Why Not Round the Decimals in Equation 1 Even More?

If we do, our final calculation will be way off. Using $y = 80x - 153,849$, for instance, gives $y = 6951$ when $x = 2010$, as compared to $y = 6865$, an increase of 86 million. The rule is: *Retain all decimal places while working a problem. Round only at the end.* We rounded the coefficients in Equation 1 enough to make it readable, but not enough to hurt the outcome. However, we knew how much we could safely round *only from first having done the entire calculation with numbers unrounded.*

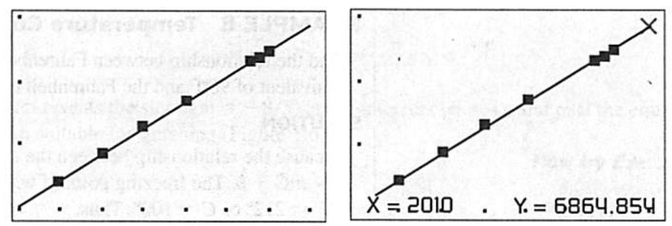
SOLUTION

Model Upon entering the data into the grapher, we find the regression equation to be approximately

$$y = 79.957x - 153848.716, \qquad (1)$$

where x represents the year and y the population *in millions.*

Figure 1.7a shows the scatter plot for Table 1.1 together with a graph of the regression line just found. You can see how well the line fits the data.



[1975, 2010] by [4000, 7000]
(a)

[1975, 2010] by [4000, 7000]
(b)

Figure 1.7 (Example 9)

Solve Graphically Our goal is to predict the population in the year 2010. Reading from the graph in Figure 1.7b, we conclude that when x is 2010, y is approximately 6865.

Confirm Algebraically Evaluating Equation 1 for $x = 2010$ gives

$$y = 79.957(2010) - 153848.716$$
$$\approx 6865.$$

Interpret The linear regression equation suggests that the world population in the year 2010 will be about 6865 million, or approximately 53 million more than the Statistical Abstract prediction of 6812 million. ***Now try Exercise 45.***

Rounding Rule

Round your answer as appropriate, but do not round the numbers in the calculations that lead to it.

Regression Analysis

Regression analysis has four steps:

1. Plot the data (scatter plot).
2. Find the regression equation. For a line, it has the form $y = mx + b$.
3. Superimpose the graph of the regression equation on the scatter plot to see the fit.
4. Use the regression equation to predict y-values for particular values of x.

Quick Review 1.1 *(For help, go to Section 1.1.)*

1. Find the value of y that corresponds to $x = 3$ in $y = -2 + 4(x - 3)$.
2. Find the value of x that corresponds to $y = 3$ in $y = 3 - 2(x + 1)$.

In Exercises 3 and 4, find the value of m that corresponds to the values of x and y.

3. $x = 5, \quad y = 2, \quad m = \frac{y - 3}{x - 4}$
4. $x = -1, \quad y = -3, \quad m = \frac{2 - y}{3 - x}$

In Exercises 5 and 6, determine whether the ordered pair is a solution to the equation.

5. $3x - 4y = 5$
   (a) $(2, 1/4)$ (b) $(3, -1)$
6. $y = -2x + 5$
   (a) $(-1, 7)$ (b) $(-2, 1)$

In Exercises 7 and 8, find the distance between the points.

7. $(1, 0), \quad (0, 1)$
8. $(2, 1), \quad (1, -1/3)$

In Exercises 9 and 10, solve for y in terms of x.

9. $4x - 3y = 7$
10. $-2x + 5y = -3$

Section 1.1 Exercises

In Exercises 1–4, find the coordinate increments from A to B.

1. $A(1, 2), \quad B(-1, -1)$
2. $A(-3, 2), \quad B(-1, -2)$
3. $A(-3, 1), \quad B(-8, 1)$
4. $A(0, 4), \quad B(0, -2)$

In Exercises 5–8, let L be the line determined by points A and B.
(a) Plot A and B. (b) Find the slope of L.
(c) Draw the graph of L.

5. $A(1, -2), \quad B(2, 1)$
6. $A(-2, -1), \quad B(1, -2)$
7. $A(2, 3), \quad B(-1, 3)$
8. $A(1, 2), \quad B(1, -3)$

In Exercise 9–12, write an equation for **(a)** the vertical line and **(b)** the horizontal line through the point P.

9. $P(3, 2)$
10. $P(-1, 4/3)$
11. $P(0, -\sqrt{2})$
12. $P(-\pi, 0)$

In Exercises 13–16, write the point-slope equation for the line through the point P with slope m.

13. $P(1, 1), \quad m = 1$
14. $P(-1, 1), \quad m = -1$
15. $P(0, 3), \quad m = 2$
16. $P(-4, 0), \quad m = -2$

In Exercises 17–20, write the slope-intercept equation for the line with slope m and y-intercept b.

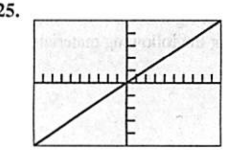
17. $m = 3, \quad b = -2$
18. $m = -1, \quad b = 2$
19. $m = -1/2, \quad b = -3$
20. $m = 1/3, \quad b = -1$

In Exercises 21–24, write a general linear equation for the line through the two points.

21. $(0, 0), \quad (2, 3)$
22. $(1, 1), \quad (2, 1)$
23. $(-2, 0), \quad (-2, -2)$
24. $(-2, 1), \quad (2, -2)$

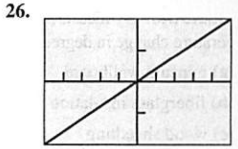
In Exercises 25 and 26, the line contains the origin and the point in the upper right corner of the grapher screen. Write an equation for the line.

25.



[–10, 10] by [–25, 25]

26.



[–5, 5] by [–2, 2]

In Exercises 27–30, find the **(a)** slope and **(b)** y-intercept, and **(c)** graph the line.

27. $3x + 4y = 12$
28. $x + y = 2$
29. $\frac{x}{3} + \frac{y}{4} = 1$
30. $y = 2x + 4$

In Exercises 31–34, write an equation for the line through P that is **(a)** parallel to L, and **(b)** perpendicular to L.

31. $P(0, 0), \quad L: y = -x + 2$
32. $P(-2, 2), \quad L: 2x + y = 4$
33. $P(-2, 4), \quad L: x = 5$
34. $P(-1, 1/2), \quad L: y = 3$

In Exercises 35 and 36, a table of values is given for the linear function $f(x) = mx + b$. Determine m and b.

35.

x	$f(x)$
1	2
3	9
5	16

36.

x	$f(x)$
2	−1
4	−4
6	−7

In Exercises 37 and 38, find the value of x or y for which the line through A and B has the given slope m.

37. $A(-2, 3)$, $B(4, y)$, $m = -2/3$

38. $A(-8, -2)$, $B(x, 2)$, $m = 2$

39. ***Revisiting Example 4*** Show that you get the same equation in Example 4 if you use the point (3, 4) to write the equation.

40. ***Writing to Learn x- and y-intercepts***

(a) Explain why c and d are the x-intercept and y-intercept, respectively, of the line

$$\frac{x}{c} + \frac{y}{d} = 1.$$

(b) How are the x-intercept and y-intercept related to c and d in the line

$$\frac{x}{c} + \frac{y}{d} = 2?$$

41. ***Parallel and Perpendicular Lines*** For what value of k are the two lines $2x + ky = 3$ and $x + y = 1$ **(a)** parallel? **(b)** perpendicular?

Group Activity In Exercises 42–44, work in groups of two or three to solve the problem.

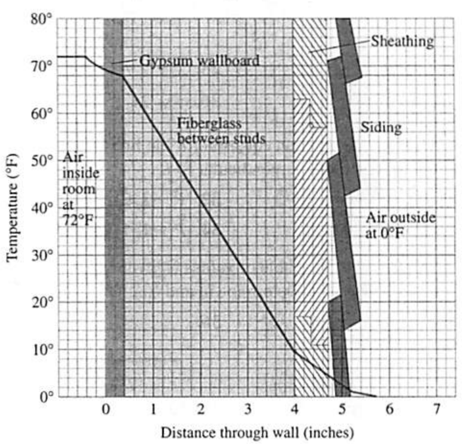
42. ***Insulation*** By measuring slopes in the figure below, find the temperature change in degrees per inch for the following materials.

(a) gypsum wallboard

(b) fiberglass insulation

(c) wood sheathing

(d) **Writing to Learn** Which of the materials in (a)–(c) is the best insulator? the poorest? Explain.



43. ***Pressure under Water*** The pressure p experienced by a diver under water is related to the diver's depth d by an equation of the form $p = kd + 1$ (k a constant). When $d = 0$ meters, the pressure is 1 atmosphere. The pressure at 100 meters is 10.94 atmospheres. Find the pressure at 50 meters.

44. ***Modeling Distance Traveled*** A car starts from point P at time $t = 0$ and travels at 45 mph.

(a) Write an expression $d(t)$ for the distance the car travels from P.

(b) Graph $y = d(t)$.

(c) What is the slope of the graph in (b)? What does it have to do with the car?

(d) **Writing to Learn** Create a scenario in which t could have negative values.

(e) **Writing to Learn** Create a scenario in which the y-intercept of $y = d(t)$ could be 30.

In Exercises 45 and 46, use linear regression analysis.

45. Table 1.2 shows the mean annual compensation of construction workers.

Table 1.2 Construction Workers' Average Annual Compensation

Year	Annual Total Compensation (dollars)
1999	42,598
2000	44,764
2001	47,822
2002	48,966

Source: U.S. Bureau of the Census, *Statistical Abstract of the United States, 2004–2005.*

(a) Find the linear regression equation for the data.

(b) Find the slope of the regression line. What does the slope represent?

(c) Superimpose the graph of the linear regression equation on a scatter plot of the data.

(d) Use the regression equation to predict the construction workers' average annual compensation in the year 2008.

46. Table 1.3 lists the ages and weights of nine girls.

Table 1.3 Girls' Ages and Weights

Age (months)	Weight (pounds)
19	22
21	23
24	25
27	28
29	31
31	28
34	32
38	34
43	39

(a) Find the linear regression equation for the data.

(b) Find the slope of the regression line. What does the slope represent?

(c) Superimpose the graph of the linear regression equation on a scatter plot of the data.

(d) Use the regression equation to predict the approximate weight of a 30-month-old girl.

Standardized Test Questions

You should solve the following problems without using a graphing calculator.

47. True or False The slope of a vertical line is zero. Justify your answer.

48. True or False The slope of a line perpendicular to the line $y = mx + b$ is $1/m$. Justify your answer.

49. Multiple Choice Which of the following is an equation of the line through $(-3, 4)$ with slope $1/2$?

(A) $y - 4 = \frac{1}{2}(x + 3)$ (B) $y + 3 = \frac{1}{2}(x - 4)$

(C) $y - 4 = -2(x + 3)$ (D) $y - 4 = 2(x + 3)$

(E) $y + 3 = 2(x - 4)$

50. Multiple Choice Which of the following is an equation of the vertical line through (–2, 4)?

(A) $y = 4$ (B) $x = 2$ (C) $y = -4$

(D) $x = 0$ (E) $x = -2$

51. Multiple Choice Which of the following is the x-intercept of the line $y = 2x - 5$?

(A) $x = -5$ (B) $x = 5$ (C) $x = 0$

(D) $x = 5/2$ (E) $x = -5/2$

52. Multiple Choice Which of the following is an equation of the line through $(-2, -1)$ parallel to the line $y = -3x + 1$?

(A) $y = -3x + 5$ (B) $y = -3x - 7$ (C) $y = \frac{1}{3}x - \frac{1}{3}$

(D) $y = -3x + 1$ (E) $y = -3x - 4$

Extending the Ideas

53. The median price of existing single-family homes has increased consistently during the past few years. However, the data in Table 1.4 show that there have been differences in various parts of the country.

Table 1.4 Median Price of Single-Family Homes

Year	South (dollars)	West (dollars)
1999	145,900	173,700
2000	148,000	196,400
2001	155,400	213,600
2002	163,400	238,500
2003	168,100	260,900

Source: U.S. Bureau of the Census, *Statistical Abstract of the United States, 2004–2005.*

(a) Find the linear regression equation for home cost in the South.

(b) What does the slope of the regression line represent?

(c) Find the linear regression equation for home cost in the West.

(d) Where is the median price increasing more rapidly, in the South or the West?

54. ***Fahrenheit versus Celsius*** We found a relationship between Fahrenheit temperature and Celsius temperature in Example 8.

(a) Is there a temperature at which a Fahrenheit thermometer and a Celsius thermometer give the same reading? If so, what is it?

(b) **Writing to Learn** Graph $y_1 = (9/5)x + 32$, $y_2 = (5/9)(x - 32)$, and $y_3 = x$ in the same viewing window. Explain how this figure is related to the question in part (a).

55. ***Parallelogram*** Three different parallelograms have vertices at $(-1, 1)$, $(2, 0)$, and $(2, 3)$. Draw the three and give the coordinates of the missing vertices.

56. ***Parallelogram*** Show that if the midpoints of consecutive sides of any quadrilateral are connected, the result is a parallelogram.

57. ***Tangent Line*** Consider the circle of radius 5 centered at (0, 0). Find an equation of the line tangent to the circle at the point (3, 4).

58. Group Activity *Distance From a Point to a Line* This activity investigates how to find the distance from a point $P(a, b)$ to a line L: $Ax + By = C$.

(a) Write an equation for the line M through P perpendicular to L.

