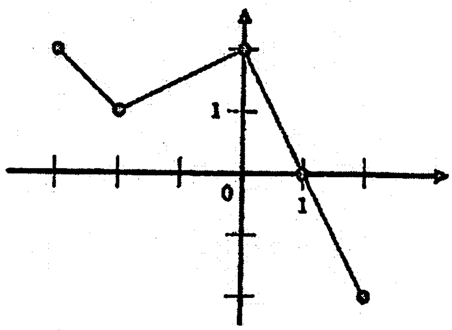
VOLUME BY KNOWN CROSS SECTIONS:

Must know volume formulas for square, rectangle, equilateral triangle, and right isosceles triangle

AVERAGE VALUE:

$$\frac{1}{b-a} \int_a^b f(x) \, dx$$

2008 Exam Non-Calculator

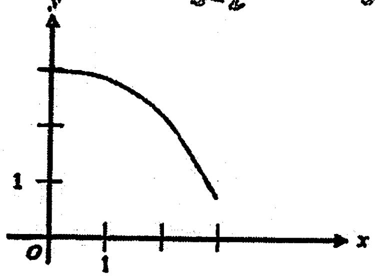


Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x)=\int_{-2}^{x} f(t)\,dt$, which of the following values is greatest? ← greatest area

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ $\boxed{\text{(D) } g(1)}$ (E) $g(2)$

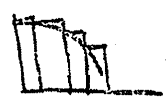
$\int_{-2}^{-3} f(x)dx$ $\int_{-2}^{-2} f(x)dx$ $\int_{-2}^{0} f(x)dx$ $\int_{-2}^{1} f(x)dx$ $\int_{-2}^{2} f(x)dx$



Graph of f

10. The graph of function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

(A) $\int_{1}^{3} f(x)\,dx$ Exact area

(B) Left Riemann sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length  over estimate

(C) Right Riemann sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length  underestimate

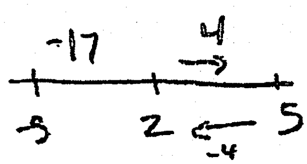
(D) Midpoint Riemann sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_{1}^{3} f(x)\,dx$ with 4 subintervals of equal length

} closer to the curve (exact area) than the left or right

79. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

(A) –21 (B) –13 (C) 0 (D) 13 (E) 21



83. What is the area enclosed by the curves $y = \underbrace{x^3 - 8x^2 + 18x - 5}_{f(x)}$ and $y = \underbrace{x + 5}_{g(x)}$?

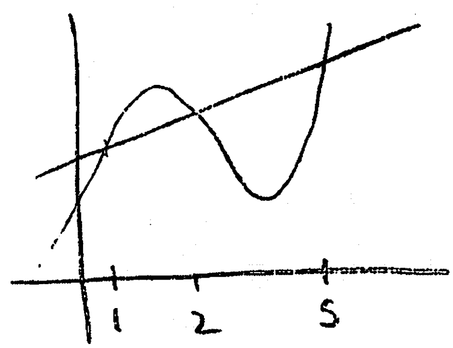
(A) 10.667

(B) 11.833

(C) 14.583

(D) 21.333

(E) 32



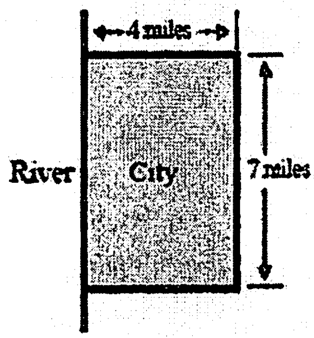
$\int_1^2 f(x) - g(x)\,dx + \int_2^5 g(x) - f(x)\,dx$