(b) Find the coordinates of the point Q in which M and L intersect.

(c) Find the distance from P to Q.

"f of g of x") is the **composite of g and f.** It is made by *composing* g and f in the order of first g, then f. The usual "stand-alone" notation for this composite is $f \circ g$, which is read as "f of g." Thus, the value of $f \circ g$ at x is $(f \circ g)(x) = f(g(x))$.

EXAMPLE 8 Composing Functions

Find a formula for $f(g(x))$ if $g(x) = x^2$ and $f(x) = x - 7$. Then find $f(g(2))$.

SOLUTION

To find $f(g(x))$, we replace x in the formula $f(x) = x - 7$ by the expression given for $g(x)$.

$$f(x) = x - 7$$
$$f(g(x)) = g(x) - 7 = x^2 - 7$$

We then find the value of $f(g(2))$ by substituting 2 for x.

$$f(g(2)) = (2)^2 - 7 = -3$$

Now try Exercise 51.

EXPLORATION 1 Composing Functions

Some graphers allow a function such as y_1 to be used as the independent variable of another function. With such a grapher, we can compose functions.

1. Enter the functions $y_1 = f(x) = 4 - x^2$, $y_2 = g(x) = \sqrt{x}$, $y_3 = y_2(y_1(x))$, and $y_4 = y_1(y_2(x))$. Which of y_3 and y_4 corresponds to $f \circ g$? to $g \circ f$?
2. Graph y_1, y_2, and y_3 and make conjectures about the domain and range of y_3.
3. Graph y_1, y_2, and y_4 and make conjectures about the domain and range of y_4.
4. Confirm your conjectures algebraically by finding formulas for y_3 and y_4.

Quick Review 1.2 *(For help, go to Appendix A1 and Section 1.2.)*

In Exercises 1–6, solve for x.

1. $3x - 1 \le 5x + 3$
2. $x(x - 2) > 0$
3. $|x - 3| \le 4$
4. $|x - 2| \ge 5$
5. $x^2 < 16$
6. $9 - x^2 \ge 0$

In Exercises 7 and 8, describe how the graph of f can be transformed to the graph of g.

7. $f(x) = x^2$, $g(x) = (x + 2)^2 - 3$
8. $f(x) = |x|$, $g(x) = |x - 5| + 2$

In Exercises 9–12, find all real solutions to the equations.

9. $f(x) = x^2 - 5$
   (a) $f(x) = 4$ (b) $f(x) = -6$
10. $f(x) = 1/x$
   (a) $f(x) = -5$ (b) $f(x) = 0$
11. $f(x) = \sqrt{x + 7}$
   (a) $f(x) = 4$ (b) $f(x) = 1$
12. $f(x) = \sqrt[3]{x - 1}$
   (a) $f(x) = -2$ (b) $f(x) = 3$

Section 1.2 Exercises

In Exercises 1–4, **(a)** write a formula for the function and **(b)** use the formula to find the indicated value of the function.

1. the area A of a circle as a function of its diameter d; the area of a circle of diameter 4 in.
2. the height h of an equilateral triangle as a function of its side length s; the height of an equilateral triangle of side length 3 m
3. the surface area S of a cube as a function of the length of the cube's edge e; the surface area of a cube of edge length 5 ft
4. the volume V of a sphere as a function of the sphere's radius r; the volume of a sphere of radius 3 cm

In Exercises 5–12, **(a)** identify the domain and range and **(b)** sketch the graph of the function.

5. $y = 4 - x^2$
6. $y = x^2 - 9$
7. $y = 2 + \sqrt{x - 1}$
8. $y = -\sqrt{-x}$
9. $y = \dfrac{1}{x - 2}$
10. $y = \sqrt[4]{-x}$
11. $y = 1 + \dfrac{1}{x}$
12. $y = 1 + \dfrac{1}{x^2}$

In Exercises 13–20, use a grapher to **(a)** identify the domain and range and **(b)** draw the graph of the function.

13. $y = \sqrt[3]{x}$
14. $y = 2\sqrt{3 - x}$
15. $y = \sqrt[3]{1 - x^2}$
16. $y = \sqrt{9 - x^2}$
17. $y = x^{2/5}$
18. $y = x^{3/2}$
19. $y = \sqrt[3]{x - 3}$
20. $y = \dfrac{1}{\sqrt{4 - x^2}}$

In Exercises 21–30, determine whether the function is even, odd, or neither. Try to answer without writing anything (except the answer).

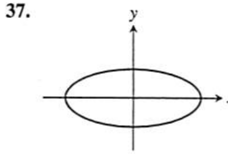
21. $y = x^4$
22. $y = x + x^2$
23. $y = x + 2$
24. $y = x^2 - 3$
25. $y = \sqrt{x^2 + 2}$
26. $y = x + x^3$
27. $y = \dfrac{x^3}{x^2 - 1}$
28. $y = \sqrt[3]{2 - x}$
29. $y = \dfrac{1}{x - 1}$
30. $y = \dfrac{1}{x^2 - 1}$

In Exercises 31–34, graph the piecewise-defined functions.

31. $f(x) = \begin{cases} 3 - x, & x \le 1 \\ 2x, & 1 < x \end{cases}$
32. $f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{x}, & x \ge 0 \end{cases}$
33. $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ (3/2)x + 3/2, & 1 \le x \le 3 \\ x + 3, & x > 3 \end{cases}$
34. $f(x) = \begin{cases} x^2, & x < 0 \\ x^3, & 0 \le x \le 1 \\ 2x - 1, & x > 1 \end{cases}$

35. **Writing to Learn** The *vertical line test* to determine whether a curve is the graph of a function states: If every vertical line in the xy-plane intersects a given curve in at most one point, then the curve is the graph of a function. Explain why this is true.
36. **Writing to Learn** For a curve to be *symmetric about the x-axis*, the point (x, y) must lie on the curve if and only if the point $(x, -y)$ lies on the curve. Explain why a curve that is symmetric about the x-axis is not the graph of a function, unless the function is $y = 0$.

In Exercises 37–40, use the vertical line test (see Exercise 35) to determine whether the curve is the graph of a function.

37.


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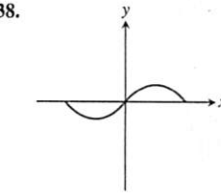

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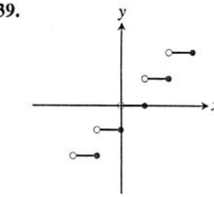

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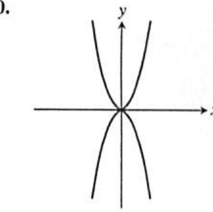
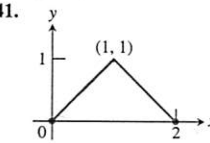


In Exercises 41–48, write a piecewise formula for the function.

41.


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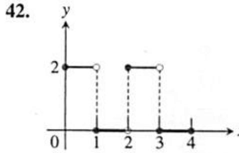

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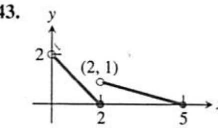

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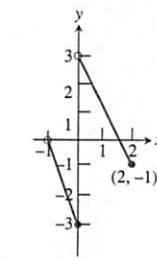

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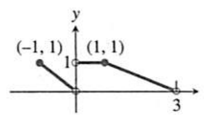

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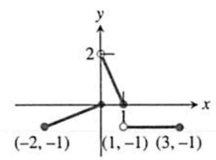

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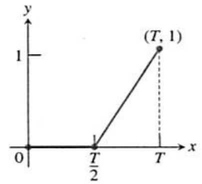

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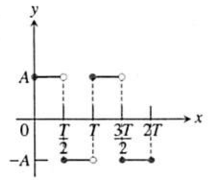


In Exercises 49 and 50, **(a)** draw the graph of the function. Then find its **(b)** domain and **(c)** range.

49. $f(x) = -|3 - x| + 2$ **50.** $f(x) = 2|x + 4| - 3$

In Exercises 51 and 52, find

(a) $f(g(x))$ **(b)** $g(f(x))$ **(c)** $f(g(0))$

(d) $g(f(0))$ **(e)** $g(g(-2))$ **(f)** $f(f(x))$

51. $f(x) = x + 5, \quad g(x) = x^2 - 3$

52. $f(x) = x + 1, \quad g(x) = x - 1$

53. Copy and complete the following table.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
(a)	?	$\sqrt{x-5}$	$\sqrt{x^2-5}$
(b)	?	$1+1/x$	x
(c)	$1/x$	?	x
(d)	$\sqrt{x}$	?	$\lvert x \rvert,\ x \ge 0$

54. ***Broadway Season Statistics*** Table 1.5 shows the gross revenue for the Broadway season in millions of dollars for several years.

Table 1.5 Broadway Season Revenue

Year	Amount ($ millions)
1997	558
1998	588
1999	603
2000	666
2001	643
2002	721
2003	771

Source: The League of American Theatres and Producers, Inc., New York, NY, as reported in *The World Almanac and Book of Facts, 2005.*

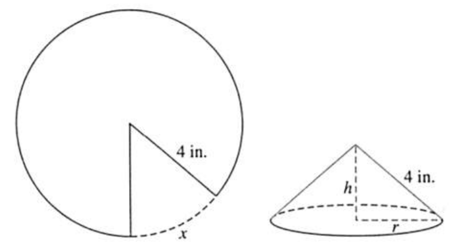
(a) Find the quadratic regression for the data in Table 1.5. Let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth.

(b) Superimpose the graph of the quadratic regression equation on a scatter plot of the data.

(c) Use the quadratic regression to predict the amount of revenue in 2008.

(d) Now find the linear regression for the data and use it to predict the amount of revenue in 2008.

55. ***The Cone Problem*** Begin with a circular piece of paper with a 4-in. radius as shown in (a). Cut out a sector with an arc length of x. Join the two edges of the remaining portion to form a cone with radius r and height h, as shown in (b).



(a) (b)

(a) Explain why the circumference of the base of the cone is $8\pi - x$.

(b) Express the radius r as a function of x.

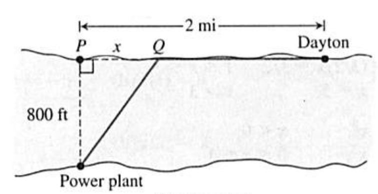
(c) Express the height h as a function of x.

(d) Express the volume V of the cone as a function of x.

56. ***Industrial Costs*** Dayton Power and Light, Inc., has a power plant on the Miami River where the river is 800 ft wide. To lay a new cable from the plant to a location in the city 2 mi downstream on the opposite side costs $180 per foot across the river and $100 per foot along the land.

(a) Suppose that the cable goes from the plant to a point Q on the opposite side that is x ft from the point P directly opposite the plant. Write a function $C(x)$ that gives the cost of laying the cable in terms of the distance x.

(b) Generate a table of values to determine if the least expensive location for point Q is less than 2000 ft or greater than 2000 ft from point P.



(Not to scale)

Standardized Test Questions

You should solve the following problems without using a graphing calculator.

57. True or False The function $f(x) = x^4 + x^2 + x$ is an even function. Justify your answer.

58. True or False The function $f(x) = x^{-3}$ is an odd function. Justify your answer.

59. Multiple Choice Which of the following gives the domain of $f(x) = \dfrac{x}{\sqrt{9 - x^2}}$?

(A) $x \ne \pm 3$ (B) $(-3, 3)$ (C) $[-3, 3]$
(D) $(-\infty, -3) \cup (3, \infty)$ (E) $(3, \infty)$

60. Multiple Choice Which of the following gives the range of $f(x) = 1 + \dfrac{1}{x - 1}$?

(A) $(-\infty, 1) \cup (1, \infty)$ (B) $x \ne 1$ (C) all real numbers
(D) $(-\infty, 0) \cup (0, \infty)$ (E) $x \ne 0$

61. Multiple Choice If $f(x) = 2x - 1$ and $g(x) = x + 3$, which of the following gives $(f \circ g)(2)$?

(A) 2 (B) 6 (C) 7 (D) 9 (E) 10

62. Multiple Choice The length L of a rectangle is twice as long as its width W. Which of the following gives the area A of the rectangle as a function of its width?

(A) $A(W) = 3W$ (B) $A(W) = \frac{1}{2}W^2$ (C) $A(W) = 2W^2$
(D) $A(W) = W^2 + 2W$ (E) $A(W) = W^2 - 2W$

Explorations

In Exercises 63–66, **(a)** graph $f \circ g$ and $g \circ f$ and make a conjecture about the domain and range of each function. **(b)** Then confirm your conjectures by finding formulas for $f \circ g$ and $g \circ f$.

63. $f(x) = x - 7, \quad g(x) = \sqrt{x}$

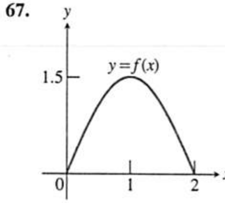
64. $f(x) = 1 - x^2, \quad g(x) = \sqrt{x}$

65. $f(x) = x^2 - 3, \quad g(x) = \sqrt{x + 2}$

66. $f(x) = \dfrac{2x - 1}{x + 3}, \quad g(x) = \dfrac{3x + 1}{2 - x}$

Group Activity In Exercises 67–70, a portion of the graph of a function defined on $[-2, 2]$ is shown. Complete each graph assuming that the graph is **(a)** even, **(b)** odd.

67.



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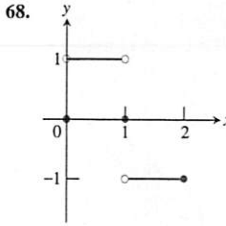

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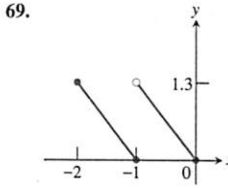

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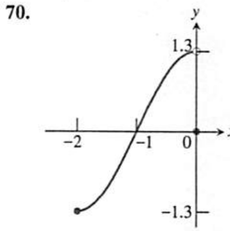


Extending the Ideas

71. Enter $y_1 = \sqrt{x}$, $y_2 = \sqrt{1 - x}$ and $y_3 = y_1 + y_2$ on your grapher.

(a) Graph y_3 in $[-3, 3]$ by $[-1, 3]$.

(b) Compare the domain of the graph of y_3 with the domains of the graphs of y_1 and y_2.

(c) Replace y_3 by

$$y_1 - y_2, \quad y_2 - y_1, \quad y_1 \cdot y_2, \quad y_1/y_2, \quad \text{and} \quad y_2/y_1,$$

in turn, and repeat the comparison of part (b).

(d) Based on your observations in (b) and (c), what would you conjecture about the domains of sums, differences, products, and quotients of functions?

72. ***Even and Odd Functions***

(a) Must the product of two even functions always be even? Give reasons for your answer.

(b) Can anything be said about the product of two odd functions? Give reasons for your answer.

Quick Review 1.3 *(For help, go to Section 1.3.)*

In Exercises 1–3, evaluate the expression. Round your answers to 3 decimal places.

1. $5^{2/3}$
2. $3^{\sqrt{2}}$
3. $3^{-1.5}$

In Exercises 4–6, solve the equation. Round your answers to 4 decimal places.

4. $x^3 = 17$
5. $x^5 = 24$
6. $x^{10} = 1.4567$

In Exercises 7 and 8, find the value of investing P dollars for n years with the interest rate r compounded annually.

7. $P = \$500, \quad r = 4.75\%, \quad n = 5$ years
8. $P = \$1000, \quad r = 6.3\%, \quad n = 3$ years

In Exercises 9 and 10, simplify the exponential expression.

9. $\dfrac{(x^{-3}y^2)^2}{(x^4y^3)^3}$
10. $\left(\dfrac{a^3b^{-2}}{c^4}\right)^2\left(\dfrac{a^4c^{-2}}{b^3}\right)^{-1}$

Section 1.3 Exercises

In Exercises 1–4, graph the function. State its domain and range.

1. $y = -2^x + 3$
2. $y = e^x + 3$
3. $y = 3 \cdot e^{-x} - 2$
4. $y = -2^{-x} - 1$

In Exercises 5–8, rewrite the exponential expression to have the indicated base.

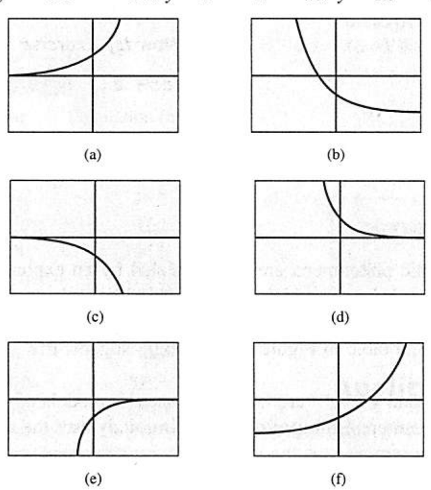
5. 9^{2x}, base 3
6. 16^{3x}, base 2
7. $(1/8)^{2x}$, base 2
8. $(1/27)^x$, base 3

In Exercises 9–12, use a graph to find the zeros of the function.

9. $f(x) = 2^x - 5$
10. $f(x) = e^x - 4$
11. $f(x) = 3^x - 0.5$
12. $f(x) = 3 - 2^x$

In Exercises 13–18, match the function with its graph. Try to do it without using your grapher.

13. $y = 2^x$
14. $y = 3^{-x}$
15. $y = -3^{-x}$
16. $y = -0.5^{-x}$
17. $y = 2^{-x} - 2$
18. $y = 1.5^x - 2$



19. ***Population of Nevada*** Table 1.9 gives the population of Nevada for several years.

Table 1.9 Population of Nevada

Year	Population (thousands)
1998	1,853
1999	1,935
2000	1,998
2001	2,095
2002	2,167
2003	2,241

Source: Statistical Abstract of the United States, 2004–2005.

(a) Compute the ratios of the population in one year by the population in the previous year.

(b) Based on part (a), create an exponential model for the population of Nevada.

(c) Use your model in part (b) to predict the population of Nevada in 2010.

20. ***Population of Virginia*** Table 1.10 gives the population of Virginia for several years.

Table 1.10 Population of Virginia

Year	Population (thousands)
1998	6,901
1999	7,000
2000	7,078
2001	7,193
2002	7,288
2003	7,386

Source: Statistical Abstract of the United States, 2004–2005.

(a) Compute the ratios of the population in one year by the population in the previous year.

(b) Based on part (a), create an exponential model for the population of Virginia.

(c) Use your model in part (b) to predict the population of Virginia in 2008.

In Exercises 21–32, use an exponential model to solve the problem.

21. ***Population Growth*** The population of Knoxville is 500,000 and is increasing at the rate of 3.75% each year. Approximately when will the population reach 1 million?

22. ***Population Growth*** The population of Silver Run in the year 1890 was 6250. Assume the population increased at a rate of 2.75% per year.

(a) Estimate the population in 1915 and 1940.

(b) Approximately when did the population reach 50,000?

23. ***Radioactive Decay*** The half-life of phosphorus-32 is about 14 days. There are 6.6 grams present initially.

(a) Express the amount of phosphorus-32 remaining as a function of time t.

(b) When will there be 1 gram remaining?

24. ***Finding Time*** If John invests \$2300 in a savings account with a 6% interest rate compounded annually, how long will it take until John's account has a balance of \$4150?

25. ***Doubling Your Money*** Determine how much time is required for an investment to double in value if interest is earned at the rate of 6.25% compounded annually.

26. ***Doubling Your Money*** Determine how much time is required for an investment to double in value if interest is earned at the rate of 6.25% compounded monthly.

27. ***Doubling Your Money*** Determine how much time is required for an investment to double in value if interest is earned at the rate of 6.25% compounded continuously.

28. ***Tripling Your Money*** Determine how much time is required for an investment to triple in value if interest is earned at the rate of 5.75% compounded annually.

29. ***Tripling Your Money*** Determine how much time is required for an investment to triple in value if interest is earned at the rate of 5.75% compounded daily.

30. ***Tripling Your Money*** Determine how much time is required for an investment to triple in value if interest is earned at the rate of 5.75% compounded continuously.

31. ***Cholera Bacteria*** Suppose that a colony of bacteria starts with 1 bacterium and doubles in number every half hour. How many bacteria will the colony contain at the end of 24 h?

32. ***Eliminating a Disease*** Suppose that in any given year, the number of cases of a disease is reduced by 20%. If there are 10,000 cases today, how many years will it take

(a) to reduce the number of cases to 1000?

(b) to eliminate the disease; that is, to reduce the number of cases to less than 1?

Group Activity In Exercises 33–36, copy and complete the table for the function.

33. $y = 2x - 3$

x	y	Change (Δy)
1	?	
2	?	?
3	?	?
4	?	?

34. $y = -3x + 4$

x	y	Change (Δy)
1	?	
2	?	?
3	?	?
4	?	?

35. $y = x^2$

x	y	Change (Δy)
1	?	
2	?	?
3	?	?
4	?	?

36. $y = 3e^x$

x	y	Ratio (y_i/y_{i-1})
1	?	
2	?	?
3	?	?
4	?	?

37. **Writing to Learn** Explain how the change Δy is related to the slopes of the lines in Exercises 33 and 34. If the changes in x are constant for a linear function, what would you conclude about the corresponding changes in y?

38. ***Bacteria Growth*** The number of bacteria in a petri dish culture after t hours is

$$B = 100e^{0.693t}.$$

(a) What was the initial number of bacteria present?

(b) How many bacteria are present after 6 hours?

(c) Approximately when will the number of bacteria be 200? Estimate the doubling time of the bacteria.

39. ***Population of Texas*** Table 1.11 gives the population of Texas for several years.

Table 1.11 Population of Texas

Year	Population (thousands)
1980	14,229
1990	16,986
1995	18,959
1998	20,158
1999	20,558
2000	20,852

Source: Statistical Abstract of the United States, 2004–2005.

(a) Let $x = 0$ represent 1980, $x = 1$ represent 1981, and so forth. Find an exponential regression for the data, and superimpose its graph on a scatter plot of the data.

(b) Use the exponential regression equation to estimate the population of Texas in 2003. How close is the estimate to the actual population of 22,119,000 in 2003?

(c) Use the exponential regression equation to estimate the annual rate of growth of the population of Texas.

40. ***Population of California*** Table 1.12 gives the population of California for several years.

Table 1.12 Population of California

Year	Population (thousands)
1980	23,668
1990	29,811
1995	31,697
1998	32,988
1999	33,499
2000	33,872

Source: Statistical Abstract of the United States, 2004–2005.

(a) Let $x = 0$ represent 1980, $x = 1$ represent 1981, and so forth. Find an exponential regression for the data, and superimpose its graph on a scatter plot of the data.

(b) Use the exponential regression equation to estimate the population of California in 2003. How close is the estimate to the actual population of 35,484,000 in 2003?

(c) Use the exponential regression equation to estimate the annual rate of growth of the population of California.

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

41. True or False The number 3^{-2} is negative. Justify your answer.

42. True or False If $4^3 = 2^a$, then $a = 6$. Justify your answer.

43. Multiple Choice John invests \$200 at 4.5% compounded annually. About how long will it take for John's investment to double in value?

(A) 6 yrs (B) 9 yrs (C) 12 yrs (D) 16 yrs (E) 20 yrs

44. Multiple Choice Which of the following gives the domain of $y = 2e^{-x} - 3$?

(A) $(-\infty, \infty)$ (B) $[-3, \infty)$ (C) $[-1, \infty)$ (D) $(-\infty, 3]$ (E) $x \neq 0$

45. Multiple Choice Which of the following gives the range of $y = 4 - 2^{-x}$?

(A) $(-\infty, \infty)$ (B) $(-\infty, 4)$ (C) $[-4, \infty)$ (D) $(-\infty, 4]$ (E) all reals

46. Multiple Choice Which of the following gives the best approximation for the zero of $f(x) = 4 - e^x$?

(A) $x = -1.386$ (B) $x = 0.386$ (C) $x = 1.386$ (D) $x = 3$ (E) there are no zeros

Exploration

47. Let $y_1 = x^2$ and $y_2 = 2^x$.

(a) Graph y_1 and y_2 in $[-5, 5]$ by $[-2, 10]$. How many times do you think the two graphs cross?

(b) Compare the corresponding changes in y_1 and y_2 as x changes from 1 to 2, 2 to 3, and so on. How large must x be for the changes in y_2 to overtake the changes in y_1?

(c) Solve for x: $x^2 = 2^x$. (d) Solve for x: $x^2 < 2^x$.

Extending the Ideas

In Exercises 48 and 49, assume that the graph of the exponential function $f(x) = k \cdot a^x$ passes through the two points. Find the values of a and k.

48. $(1, 4.5), (-1, 0.5)$ **49.** $(1, 1.5), (-1, 6)$

Quick Quiz for AP* Preparation: Sections 1.1–1.3

You may use graphing calculator to solve the following problems.

1. Multiple Choice Which of the following gives an equation for the line through $(3, -1)$ and parallel to the line $y = -2x + 1$?

(A) $= \frac{1}{2}x + \frac{7}{2}$ (B) $y = \frac{1}{2}x - \frac{5}{2}$ (C) $y = -2x + 5$ (D) $y = -2x - 7$ (E) $y = -2x + 1$

2. Multiple Choice If $f(x) = x^2 + 1$ and $g(x) = 2x - 1$, which of the following gives $f \circ g(2)$?

(A) 2 (B) 5 (C) 9 (D) 10 (E) 15

3. Multiple Choice The half-life of a certain radioactive substance is 8 hrs. There are 5 grams present initially. Which of the following gives the best approximation when there will be 1 gram remaining?

(A) 2 (B) 10 (C) 15 (D) 16 (E) 19

4. Free Response Let $f(x) = e^{-x} - 2$.

(a) Find the domain of f. (b) Find the range of f.

(c) Find the zeros of f.

This line goes through the point $(-2, 1)$ when $t = 0$. We determine a and b so that the line goes through $(3, 5)$ when $t = 1$.

$$3 = -2 + a \quad \Rightarrow \quad a = 5 \qquad x = 3 \text{ when } t = 1.$$
$$5 = 1 + b \quad \Rightarrow \quad b = 4 \qquad y = 5 \text{ when } t = 1.$$

Therefore,

$$x = -2 + 5t, \quad y = 1 + 4t, \quad 0 \le t \le 1$$

is a parametrization of the line segment with initial point $(-2, 1)$ and terminal point $(3, 5)$.

Now try Exercise 23.

Quick Review 1.4 *(For help, go to Section 1.1 and Appendix A1.)*

In Exercises 1–3, write an equation for the line.

1. the line through the points $(1, 8)$ and $(4, 3)$
2. the horizontal line through the point $(3, -4)$
3. the vertical line through the point $(2, -3)$

In Exercises 4–6, find the x- and y-intercepts of the graph of the relation.