91. What is the average value of $y = \dfrac{\cos x}{x^2 + x + 2}$ on the closed interval [–1, 3]?

(A) –0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732

$\dfrac{1}{3+1} \int_{-1}^{3} \dfrac{\cos x}{x^2+x+2}\,dx$

this is the answer if you forget the $\frac{1}{4}$



92. A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?

(A) $\int_0^4 f(x)\,dx$

(B) $7\int_0^4 f(x)\,dx$

(C) $28\int_0^4 f(x)\,dx$

(D) $\int_0^7 f(x)\,dx$

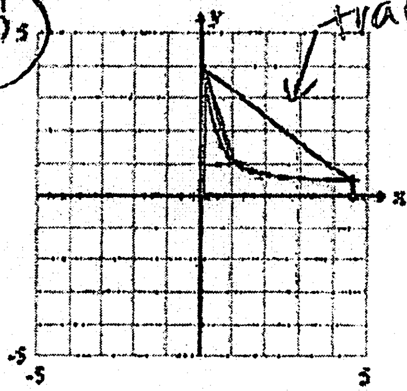
(E) $4\int_0^7 f(x)\,dx$

2003 Exam Calculator

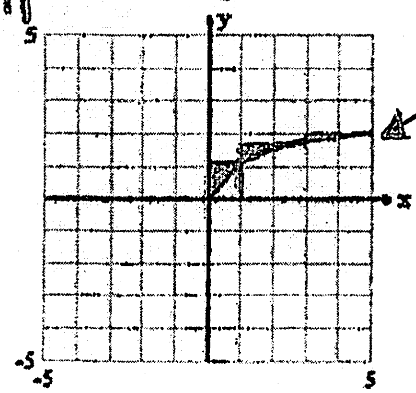
85. If a trapezoidal sum overapproximates $\int_0^4 f(x)dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?

trap overapproximates

A)

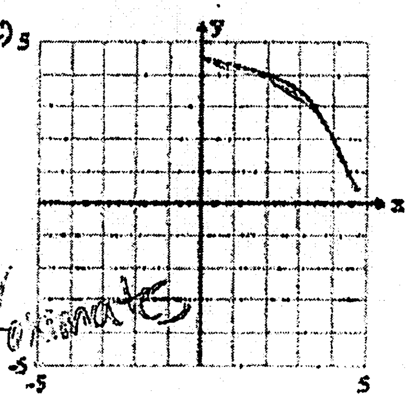


B)



Rt overapproximated

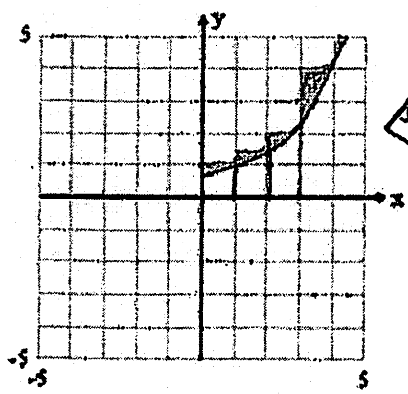
C)



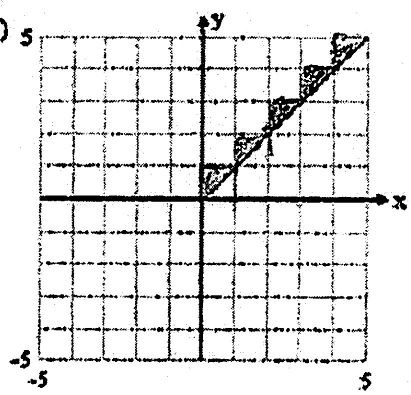
→D

Trap under-approximates

D)



E)

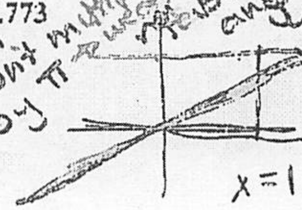


86. The base of a solid is the region in the first quadrant bounded by the y- axis. the graph of $y = \tan^{-1}(x)$, the horizontal line y=3. and the vertical line x=1. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

A) 2.561 B) 6.612 C) 8.046 D) 8.755 E) 20.773

↑ don't multiply by π we're [illegible] revolving anything

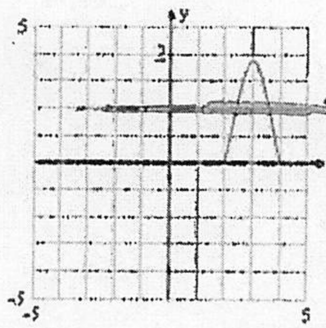
$$\int_0^1 (3 - \tan^{-1} x)^2 dx$$



88. On the closed interval [2,4], which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_2^4 f(t)dt = 1$?

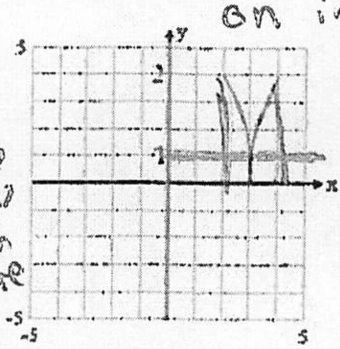
← average value is 1 on int (2,4)