4. $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$ 5. $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$

6. $2y^2 = x + 1$

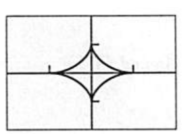
In Exercises 7 and 8, determine whether the given points lie on the graph of the relation.

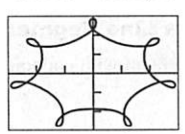
7. $2x^2y + y^2 = 3$
   (a) $(1, 1)$ (b) $(-1, -1)$ (c) $(1/2, -2)$
8. $9x^2 - 18x + 4y^2 = 27$
   (a) $(1, 3)$ (b) $(1, -3)$ (c) $(-1, 3)$
9. Solve for t.
   (a) $2x + 3t = -5$ (b) $3y - 2t = -1$
10. For what values of a is each equation true?
   (a) $\sqrt{a^2} = a$ (b) $\sqrt{a^2} = \pm a$ (c) $\sqrt{4a^2} = 2|a|$

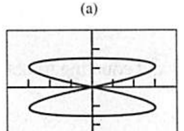
Section 1.4 Exercises

In Exercises 1–4, match the parametric equations with their graph. State the approximate dimensions of the viewing window. Give a parameter interval that traces the curve exactly once.

1. $x = 3\sin(2t), \quad y = 1.5\cos t$
2. $x = \sin^3 t, \quad y = \cos^3 t$
3. $x = 7\sin t - \sin(7t), \quad y = 7\cos t - \cos(7t)$
4. $x = 12\sin t - 3\sin(6t), \quad y = 12\cos t + 3\cos(6t)$


(a)


(b)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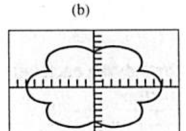

(d)

In Exercises 5–22, a parametrization is given for a curve.

(a) Graph the curve. What are the initial and terminal points, if any? Indicate the direction in which the curve is traced.

(b) Find a Cartesian equation for a curve that contains the parametrized curve. What portion of the graph of the Cartesian equation is traced by the parametrized curve?

5. $x = 3t, \quad y = 9t^2, \quad -\infty < t < \infty$
6. $x = -\sqrt{t}, \quad y = t, \quad t \ge 0$
7. $x = t, \quad y = \sqrt{t}, \quad t \ge 0$
8. $x = (\sec^2 t) - 1, \quad y = \tan t, \quad -\pi/2 < t < \pi/2$
9. $x = \cos t, \quad y = \sin t, \quad 0 \le t \le \pi$
10. $x = \sin(2\pi t), \quad y = \cos(2\pi t), \quad 0 \le t \le 1$
11. $x = \cos(\pi - t), \quad y = \sin(\pi - t), \quad 0 \le t \le \pi$
12. $x = 4\cos t, \quad y = 2\sin t, \quad 0 \le t \le 2\pi$
13. $x = 4\sin t, \quad y = 2\cos t, \quad 0 \le t \le \pi$
14. $x = 4\sin t, \quad y = 5\cos t, \quad 0 \le t \le 2\pi$
15. $x = 2t - 5, \quad y = 4t - 7, \quad -\infty < t < \infty$
16. $x = 1 - t, \quad y = 1 + t, \quad -\infty < t < \infty$
17. $x = t, \quad y = 1 - t, \quad 0 \le t \le 1$
18. $x = 3 - 3t, \quad y = 2t, \quad 0 \le t \le 1$
19. $x = 4 - \sqrt{t}, \quad y = \sqrt{t}, \quad 0 \le t$
20. $x = t^2, \quad y = \sqrt{4 - t^2}, \quad 0 \le t \le 2$
21. $x = \sin t, \quad y = \cos 2t, \quad -\infty < t < \infty$
22. $x = t^2 - 3, \quad y = t, \quad t \le 0$

In Exercises 23–28, find a parametrization for the curve.

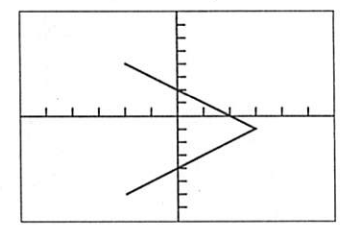
23. the line segment with endpoints $(-1, -3)$ and $(4, 1)$
24. the line segment with endpoints $(-1, 3)$ and $(3, -2)$
25. the lower half of the parabola $x - 1 = y^2$
26. the left half of the parabola $y = x^2 + 2x$
27. the ray (half line) with initial point $(2, 3)$ that passes through the point $(-1, -1)$
28. the ray (half line) with initial point $(-1, 2)$ that passes through the point $(0, 0)$

Group Activity In Exercises 29–32, refer to the graph of

$$x = 3 - |t|, \quad y = t - 1, \quad -5 \le t \le 5,$$

shown in the figure. Find the values of t that produce the graph in the given quadrant.

29. Quadrant I
30. Quadrant II
31. Quadrant III
32. Quadrant IV



[−6, 6] by [−8, 8]

In Exercises 33 and 34, find a parametrization for the part of the graph that lies in Quadrant I.

33. $y = x^2 + 2x + 2$
34. $y = \sqrt{x + 3}$
35. ***Circles*** Find parametrizations to model the motion of a particle that starts at $(a, 0)$ and traces the circle $x^2 + y^2 = a^2$, $a > 0$, as indicated.
   (a) once clockwise (b) once counterclockwise
   (c) twice clockwise (d) twice counterclockwise
36. ***Ellipses*** Find parametrizations to model the motion of a particle that starts at $(-a, 0)$ and traces the ellipse
   $$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1, \quad a > 0, b > 0,$$
   as indicated.
   (a) once clockwise (b) once counterclockwise
   (c) twice clockwise (d) twice counterclockwise

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

37. **True or False** The graph of the parametric curve $x = 3\cos t$, $y = 4\sin t$ is a circle. Justify your answer.
38. **True or False** The parametric curve $x = 2\cos(-t)$, $y = 2\sin(-t)$, $0 \le t \le 2\pi$ is traced clockwise. Justify your answer.

In Exercises 39 and 40, use the parametric curve $x = 5t$, $y = 3 - 3t$, $0 \le t \le 1$.

39. **Multiple Choice** Which of the following describes its graph?
   (A) circle (B) parabola (C) ellipse
   (D) line segment (E) line
40. **Multiple Choice** Which of the following is the initial point of the curve?
   (A) $(-5, 6)$ (B) $(0, -3)$ (C) $(0, 3)$ (D) $(5, 0)$
   (E) $(10, -3)$
41. **Multiple Choice** Which of the following describes the graph of the parametric curve $x = -3\sin t$, $y = -3\cos t$?
   (A) circle (B) parabola (C) ellipse
   (D) hyperbola (E) line
42. **Multiple Choice** Which of the following describes the graph of the parametric curve $x = 3t$, $y = 2t$, $t \ge 1$?
   (A) circle (B) parabola (C) line segment
   (D) line (E) ray

Explorations

43. ***Hyperbolas*** Let $x = a\sec t$ and $y = b\tan t$.
   (a) **Writing to Learn** Let $a = 1, 2,$ or 3, $b = 1, 2,$ or 3, and graph using the parameter interval $(-\pi/2, \pi/2)$. Explain what you see, and describe the role of a and b in these parametric equations. (Caution: If you get what appear to be asymptotes, try using the approximation $[-1.57, 1.57]$ for the parameter interval.)
   (b) Let $a = 2$, $b = 3$, and graph in the parameter interval $(\pi/2, 3\pi/2)$. Explain what you see.
   (c) **Writing to Learn** Let $a = 2$, $b = 3$, and graph using the parameter interval $(-\pi/2, 3\pi/2)$. Explain why you must be careful about graphing in this interval or any interval that contains $\pm\pi/2$.
   (d) Use algebra to explain why
   $$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1.$$
   (e) Let $x = a\tan t$ and $y = b\sec t$. Repeat (a), (b), and (d) using an appropriate version of (d).
44. ***Transformations*** Let $x = (2\cos t) + h$ and $y = (2\sin t) + k$.
   (a) **Writing to Learn** Let $k = 0$ and $h = -2, -1, 1,$ and 2, in turn. Graph using the parameter interval $[0, 2\pi]$. Describe the role of h.

(b) Writing to Learn Let $h = 0$ and $k = -2, -1, 1,$ and 2, in turn. Graph using the parameter interval $[0, 2\pi]$. Describe the role of k.

(c) Find a parametrization for the circle with radius 5 and center at $(2, -3)$.

(d) Find a parametrization for the ellipse centered at $(-3, 4)$ with semimajor axis of length 5 parallel to the x-axis and semiminor axis of length 2 parallel to the y-axis.

In Exercises 45 and 46, a parametrization is given for a curve.

(a) Graph the curve. What are the initial and terminal points, if any? Indicate the direction in which the curve is traced.

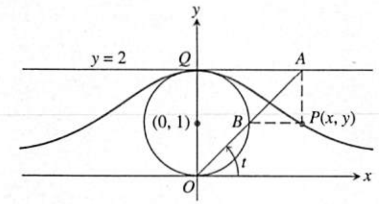
(b) Find a Cartesian equation for a curve that contains the parametrized curve. What portion of the graph of the Cartesian equation is traced by the parametrized curve?

45. $x = -\sec t, \quad y = \tan t, \quad -\pi/2 < t < \pi/2$

46. $x = \tan t, \quad y = -2 \sec t, \quad -\pi/2 < t < \pi/2$

Extending the Ideas

47. *The Witch of Agnesi* The bell-shaped witch of Agnesi can be constructed as follows. Start with the circle of radius 1, centered at the point $(0, 1)$ as shown in the figure.



Choose a point A on the line $y = 2$, and connect it to the origin with a line segment. Call the point where the segment crosses the circle B. Let P be the point where the vertical line through A crosses the horizontal line through B. The witch is the curve traced by P as A moves along the line $y = 2$.

Find a parametrization for the witch by expressing the coordinates of P in terms of t, the radian measure of the angle that segment OA makes with the positive x-axis. The following equalities (which you may assume) will help:

(i) $x = AQ$ **(ii)** $y = 2 - AB \sin t$ **(iii)** $AB \cdot AO = (AQ)^2$

48. *Parametrizing Lines and Segments*

(a) Show that $x = x_1 + (x_2 - x_1)t, \quad y = y_1 + (y_2 - y_1)t,$ $-\infty < t < \infty$ is a parametrization for the line through the points (x_1, y_1) and (x_2, y_2).

(b) Find a parametrization for the line segment with endpoints (x_1, y_1) and (x_2, y_2).

1.5 Functions and Logarithms

What you'll learn about

- One-to-One Functions
- Inverses
- Finding Inverses
- Logarithmic Functions
- Properties of Logarithms
- Applications

. . . and why

Logarithmic functions are used in many applications, including finding time in investment problems.

One-to-One Functions

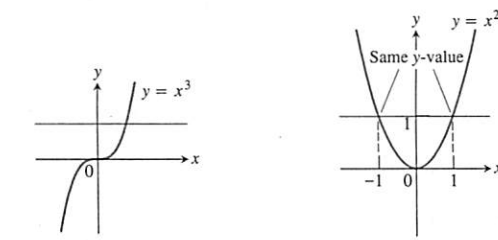
As you know, a function is a rule that assigns a single value in its range to each point in its domain. Some functions assign the same output to more than one input. For example, $f(x) = x^2$ assigns the output 4 to both 2 and -2. Other functions never output a given value more than once. For example, the cubes of different numbers are always different.

If each output value of a function is associated with exactly one input value, the function is *one-to-one.*

DEFINITION One-to-One Function

A function $f(x)$ is **one-to-one** on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$.

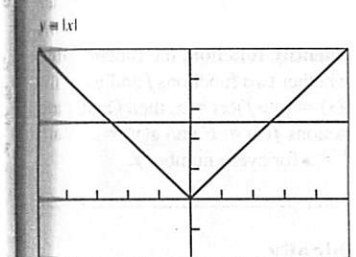
The graph of a one-to-one function $y = f(x)$ can intersect any horizontal line at most once (the *horizontal line test*). If it intersects such a line more than once it assumes the same y-value more than once, and is therefore not one-to-one (Figure 1.33).



One-to-one: Graph meets each horizontal line once.

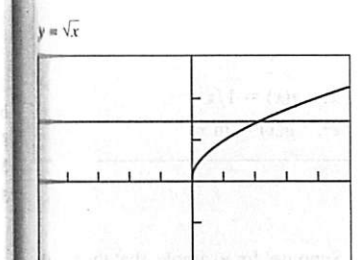
Not one-to-one: Graph meets some horizontal lines more than once.

Figure 1.33 Using the horizontal line test, we see that $y = x^3$ is one-to-one and $y = x^2$ is not.



[–5, 5] by [–2, 5]

(a)



[–5, 5] by [–2, 3]

(b)

Figure 1.34 (a) The graph of $f(x) = |x|$ and a horizontal line. (b) The graph of $g(x) = \sqrt{x}$ and a horizontal line. (Example 1)

EXAMPLE 1 Using the Horizontal Line Test

Determine whether the functions are one-to-one.

(a) $f(x) = |x|$ **(b)** $g(x) = \sqrt{x}$

SOLUTION

(a) As Figure 1.34a suggests, each horizontal line $y = c, c > 0$, intersects the graph of $f(x) = |x|$ twice. So f is not one-to-one.

(b) As Figure 1.34b suggests, each horizontal line intersects the graph of $g(x) = \sqrt{x}$ either once or not at all. The function g is one-to-one.

Now try Exercise 1.

39. ***Population of Texas*** Table 1.11 gives the population of Texas for several years.

Table 1.11 Population of Texas

Year	Population (thousands)
1980	14,229
1990	16,986
1995	18,959
1998	20,158
1999	20,558
2000	20,852

Source: Statistical Abstract of the United States, 2004–2005.

(a) Let $x = 0$ represent 1980, $x = 1$ represent 1981, and so forth. Find an exponential regression for the data, and superimpose its graph on a scatter plot of the data.

(b) Use the exponential regression equation to estimate the population of Texas in 2003. How close is the estimate to the actual population of 22,119,000 in 2003?

(c) Use the exponential regression equation to estimate the annual rate of growth of the population of Texas.

40. ***Population of California*** Table 1.12 gives the population of California for several years.

Table 1.12 Population of California

Year	Population (thousands)
1980	23,668
1990	29,811
1995	31,697
1998	32,988
1999	33,499
2000	33,872

Source: Statistical Abstract of the United States, 2004–2005.

(a) Let $x = 0$ represent 1980, $x = 1$ represent 1981, and so forth. Find an exponential regression for the data, and superimpose its graph on a scatter plot of the data.

(b) Use the exponential regression equation to estimate the population of California in 2003. How close is the estimate to the actual population of 35,484,000 in 2003?

(c) Use the exponential regression equation to estimate the annual rate of growth of the population of California.

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

41. True or False The number 3^{-2} is negative. Justify your answer.

42. True or False If $4^3 = 2^a$, then $a = 6$. Justify your answer.

43. Multiple Choice John invests $200 at 4.5% compounded annually. About how long will it take for John's investment to double in value?

(A) 6 yrs (B) 9 yrs (C) 12 yrs (D) 16 yrs (E) 20 yrs

44. Multiple Choice Which of the following gives the domain of $y = 2e^{-x} - 3$?

(A) $(-\infty, \infty)$ (B) $[-3, \infty)$ (C) $[-1, \infty)$ (D) $(-\infty, 3]$ (E) $x \neq 0$

45. Multiple Choice Which of the following gives the range of $y = 4 - 2^{-x}$?

(A) $(-\infty, \infty)$ (B) $(-\infty, 4)$ (C) $[-4, \infty)$ (D) $(-\infty, 4]$ (E) all reals

46. Multiple Choice Which of the following gives the best approximation for the zero of $f(x) = 4 - e^x$?

(A) $x = -1.386$ (B) $x = 0.386$ (C) $x = 1.386$ (D) $x = 3$ (E) there are no zeros

Exploration

47. Let $y_1 = x^2$ and $y_2 = 2^x$.

(a) Graph y_1 and y_2 in $[-5, 5]$ by $[-2, 10]$. How many times do you think the two graphs cross?

(b) Compare the corresponding changes in y_1 and y_2 as x changes from 1 to 2, 2 to 3, and so on. How large must x be for the changes in y_2 to overtake the changes in y_1?

(c) Solve for x: $x^2 = 2^x$. **(d)** Solve for x: $x^2 < 2^x$.

Extending the Ideas

In Exercises 48 and 49, assume that the graph of the exponential function $f(x) = k \cdot a^x$ passes through the two points. Find the values of a and k.

48. $(1, 4.5), (-1, 0.5)$ **49.** $(1, 1.5), (-1, 6)$

Quick Quiz for AP* Preparation: Sections 1.1–1.3

You may use graphing calculator to solve the following problems.

1. Multiple Choice Which of the following gives an equation for the line through $(3, -1)$ and parallel to the line $y = -2x + 1$?

(A) $= \frac{1}{2}x + \frac{7}{2}$ (B) $y = \frac{1}{2}x - \frac{5}{2}$ (C) $y = -2x + 5$ (D) $y = -2x - 7$ (E) $y = -2x + 1$

2. Multiple Choice If $f(x) = x^2 + 1$ and $g(x) = 2x - 1$, which of the following gives $f \circ g(2)$?

(A) 2 (B) 5 (C) 9 (D) 10 (E) 15

3. Multiple Choice The half-life of a certain radioactive substance is 8 hrs. There are 5 grams present initially. Which of the following gives the best approximation when there will be 1 gram remaining?

(A) 2 (B)10 (C) 15 (D) 16 (E) 19

4. Free Response Let $f(x) = e^{-x} - 2$.

(a) Find the domain of f. **(b)** Find the range of f.

(c) Find the zeros of f.

Quick Review 1.3 *(For help, go to Section 1.3.)*

In Exercises 1–3, evaluate the expression. Round your answers to 3 decimal places.

1. $5^{2/3}$ **2.** $3^{\sqrt{2}}$

3. $3^{-1.5}$

In Exercises 4–6, solve the equation. Round your answers to 4 decimal places.

4. $x^3 = 17$ **5.** $x^5 = 24$

6. $x^{10} = 1.4567$

In Exercises 7 and 8, find the value of investing P dollars for n years with the interest rate r compounded annually.

7. $P = \$500,\quad r = 4.75\%,\quad n = 5$ years

8. $P = \$1000,\quad r = 6.3\%,\quad n = 3$ years

In Exercises 9 and 10, simplify the exponential expression.

9. $\dfrac{(x^{-3}y^2)^2}{(x^4y^3)^3}$ **10.** $\left(\dfrac{a^3b^{-2}}{c^4}\right)^2\left(\dfrac{a^4c^{-2}}{b^3}\right)^{-1}$

Section 1.3 Exercises

In Exercises 1–4, graph the function. State its domain and range.

1. $y = -2^x + 3$ **2.** $y = e^x + 3$

3. $y = 3 \cdot e^{-x} - 2$ **4.** $y = -2^{-x} - 1$

In Exercises 5–8, rewrite the exponential expression to have the indicated base.

5. 9^{2x}, base 3 **6.** 16^{3x}, base 2

7. $(1/8)^{2x}$, base 2 **8.** $(1/27)^x$, base 3

In Exercises 9–12, use a graph to find the zeros of the function.

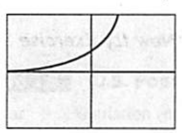
9. $f(x) = 2^x - 5$ **10.** $f(x) = e^x - 4$

11. $f(x) = 3^x - 0.5$ **12.** $f(x) = 3 - 2^x$

In Exercises 13–18, match the function with its graph. Try to do it without using your grapher.

13. $y = 2^x$ **14.** $y = 3^{-x}$ **15.** $y = -3^{-x}$

16. $y = -0.5^{-x}$ **17.** $y = 2^{-x} - 2$ **18.** $y = 1.5^x - 2$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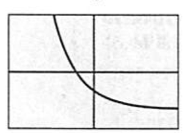
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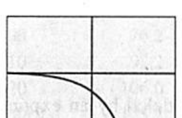
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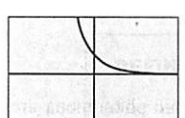
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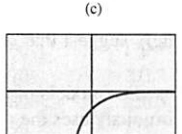
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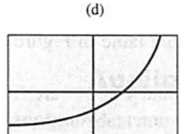
(f)

19. ***Population of Nevada*** Table 1.9 gives the population of Nevada for several years.

Table 1.9 Population of Nevada

Year	Population (thousands)
1998	1,853
1999	1,935
2000	1,998
2001	2,095
2002	2,167
2003	2,241

Source: Statistical Abstract of the United States, 2004–2005.

(a) Compute the ratios of the population in one year by the population in the previous year.

(b) Based on part (a), create an exponential model for the population of Nevada.

(c) Use your model in part (b) to predict the population of Nevada in 2010.

20. ***Population of Virginia*** Table 1.10 gives the population of Virginia for several years.

Table 1.10 Population of Virginia

Year	Population (thousands)
1998	6,901
1999	7,000
2000	7,078
2001	7,193
2002	7,288
2003	7,386

Source: Statistical Abstract of the United States, 2004–2005.

(a) Compute the ratios of the population in one year by the population in the previous year.

(b) Based on part (a), create an exponential model for the population of Virginia.

(c) Use your model in part (b) to predict the population of Virginia in 2008.

In Exercises 21–32, use an exponential model to solve the problem.

21. ***Population Growth*** The population of Knoxville is 500,000 and is increasing at the rate of 3.75% each year. Approximately when will the population reach 1 million?

22. ***Population Growth*** The population of Silver Run in the year 1890 was 6250. Assume the population increased at a rate of 2.75% per year.

(a) Estimate the population in 1915 and 1940.

(b) Approximately when did the population reach 50,000?

23. ***Radioactive Decay*** The half-life of phosphorus-32 is about 14 days. There are 6.6 grams present initially.

(a) Express the amount of phosphorus-32 remaining as a function of time t.

(b) When will there be 1 gram remaining?

24. ***Finding Time*** If John invests \$2300 in a savings account with a 6% interest rate compounded annually, how long will it take until John's account has a balance of \$4150?

25. ***Doubling Your Money*** Determine how much time is required for an investment to double in value if interest is earned at the rate of 6.25% compounded annually.

26. ***Doubling Your Money*** Determine how much time is required for an investment to double in value if interest is earned at the rate of 6.25% compounded monthly.

27. ***Doubling Your Money*** Determine how much time is required for an investment to double in value if interest is earned at the rate of 6.25% compounded continuously.

28. ***Tripling Your Money*** Determine how much time is required for an investment to triple in value if interest is earned at the rate of 5.75% compounded annually.

29. ***Tripling Your Money*** Determine how much time is required for an investment to triple in value if interest is earned at the rate of 5.75% compounded daily.

30. ***Tripling Your Money*** Determine how much time is required for an investment to triple in value if interest is earned at the rate of 5.75% compounded continuously.

31. ***Cholera Bacteria*** Suppose that a colony of bacteria starts with 1 bacterium and doubles in number every half hour. How many bacteria will the colony contain at the end of 24 h?

32. ***Eliminating a Disease*** Suppose that in any given year, the number of cases of a disease is reduced by 20%. If there are 10,000 cases today, how many years will it take

(a) to reduce the number of cases to 1000?

(b) to eliminate the disease; that is, to reduce the number of cases to less than 1?

Group Activity In Exercises 33–36, copy and complete the table for the function.

33. $y = 2x - 3$

x	y	Change (Δy)
1	?	
2	?	?
3	?	?
4	?	?

34. $y = -3x + 4$

x	y	Change (Δy)
1	?	
2	?	?
3	?	?
4	?	?

35. $y = x^2$

x	y	Change (Δy)
1	?	
2	?	?
3	?	?
4	?	?

36. $y = 3e^x$

x	y	Ratio (y_i/y_{i-1})
1	?	
2	?	?
3	?	?
4	?	?

37. **Writing to Learn** Explain how the change Δy is related to the slopes of the lines in Exercises 33 and 34. If the changes in x are constant for a linear function, what would you conclude about the corresponding changes in y?

38. ***Bacteria Growth*** The number of bacteria in a petri dish culture after t hours is

$$B = 100e^{0.693t}.$$

(a) What was the initial number of bacteria present?

(b) How many bacteria are present after 6 hours?

(c) Approximately when will the number of bacteria be 200? Estimate the doubling time of the bacteria.

EXPLORATION 2 Supporting the Product Rule

Let $y_1 = \ln(ax)$, $y_2 = \ln x$, and $y_3 = y_1 - y_2$.

1. Graph y_1 and y_2 for $a = 2, 3, 4$, and 5. How do the graphs of y_1 and y_2 appear to be related?
2. Support your finding by graphing y_3.
3. Confirm your finding algebraically.

The following formula allows us to evaluate $\log_a x$ for any base $a > 0$, $a \neq 1$, and to obtain its graph using the natural logarithm function on our grapher.

Change of Base Formula

$$\log_a x = \frac{\ln x}{\ln a}$$

EXAMPLE 5 Graphing a Base *a* Logarithm Function

Graph $f(x) = \log_2 x$.

SOLUTION

We use the change of base formula to rewrite $f(x)$.

$$f(x) = \log_2 x = \frac{\ln x}{\ln 2}.$$

Figure 1.38 gives the graph of f. *Now try Exercise 41.*

$y = \dfrac{\ln x}{\ln 2}$

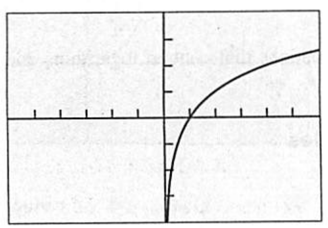

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Figure 1.38 The graph of $f(x) = \log_2 x$ using $f(x) = (\ln x)/(\ln 2)$. (Example 5)

Applications

In Section 1.3 we used graphical methods to solve exponential growth and decay problems. Now we can use the properties of logarithms to solve the same problems algebraically.

EXAMPLE 6 Finding Time

Sarah invests \$1000 in an account that earns 5.25% interest compounded annually. How long will it take the account to reach \$2500?

SOLUTION

Model The amount in the account at any time t in years is $1000(1.0525)^t$, so we need to solve the equation

$$1000(1.0525)^t = 2500.$$

continued

Table 1.15 Saudi Arabia's Natural Gas Production

Year	Cubic Feet (trillions)
1997	1.60
1998	1.65
1999	1.63
2000	1.76
2001	1.90

Source: Statistical Abstract of the United States, 2004–2005.

Solve Algebraically

$$(1.0525)^t = 2.5 \quad \text{Divide by 1000.}$$
$$\ln(1.0525)^t = \ln 2.5 \quad \text{Take logarithms of both sides.}$$
$$t \ln 1.0525 = \ln 2.5 \quad \text{Power Rule}$$
$$t = \frac{\ln 2.5}{\ln 1.0525} \approx 17.9$$

Interpret The amount in Sarah's account will be \$2500 in about 17.9 years, or about 17 years and 11 months. *Now try Exercise 47.*

EXAMPLE 7 Estimating Natural Gas Production

Table 1.15 shows the annual number of cubic feet in trillions of natural gas produced by Saudi Arabia for several years.