A)



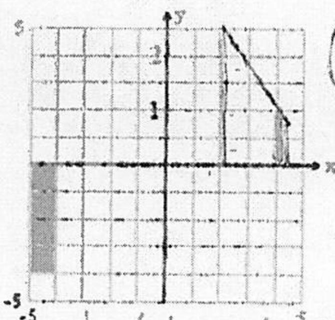
more below than above

B)



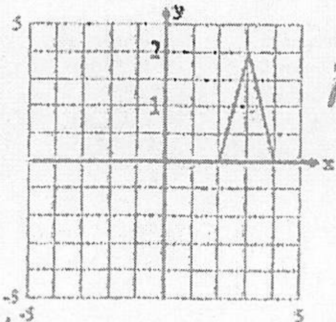
more above than below

C)



$\frac{1}{2} \cdot \left(\frac{1}{2}(2)(2.5+1)\right) > 1$

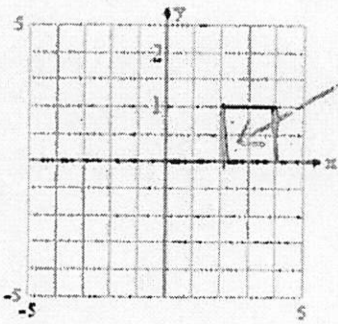
D)



$A = \frac{1}{2}(2)(2) = 2$

$\frac{1}{2} \cdot A = 1$

E)



$\frac{1}{2} \cdot 4 = 2 \neq 1$

↓

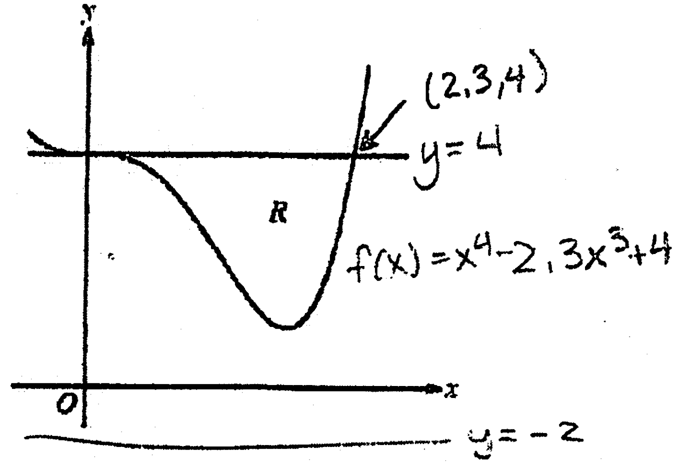
area

AP Calculus AB Name ____________________

Applications of Integration: Open Ended Area and Volume Problems

Calculator:

2014 AP® CALCULUS AB FREE-RESPONSE QUESTIONS



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

$$V = \pi \int_0^{2.3} \underbrace{[-2-4]^2}_{+1} - \underbrace{[-2-(x^4-2.3x^3+4)]^2}_{+1} dx$$

limits +1

31.470268π or 98.867 or 98.868

answer +1

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

$A = \frac{1}{2}x^2$

+2 integrand

$$\frac{1}{2}\int_0^{2.3} (4-(x^4-2.3x^3+4))^2 dx$$

$= 3.573$ or 3.574

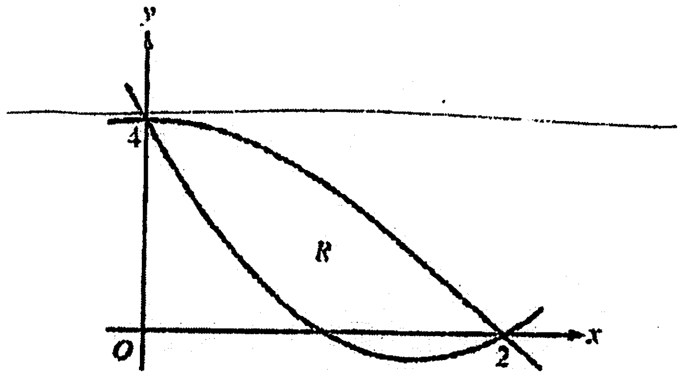
+1 answer

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

$$\int_0^K (4-f(x))dx = \int_K^{2.3} (4-f(x))dx$$

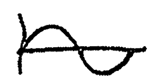
+1 area of 1 region
+1 equation

Non- Calculator



5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.