Find the natural logarithm regression equation for the data in Table 1.15 and use it to estimate the number of cubic feet of natural gas produced by Saudi Arabia in 2002. Compare with the actual amount of 2.00 trillion cubic feet in 2002.

SOLUTION

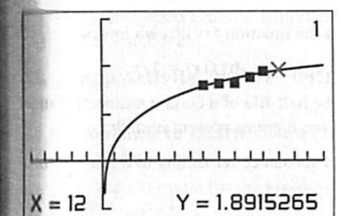
Model We let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth. We compute the natural logarithm regression equation to be

$$f(x) = 0.3730 + (0.611)\ln(x).$$

Solve Graphically Figure 1.39 shows the graph of f superimposed on the scatter plot of the data. The year 2002 is represented by $x = 12$. Reading from the graph we find $f(12) = 1.89$ trillion cubic feet.

Interpret The natural logarithmic model gives an underestimate of 0.11 trillion cubic feet of the 2002 natural gas production. *Now try Exercise 49.*

$f(x) = 0.3730 + (0.611)\ln x$



[–5, 15] by [–1, 3]

Figure 1.39 The value of f at $x = 12$ is about 1.89. (Example 7)

Quick Review 1.5 *(For help, go to Sections 1.2, 1.3, and 1.4.)*

In Exercises 1–4, let $f(x) = \sqrt[3]{x-1}$, $g(x) = x^2 + 1$, and evaluate the expression.

1. $(f \circ g)(1)$
2. $(g \circ f)(-7)$
3. $(f \circ g)(x)$
4. $(g \circ f)(x)$

In Exercises 5 and 6, choose parametric equations and a parameter interval to represent the function on the interval specified.

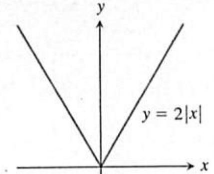
5. $y = \dfrac{1}{x-1}$, $x \geq 2$
6. $y = x$, $x < -3$

In Exercises 7–10, find the points of intersection of the two curves. Round your answers to 2 decimal places.

7. $y = 2x - 3$, $y = 5$
8. $y = -3x + 5$, $y = -3$
9. (a) $y = 2^x$, $y = 3$
   (b) $y = 2^x$, $y = -1$
10. (a) $y = e^{-x}$, $y = 4$
    (b) $y = e^{-x}$, $y = -1$

Section 1.5 Exercises

In Exercises 1–6, determine whether the function is one-to-one.

1.


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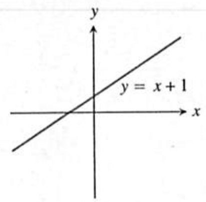

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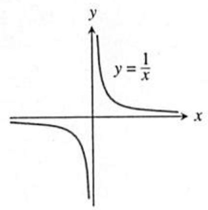

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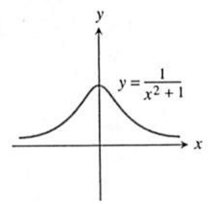

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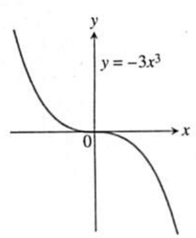
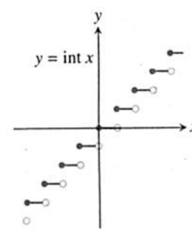


6.


In Exercises 7–12, determine whether the function has an inverse function.

7. $y = \frac{3}{x-2} - 1$ **8.** $y = x^2 + 5x$ **9.** $y = x^3 - 4x + 6$

10. $y = x^3 + x$ **11.** $y = \ln x^2$ **12.** $y = 2^{3-x}$

In Exercises 13–24, find f^{-1} and verify that

$$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x.$$

13. $f(x) = 2x + 3$ **14.** $f(x) = 5 - 4x$

15. $f(x) = x^3 - 1$ **16.** $f(x) = x^2 + 1, \quad x \geq 0$

17. $f(x) = x^2, \quad x \leq 0$ **18.** $f(x) = x^{2/3}, \quad x \geq 0$

19. $f(x) = -(x-2)^2, \quad x \leq 2$

20. $f(x) = x^2 + 2x + 1, \quad x \geq -1$

21. $f(x) = \frac{1}{x^2}, \quad x > 0$ **22.** $f(x) = \frac{1}{x^3}$

23. $f(x) = \frac{2x+1}{x+3}$ **24.** $f(x) = \frac{x+3}{x-2}$

In Exercises 25–32, use parametric graphing to graph f, f^{-1}, and $y = x$.

25. $f(x) = e^x$ **26.** $f(x) = 3^x$ **27.** $f(x) = 2^{-x}$

28. $f(x) = 3^{-x}$ **29.** $f(x) = \ln x$ **30.** $f(x) = \log x$

31. $f(x) = \sin^{-1} x$ **32.** $f(x) = \tan^{-1} x$

In Exercises 33–36, solve the equation algebraically. Support your solution graphically.

33. $(1.045)^t = 2$ **34.** $e^{0.05t} = 3$

35. $e^x + e^{-x} = 3$ **36.** $2^x + 2^{-x} = 5$

In Exercises 37 and 38, solve for y.

37. $\ln y = 2t + 4$ **38.** $\ln(y-1) - \ln 2 = x + \ln x$

In Exercises 39–42, draw the graph and determine the domain and range of the function.

39. $y = 2\ln(3-x) - 4$ **40.** $y = -3\log(x+2) + 1$

41. $y = \log_2(x+1)$ **42.** $y = \log_3(x-4)$

In Exercises 43 and 44, find a formula for f^{-1} and verify that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.

43. $f(x) = \frac{100}{1 + 2^{-x}}$ **44.** $f(x) = \frac{50}{1 + 1.1^{-x}}$

45. ***Self-inverse*** Prove that the function f is its own inverse.

(a) $f(x) = \sqrt{1 - x^2}, \quad x \geq 0$ (b) $f(x) = 1/x$

46. ***Radioactive Decay*** The half-life of a certain radioactive substance is 12 hours. There are 8 grams present initially.

(a) Express the amount of substance remaining as a function of time t.

(b) When will there be 1 gram remaining?

47. ***Doubling Your Money*** Determine how much time is required for a $500 investment to double in value if interest is earned at the rate of 4.75% compounded annually.

48. ***Population Growth*** The population of Glenbrook is 375,000 and is increasing at the rate of 2.25% per year. Predict when the population will be 1 million.

In Exercises 49 and 50, let $x = 0$ represent 1990, $x = 1$ represent 1991, and so forth.

49. ***Natural Gas Production***

(a) Find a natural logarithm regression equation for the data in Table 1.16 and superimpose its graph on a scatter plot of the data.

Table 1.16 **Canada's Natural Gas Production**

Year	Cubic Feet (trillions)
1997	5.76
1998	5.98
1999	6.26
2000	6.47
2001	6.60

Source: Statistical Abstract of the United States, 2004–2005.

(b) Estimate the number of cubic feet of natural gas produced by Canada in 2002. Compare with the actual amount of 6.63 trillion cubic feet in 2002.

(c) Predict when Canadian natural gas production will reach 7 trillion cubic feet.

50. ***Natural Gas Production***

(a) Find a natural logarithm regression equation for the data in Table 1.17 and superimpose its graph on a scatter plot of the data.

Table 1.17 **China's Natural Gas Production**

Year	Cubic Feet (trillions)
1997	0.75
1998	0.78
1999	0.85
2000	0.96
2001	1.07

Source: Statistical Abstract of the United States, 2004–2005.

(b) Estimate the number of cubic feet of natural gas produced by China in 2002. Compare with the actual amount of 1.15 trillion cubic feet in 2002.

(c) Predict when China's natural gas production will reach 1.5 trillion cubic feet.

51. **Group Activity** ***Inverse Functions*** Let $y = f(x) = mx + b$, $m \neq 0$.

(a) **Writing to Learn** Give a convincing argument that f is a one-to-one function.

(b) Find a formula for the inverse of f. How are the slopes of f and f^{-1} related?

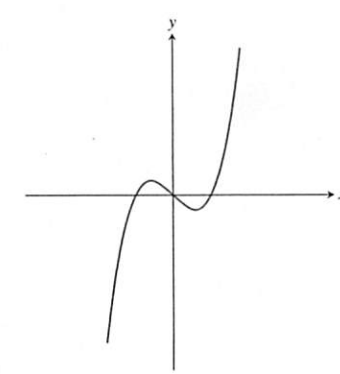
(c) If the graphs of two functions are parallel lines with a nonzero slope, what can you say about the graphs of the inverses of the functions?

(d) If the graphs of two functions are perpendicular lines with a nonzero slope, what can you say about the graphs of the inverses of the functions?

Standardized Test Questions

You should solve the following problems without using a graphing calculator.

52. **True or False** The function displayed in the graph below is one-to-one. Justify your answer.



53. **True or False** If $(f \circ g)(x) = x$, then g is the inverse function of f. Justify your answer.

In Exercises 54 and 55, use the function $f(x) = 3 - \ln(x+2)$.

54. **Multiple Choice** Which of the following is the domain of f?

(A) $x \neq -2$ (B) $(-\infty, \infty)$ (C) $(-2, \infty)$
(D) $[-1.9, \infty)$ (E) $(0, \infty)$

55. **Multiple Choice** Which of the following is the range of f?

(A) $(-\infty, \infty)$ (B) $(-\infty, 0)$ (C) $(-2, \infty)$
(D) $(0, \infty)$ (E) $(0, 5.3)$

56. **Multiple Choice** Which of the following is the inverse of $f(x) = 3x - 2$?

(A) $g(x) = \frac{1}{3x-2}$ (B) $g(x) = x$ (C) $g(x) = 3x - 2$
(D) $g(x) = \frac{x-2}{3}$ (E) $g(x) = \frac{x+2}{3}$

57. **Multiple Choice** Which of the following is a solution of the equation $2 - 3^{-x} = -1$?

(A) $x = -2$ (B) $x = -1$ (C) $x = 0$
(D) $x = 1$ (E) There are no solutions.

Exploration

58. ***Supporting the Quotient Rule*** Let $y_1 = \ln(x/a)$, $y_2 = \ln x$, $y_3 = y_2 - y_1$, and $y_4 = e^{y_3}$.

(a) Graph y_1 and y_2 for $a = 2, 3, 4$, and 5. How are the graphs of y_1 and y_2 related?

(b) Graph y_3 for $a = 2, 3, 4$, and 5. Describe the graphs.

(c) Graph y_4 for $a = 2, 3, 4$, and 5. Compare the graphs to the graph of $y = a$.

(d) Use $e^{y_3} = e^{y_2 - y_1} = a$ to solve for y_1.

Extending the Ideas

59. ***One-to-One Functions*** If f is a one-to-one function, prove that $g(x) = -f(x)$ is also one-to-one.

60. ***One-to-One Functions*** If f is a one-to-one function and $f(x)$ is never zero, prove that $g(x) = 1/f(x)$ is also one-to-one.

61. ***Domain and Range*** Suppose that $a \neq 0$, $b \neq 1$, and $b > 0$. Determine the domain and range of the function.

(a) $y = a(b^{c-x}) + d$ (b) $y = a\log_b(x - c) + d$

62. **Group Activity** ***Inverse Functions***

Let $f(x) = \frac{ax+b}{cx+d}, \quad c \neq 0, \quad ad - bc \neq 0.$

(a) **Writing to Learn** Give a convincing argument that f is one-to-one.

(b) Find a formula for the inverse of f.

(c) Find the horizontal and vertical asymptotes of f.

(d) Find the horizontal and vertical asymptotes of f^{-1}. How are they related to those of f?

Quick Review 1.6 *(For help, go to Sections 1.2 and 1.6.)*

In Exercises 1–4, convert from radians to degrees or degrees to radians.

1. $\pi/3$ **2.** -2.5 **3.** $-40°$ **4.** $45°$

In Exercises 5–7, solve the equation graphically in the given interval.

5. $\sin x = 0.6, \quad 0 \le x \le 2\pi$ **6.** $\cos x = -0.4, \quad 0 \le x \le 2\pi$

7. $\tan x = 1, \quad -\frac{\pi}{2} \le x < \frac{3\pi}{2}$

8. Show that $f(x) = 2x^2 - 3$ is an even function. Explain why its graph is symmetric about the y-axis.

9. Show that $f(x) = x^3 - 3x$ is an odd function. Explain why its graph is symmetric about the origin.

10. Give one way to restrict the domain of the function $f(x) = x^4 - 2$ to make the resulting function one-to-one.

Section 1.6 Exercises

In Exercises 1–4, the angle lies at the center of a circle and subtends an arc of the circle. Find the missing angle measure, circle radius, or arc length.

	Angle	Radius	Arc Length
1.	$5\pi/8$	2	?
2.	175°	?	10
3.	?	14	7
4.	?	6	$3\pi/2$

In Exercises 5–8, determine if the function is even or odd.

5. secant **6.** tangent

7. cosecant **8.** cotangent

In Exercises 9 and 10, find all the trigonometric values of θ with the given conditions.

9. $\cos\theta = -\frac{15}{17}, \quad \sin\theta > 0$

10. $\tan\theta = -1, \quad \sin\theta < 0$

In Exercises 11–14, determine **(a)** the period, **(b)** the domain, **(c)** the range, and **(d)** draw the graph of the function.

11. $y = 3\csc(3x + \pi) - 2$ **12.** $y = 2\sin(4x + \pi) + 3$

13. $y = -3\tan(3x + \pi) + 2$

14. $y = 2\sin\left(2x + \frac{\pi}{3}\right)$

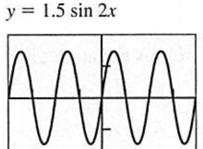
In Exercises 15 and 16, choose an appropriate viewing window to display two complete periods of each trigonometric function in radian mode.

15. **(a)** $y = \sec x$ **(b)** $y = \csc x$ **(c)** $y = \cot x$

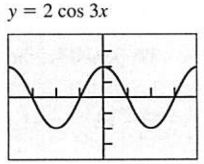
16. **(a)** $y = \sin x$ **(b)** $y = \cos x$ **(c)** $y = \tan x$

In Exercises 17–22, specify **(a)** the period, **(b)** the amplitude, and **(c)** identify the viewing window that is shown.

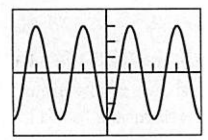
17. $y = 1.5\sin 2x$



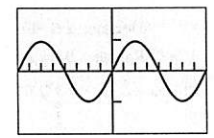
18. $y = 2\cos 3x$



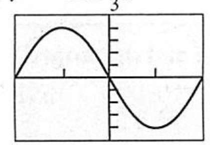
19. $y = -3\cos 2x$



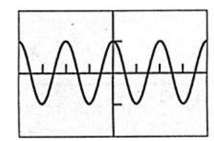
20. $y = 5\sin\frac{x}{2}$



21. $y = -4\sin\frac{\pi}{3}x$



22. $y = \cos\pi x$



23. **Group Activity** A musical note like that produced with a tuning fork or pitch meter is a pressure wave. Table 1.19 gives frequencies (in Hz) of musical notes on the tempered scale. The pressure versus time tuning fork data in Table 1.20 were collected using a CBL™ and a microphone.

Table 1.19 Frequencies of Notes

Note	Frequency (Hz)
C	262
C♯ or D♭	277
D	294
D♯ or E♭	311
E	330
F	349
F♯ or G♭	370
G	392
G♯ or A♭	415
A	440
A♯ or B♭	466
B	494
C (next octave)	524

Source: CBL™ System Experimental Workbook, Texas Instruments, Inc., 1994.

Table 1.20 Tuning Fork Data

Time (s)	Pressure	Time (s)	Pressure
0.0002368	1.29021	0.0049024	−1.06632
0.0005664	1.50851	0.0051520	0.09235
0.0008256	1.51971	0.0054112	1.44694
0.0010752	1.51411	0.0056608	1.51411
0.0013344	1.47493	0.0059200	1.51971
0.0015840	0.45619	0.0061696	1.51411
0.0018432	−0.89280	0.0064288	1.43015
0.0020928	−1.51412	0.0066784	0.19871
0.0023520	−1.15588	0.0069408	−1.06072
0.0026016	−0.04758	0.0071904	−1.51412
0.0028640	1.36858	0.0074496	−0.97116
0.0031136	1.50851	0.0076992	0.23229
0.0033728	1.51971	0.0079584	1.46933
0.0036224	1.51411	0.0082080	1.51411
0.0038816	1.45813	0.0084672	1.51971
0.0041312	0.32185	0.0087168	1.50851
0.0043904	−0.97676	0.0089792	1.36298
0.0046400	−1.51971		

(a) Find a sinusoidal regression equation for the data in Table 1.20 and superimpose its graph on a scatter plot of the data.

(b) Determine the frequency of and identify the musical note produced by the tuning fork.

24. ***Temperature Data*** Table 1.21 gives the average monthly temperatures for St. Louis for a 12-month period starting with January. Model the monthly temperature with an equation of the form

$$y = a\sin[b(t - h)] + k,$$

y in degrees Fahrenheit, t in months, as follows:

Table 1.21 Temperature Data for St. Louis

Time (months)	Temperature (°F)
1	34
2	30
3	39
4	44
5	58
6	67
7	78
8	80
9	72
10	63
11	51
12	40

(a) Find the value of b assuming that the period is 12 months.

(b) How is the amplitude a related to the difference $80° - 30°$?

(c) Use the information in (b) to find k.

(d) Find h, and write an equation for y.

(e) Superimpose a graph of y on a scatter plot of the data.

In Exercises 25–26, show that the function is one-to-one, and graph its inverse.

25. $y = \cos x, \quad 0 \le x \le \pi$ **26.** $y = \tan x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

In Exercises 27–30, give the measure of the angle in radians and degrees. Give exact answers whenever possible.

27. $\sin^{-1}(0.5)$ **28.** $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

29. $\tan^{-1}(-5)$ **30.** $\cos^{-1}(0.7)$

In Exercises 31–36, solve the equation in the specified interval.

31. $\tan x = 2.5, \quad 0 \le x \le 2\pi$

32. $\cos x = -0.7, \quad 2\pi \le x < 4\pi$

33. $\csc x = 2, \quad 0 < x < 2\pi$ **34.** $\sec x = -3, \quad -\pi \le x < \pi$

35. $\sin x = -0.5, \quad -\infty < x < \infty$ **36.** $\cot x = -1, \quad -\infty < x < \infty$

In Exercises 37–40, use the given information to find the values of the six trigonometric functions at the angle θ. Give exact answers.

37. $\theta = \sin^{-1}\left(\frac{8}{17}\right)$ **38.** $\theta = \tan^{-1}\left(-\frac{5}{12}\right)$

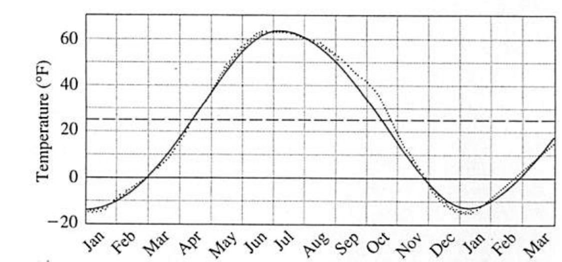
39. The point $P(-3, 4)$ is on the terminal side of θ.

40. The point $P(-2, 2)$ is on the terminal side of θ.

In Exercises 41 and 42, evaluate the expression.

41. $\sin\left(\cos^{-1}\left(\frac{7}{11}\right)\right)$ **42.** $\tan\left(\sin^{-1}\left(\frac{9}{13}\right)\right)$

43. ***Temperatures in Fairbanks, Alaska*** Find the **(a)** amplitude, **(b)** period, **(c)** horizontal shift, and **(d)** vertical shift of the model used in the figure below. **(e)** Then write the equation for the model.



Normal mean air temperature for Fairbanks, Alaska, plotted as data points (red). The approximating sine function $f(x)$ is drawn in blue.
Source: "Is the Curve of Temperature Variation a Sine Curve?" by B. M. Lando and C. A. Lando, *The Mathematics Teacher, 7.6,* Fig. 2, p. 535 (Sept. 1977).

44. ***Temperatures in Fairbanks, Alaska*** Use the equation of Exercise 43 to approximate the answers to the following questions about the temperatures in Fairbanks, Alaska, shown in the figure in Exercise 43. Assume that the year has 365 days.

(a) What are the highest and lowest mean daily temperatures?

(b) What is the average of the highest and lowest mean daily temperatures? Why is this average the vertical shift of the function?

45. ***Even-Odd***
(a) Show that cot x is an odd function of x.
(b) Show that the quotient of an even function and an odd function is an odd function.

46. ***Even-Odd***
(a) Show that csc x is an odd function of x.
(b) Show that the reciprocal of an odd function is odd.

47. ***Even-Odd*** Show that the product of an even function and an odd function is an odd function.

48. ***Finding the Period*** Give a convincing argument that the period of tan x is π.

49. ***Sinusoidal Regression*** Table 1.22 gives the values of the function

$$f(x) = a \sin (bx + c) + d$$

accurate to two decimals.

Table 1.22 Values of a Function

x	$f(x)$
1	3.42
2	0.73
3	0.12
4	2.16
5	4.97
6	5.97

(a) Find a sinusoidal regression equation for the data.
(b) Rewrite the equation with a, b, c, and d rounded to the nearest integer.

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

50. **True or False** The period of $y = \sin(x/2)$ is π. Justify your answer.

51. **True or False** The amplitude of $y = \frac{1}{2}\cos x$ is 1. Justify your answer.

In Exercises 52–54, $f(x) = 2\cos(4x + \pi) - 1$.

52. **Multiple Choice** Which of the following is the domain of f?
(A) $[-\pi, \pi]$ (B) $[-3, 1]$ (C) $[-1, 4]$
(D) $(-\infty, \infty)$ (E) $x \neq 0$

53. **Multiple Choice** Which of the following is the range of f?
(A) $(-3, 1)$ (B) $[-3, 1]$ (C) $(-1, 4)$
(D) $[-1, 4]$ (E) $(-\infty, \infty)$

54. **Multiple Choice** Which of the following is the period of f?
(A) 4π (B) 3π (C) 2π (D) π (E) $\pi/2$

55. **Multiple Choice** Which of the following is the measure of $\tan^{-1}(-\sqrt{3})$ in degrees?
(A) $-60°$ (B) $-30°$ (C) $30°$ (D) $60°$ (E) $120°$

Exploration

56. ***Trigonometric Identities*** Let $f(x) = \sin x + \cos x$.
(a) Graph $y = f(x)$. Describe the graph.
(b) Use the graph to identify the amplitude, period, horizontal shift, and vertical shift.
(c) Use the formula

$$\sin \alpha \cos \beta + \cos \alpha \sin \beta = \sin (\alpha + \beta)$$

for the sine of the sum of two angles to confirm your answers.

Extending the Ideas

57. **Exploration** Let $y = \sin(ax) + \cos(ax)$.
Use the symbolic manipulator of a computer algebra system (CAS) to help you with the following:
(a) Express y as a sinusoid for $a = 2, 3, 4$, and 5.
(b) Conjecture another formula for y for a equal to any positive integer n.
(c) Check your conjecture with a CAS.
(d) Use the formula for the sine of the sum of two angles (see Exercise 56c) to confirm your conjecture.

58. **Exploration** Let $y = a \sin x + b \cos x$.
Use the symbolic manipulator of a computer algebra system (CAS) to help you with the following:
(a) Express y as a sinusoid for the following pairs of values: $a = 2, b = 1$; $a = 1, b = 2$; $a = 5, b = 2$; $a = 2, b = 5$; $a = 3, b = 4$.
(b) Conjecture another formula for y for any pair of positive integers. Try other values if necessary.
(c) Check your conjecture with a CAS.
(d) Use the following formulas for the sine or cosine of a sum or difference of two angles to confirm your conjecture.

$$\sin \alpha \cos \beta \pm \cos \alpha \sin \beta = \sin (\alpha \pm \beta)$$

$$\cos \alpha \cos \beta \pm \sin \alpha \sin \beta = \cos (\alpha \mp \beta)$$

In Exercises 59 and 60, show that the function is periodic and find its period.

59. $y = \sin^3 x$

60. $y = |\tan x|$

In Exercises 61 and 62, graph one period of the function.

61. $f(x) = \sin (60x)$

62. $f(x) = \cos (60\pi x)$

Quick Quiz for AP* Preparation: Sections 1.4–1.6

You should solve the following problems without using a graphing calculator.

1. **Multiple Choice** Which of the following is the domain of $f(x) = -\log_2(x + 3)$?
(A) $(-\infty, \infty)$ (B) $(-\infty, 3)$ (C) $(-3, \infty)$
(D) $[-3, \infty)$ (E) $(-\infty, 3]$

2. **Multiple Choice** Which of the following is the range of $f(x) = 5\cos(x + \pi) + 3$?
(A) $(-\infty, \infty)$ (B) $[2, 4]$ (C) $[-8, 2]$
(D) $[-2, 8]$ (E) $\left[-\frac{2}{5}, \frac{8}{5}\right]$

3. **Multiple Choice** Which of the following gives the solution of $\tan x = -1$ in $\pi < x < \frac{3\pi}{2}$?
(A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{3\pi}{4}$ (E) $\frac{5\pi}{4}$

4. **Free Response** Let $f(x) = 5x - 3$.
(a) Find the inverse g of f.
(b) Compute $f \circ g(x)$. Show your work.
(c) Compute $g \circ f(x)$. Show your work.