$$\underbrace{\int_0^2 4\cos\left(\tfrac{1}{4}\pi x\right) - (2x^2-6x+4)\,dx}_{+1}$$

$$\longrightarrow \underbrace{\frac{16}{\pi}\sin\left(\tfrac{1}{4}\pi x\right) - \frac{2}{3}x^3 + 3x^2 - 4x}_{+1}\Big|_{x=0}^{x=2}$$

Aside

$\int 4\cos\left(\frac{1}{4}\pi x\right)dx \to \int 4\cos u \cdot \frac{4}{\pi}\,du$

let $u = \frac{1}{4}\pi x$ $\quad = \frac{16}{\pi}\sin u + c$

$\frac{du}{dx} = \frac{1}{4}\pi \to \boxed{\frac{4}{\pi}du = dx} = \frac{16}{\pi}\sin\left(\frac{1}{4}\pi x\right) + c$

$$= \frac{16}{\pi}\sin\left(\frac{\pi}{2}\right) - \frac{2}{3}(2)^3 + 3(2)^2 - 4(2) - \frac{16}{\pi}\sin 0 \approx 0$$

$$= \frac{16}{\pi} - \frac{16}{3} + 12 - 8 - 0$$

$$= \boxed{\frac{16}{\pi} - \frac{4}{3}} \quad +1$$

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

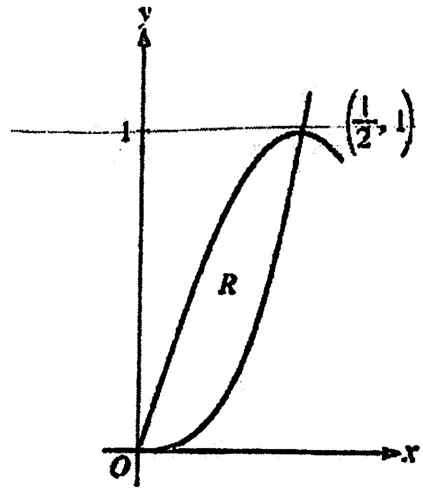
$$V = \pi \underbrace{\int_0^2}_{+1} \underbrace{\left(4 - (2x^2-6x+4)\right)^2}_{+1} - \underbrace{\left(4 - 4\cos\left(\tfrac{1}{4}\pi x\right)\right)^2}_{+1} dx$$

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$$V = \underbrace{\int_0^2}_{\substack{+1 \\ \text{no } \pi}} \Big[\underbrace{4\cos\left(\tfrac{1}{4}\pi x\right) - (2x^2-6x+4)}_{\text{integrand } +1}\Big]^2 dx$$

Non Calculator

2011 AP® CALCULUS AB FREE-RESPONSE QUESTIONS



3. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

$f'(x) = 24x^2$

$f'(\frac{1}{2}) = 24(\frac{1}{2})^2 = 6 = m$ +1

$f(\frac{1}{2}) = 8(\frac{1}{2})^3 = 1$

pt $(\frac{1}{2}, 1)$

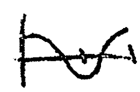
$1 = \frac{1}{2}(6) + b$

$-2 = b$

$\boxed{y = 6x - 2}$ +1

(b) Find the area of R.

+1 integrand



$\int_0^1 \sin(\pi x) - 8x^3\, dx =$

+1

$\to \left. -\frac{1}{\pi}\cos(\pi x) - 2x^4 \right|_{x=0}^{x=\frac{1}{2}}$

$\int \sin \pi x$

let $u = \pi x$

$\frac{1}{\pi} du = dx$

$\int \frac{1}{\pi} \sin u\, du$

$= \frac{-1}{\pi}\cos u + c = \frac{-1}{\pi}\cos(\pi x) + c$ +1

$-\frac{1}{\pi}\cos\frac{\pi}{2} - 2(\frac{1}{2})^4 - (-\frac{1}{\pi}\cos 0 - 0)$

$= 0 - \frac{1}{8} + \frac{1}{\pi} = -\frac{1}{8} + \frac{1}{\pi}$ +1

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

$V = \pi \int_0^{1/2} (1 - 8x^3)^2 - (1 - \sin(\pi x))^2\, dx$

+1 +1 +1