Chapter 1 Key Terms

absolute value function (p. 17)
base a logarithm function (p. 40)
boundary of an interval (p. 13)
boundary points (p. 13)
change of base formula (p. 42)
closed interval (p. 13)
common logarithm function (p. 41)
composing (p. 18)
composite function (p. 17)
compounded continuously (p. 25)
cosecant function (p. 46)
cosine function (p. 46)
cotangent function (p. 46)
dependent variable (p. 12)
domain (p. 12)
even function (p. 15)
exponential decay (p. 24)
exponential function base a (p. 22)
exponential growth (p. 24)
function (p. 12)
general linear equation (p. 5)
graph of a function (p. 13)
graph of a relation (p. 30)
grapher failure (p. 15)
half-life (p. 24)
half-open interval (p. 13)
identity function (p. 38)
increments (p. 3)
independent variable (p. 12)
initial point of parametrized curve (p. 30)
interior of an interval (p. 13)
interior points of an interval (p. 13)
inverse cosecant function (p. 50)
inverse cosine function (p. 50)
inverse cotangent function (p. 50)
inverse function (p. 38)
inverse properties for a^x and $\log_a x$ (p. 41)
inverse secant function (p. 50)
inverse sine function (p. 50)
inverse tangent function (p. 50)
linear regression (p. 7)
natural domain (p. 13)
natural logarithm function (p. 41)
odd function (p. 15)
one-to-one function (p. 37)
open interval (p. 13)
parallel lines (p. 4)
parameter (p. 30)
parameter interval (p. 30)
parametric curve (p. 30)
parametric equations (p. 30)
parametrization of a curve (p. 30)
parametrize (p. 30)
period of a function (p. 47)
periodic function (p. 47)
perpendicular lines (p. 4)
piecewise defined function (p. 16)
point-slope equation (p. 4)
power rule for logarithms (p. 41)
product rule for logarithms (p. 41)
quotient rule for logarithms (p. 41)
radian measure (p. 46)
range (p. 12)
regression analysis (p. 7)
regression curve (p. 7)
relation (p. 30)
rise (p. 3)
rules for exponents (p. 23)
run (p. 3)
scatter plot (p. 7)
secant function (p. 46)
sine function (p. 46)
sinusoid (p. 48)
sinusoidal regression (p. 49)
slope (p. 4)
slope-intercept equation (p. 5)
symmetry about the origin (p. 15)
symmetry about the y-axis (p. 15)
tangent function (p. 46)
terminal point of parametrized curve (p. 30)
witch of Agnesi (p. 33)
x-intercept (p. 5)
y-intercept (p. 5)

Chapter 1 Review Exercises

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–14, write an equation for the specified line.

1. through $(1, -6)$ with slope 3
2. through $(-1, 2)$ with slope $-1/2$
3. the vertical line through $(0, -3)$
4. through $(-3, 6)$ and $(1, -2)$
5. the horizontal line through $(0, 2)$
6. through $(3, 3)$ and $(-2, 5)$
7. with slope -3 and y-intercept 3
8. through $(3, 1)$ and parallel to $2x - y = -2$
9. through $(4, -12)$ and parallel to $4x + 3y = 12$
10. through $(-2, -3)$ and perpendicular to $3x - 5y = 1$
11. through $(-1, 2)$ and perpendicular to $\frac{1}{2}x + \frac{1}{3}y = 1$
12. with x-intercept 3 and y-intercept -5
13. the line $y = f(x)$, where f has the following values:

x	-2	2	4
$f(x)$	4	2	1

14. through $(4, -2)$ with x-intercept -3

In Exercises 15–18, determine whether the graph of the function is symmetric about the y-axis, the origin, or neither.

15. $y = x^{1/5}$
16. $y = x^{2/5}$
17. $y = x^2 - 2x - 1$
18. $y = e^{-x^2}$

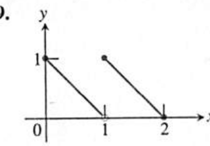
In Exercises 19–26, determine whether the function is even, odd, or neither.

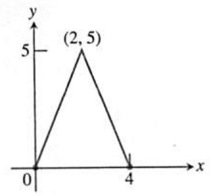
19. $y = x^2 + 1$
20. $y = x^5 - x^3 - x$
21. $y = 1 - \cos x$
22. $y = \sec x \tan x$
23. $y = \dfrac{x^4 + 1}{x^3 - 2x}$
24. $y = 1 - \sin x$
25. $y = x + \cos x$
26. $y = \sqrt{x^4 - 1}$

In Exercises 27–38, find the (a) domain and (b) range, and (c) graph the function.

27. $y = |x| - 2$
28. $y = -2 + \sqrt{1 - x}$
29. $y = \sqrt{16 - x^2}$
30. $y = 3^{2-x} + 1$
31. $y = 2e^{-x} - 3$
32. $y = \tan(2x - \pi)$
33. $y = 2\sin(3x + \pi) - 1$
34. $y = x^{2/5}$
35. $y = \ln(x - 3) + 1$
36. $y = -1 + \sqrt[3]{2 - x}$
37. $y = \begin{cases} \sqrt{-x}, & -4 \le x \le 0 \\ \sqrt{x}, & 0 < x \le 4 \end{cases}$
38. $y = \begin{cases} -x - 2, & -2 \le x \le -1 \\ x, & -1 < x \le 1 \\ -x + 2, & 1 < x \le 2 \end{cases}$

In Exercises 39 and 40, write a piecewise formula for the function.

39. 

40. 

In Exercises 41 and 42, find

(a) $(f \circ g)(-1)$ (b) $(g \circ f)(2)$ (c) $(f \circ f)(x)$ (d) $(g \circ g)(x)$

41. $f(x) = \dfrac{1}{x}$, $g(x) = \dfrac{1}{\sqrt{x + 2}}$
42. $f(x) = 2 - x$, $g(x) = \sqrt[3]{x + 1}$

In Exercises 43 and 44, (a) write a formula for $f \circ g$ and $g \circ f$ and find the (b) domain and (c) range of each.

43. $f(x) = 2 - x^2$, $g(x) = \sqrt{x + 2}$
44. $f(x) = \sqrt{x}$, $g(x) = \sqrt{1 - x}$

In Exercises 45–48, a parametrization is given for a curve.

(a) Graph the curve. Identify the initial and terminal points, if any. Indicate the direction in which the curve is traced.

(b) Find a Cartesian equation for a curve that contains the parametrized curve. What portion of the graph of the Cartesian equation is traced by the parametrized curve?

45. $x = 5\cos t$, $y = 2\sin t$, $0 \le t \le 2\pi$
46. $x = 4\cos t$, $y = 4\sin t$, $\pi/2 \le t < 3\pi/2$
47. $x = 2 - t$, $y = 11 - 2t$, $-2 \le t \le 4$
48. $x = 1 + t$, $y = \sqrt{4 - 2t}$, $t \le 2$

In Exercises 49–52, give a parametrization for the curve.

49. the line segment with endpoints $(-2, 5)$ and $(4, 3)$
50. the line through $(-3, -2)$ and $(4, -1)$
51. the ray with initial point $(2, 5)$ that passes through $(-1, 0)$
52. $y = x(x - 4)$, $x \le 2$

Group Activity In Exercises 53 and 54, do the following.

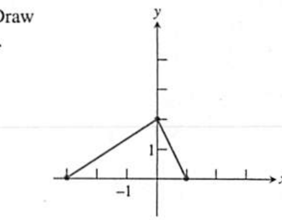
(a) Find f^{-1} and show that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.
(b) Graph f and f^{-1} in the same viewing window.

53. $f(x) = 2 - 3x$
54. $f(x) = (x + 2)^2$, $x \ge -2$

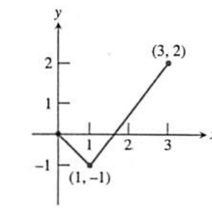
In Exercises 55 and 56, find the measure of the angle in radians and degrees.

55. $\sin^{-1}(0.6)$
56. $\tan^{-1}(-2.3)$
57. Find the six trigonometric values of $\theta = \cos^{-1}(3/7)$. Give exact answers.
58. Solve the equation $\sin x = -0.2$ in the following intervals.
(a) $0 \le x < 2\pi$ (b) $-\infty < x < \infty$
59. Solve for x: $e^{-0.2x} = 4$

60. The graph of f is shown. Draw the graph of each function.
(a) $y = f(-x)$
(b) $y = -f(x)$
(c) $y = -2f(x + 1) + 1$
(d) $y = 3f(x - 2) - 2$



61. A portion of the graph of a function defined on $[-3, 3]$ is shown. Complete the graph assuming that the function is
(a) even.
(b) odd.



62. ***Depreciation*** Smith Hauling purchased an 18-wheel truck for \$100,000. The truck depreciates at the constant rate of \$10,000 per year for 10 years.
(a) Write an expression that gives the value y after x years.
(b) When is the value of the truck \$55,000?

63. ***Drug Absorption*** A drug is administered intravenously for pain. The function

$$f(t) = 90 - 52\ln(1 + t), \quad 0 \le t \le 4$$

gives the number of units of the drug in the body after t hours.
(a) What was the initial number of units of the drug administered?
(b) How much is present after 2 hours? (c) Draw the graph of f.

64. ***Finding Time*** If Joenita invests \$1500 in a retirement account that earns 8% compounded annually, how long will it take this single payment to grow to \$5000?

65. ***Guppy Population*** The number of guppies in Susan's aquarium doubles every day. There are four guppies initially.
(a) Write the number of guppies as a function of time t.
(b) How many guppies were present after 4 days? after 1 week?
(c) When will there be 2000 guppies?
(d) **Writing to Learn** Give reasons why this might not be a good model for the growth of Susan's guppy population.

66. ***Doctoral Degrees*** Table 1.23 shows the number of doctoral degrees earned by Hispanic students for several years. Let $x = 0$ represent 1980, $x = 1$ represent 1981, and so forth.

Table 1.23 Doctorates Earned by Hispanic Americans

Year	Number of Degrees
1981	456
1985	677
1990	780
1995	984
2000	1305

Source: Statistical Abstract of the United States, 2004–2005.

(a) Find a linear regression equation for the data and superimpose its graph on a scatter plot of the data.
(b) Use the regression equation to predict the number of doctoral degrees that will be earned by Hispanic Americans in 2002. How close is the estimate to the actual number in 2002 of 1432?
(c) **Writing to Learn** Find the slope of the regression line. What does the slope represent?

67. ***Population of New York*** Table 1.24 shows the population of New York State for several years. Let $x = 0$ represent 1980, $x = 1$ represent 1981, and so forth.

Table 1.24 Population of New York State

Year	Population (thousands)
1980	17,558
1990	17,991
1995	18,524
1998	18,756
1999	18,883
2000	18,977

Source: Statistical Abstract of the United States, 2004–2005.

(a) Find the exponential regression equation for the data and superimpose its graph on a scatter plot of the data.
(b) Use the regression equation to predict the population in 2003. How close is the estimate to the actual number in 2003 of 19,190 thousand?
(c) Use the exponential regression equation to estimate the annual rate of growth of the population of New York State.

AP* Examination Preparation

You may use a graphing calculator to solve the following problems.

68. Consider the point $P(-2, 1)$ and the line L: $x + y = 2$.
(a) Find the slope of L.
(b) Write an equation for the line through P and parallel to L.
(c) Write an equation for the line through P and perpendicular to L.
(d) What is the x-intercept of L?

69. Let $f(x) = 1 - \ln(x - 2)$.
(a) What is the domain of f? (b) What is the range of f?
(c) What are the x-intercepts of the graph of f?
(d) Find f^{-1}. (e) Confirm your answer algebraically in part (d).

70. Let $f(x) = 1 - 3\cos(2x)$.
(a) What is the domain of f? (b) What is the range of f?
(c) What is the period of f?
(d) Is f an even function, odd function, or neither?
(e) Find all the zeros of f in $\pi/2 \le x \le \pi$.

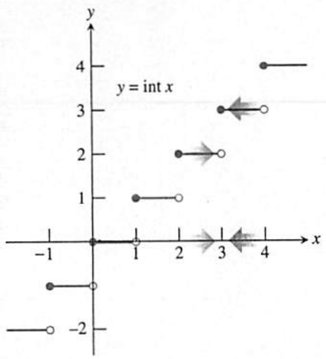


Figure 2.5 At each integer, the greatest integer function $y = \text{int } x$ has different right-hand and left-hand limits. (Example 7)

right the **right-hand limit** of f at c and the limit as x approaches c from the left the **left-hand limit** of f at c. Here is the notation we use:

right-hand: $\lim_{x \to c^+} f(x)$ *The limit of f as x approaches c from the right.*

left-hand: $\lim_{x \to c^-} f(x)$ *The limit of f as x approaches c from the left.*

EXAMPLE 7 Function Values Approach Two Numbers

The greatest integer function $f(x) = \text{int } x$ has different right-hand and left-hand limits at each integer, as we can see in Figure 2.5. For example,

$$\lim_{x \to 3^+} \text{int } x = 3 \quad \text{and} \quad \lim_{x \to 3^-} \text{int } x = 2.$$

The limit of int x as x approaches an integer n from the right is n, while the limit as x approaches n from the left is $n - 1$.

Now try Exercises 31 and 32.

We sometimes call $\lim_{x \to c} f(x)$ the **two-sided limit** of f at c to distinguish it from the *one-sided* right-hand and left-hand limits of f at c. Theorem 3 shows how these limits are related.

On the Far Side

If f is not defined to the left of $x = c$, then f does not have a left-hand limit at c. Similarly, if f is not defined to the right of $x = c$, then f does not have a right-hand limit at c.

THEOREM 3 One-sided and Two-sided Limits

A function $f(x)$ has a limit as x approaches c if and only if the right-hand and left-hand limits at c exist and are equal. In symbols,

$$\lim_{x \to c} f(x) = L \Leftrightarrow \lim_{x \to c^+} f(x) = L \quad \text{and} \quad \lim_{x \to c^-} f(x) = L.$$

Thus, the greatest integer function $f(x) = \text{int } x$ of Example 7 does not have a limit as $x \to 3$ even though each one-sided limit exists.

EXAMPLE 8 Exploring Right- and Left-Hand Limits

All the following statements about the function $y = f(x)$ graphed in Figure 2.6 are true.

At $x = 0$: $\lim_{x \to 0^+} f(x) = 1$.

At $x = 1$: $\lim_{x \to 1^-} f(x) = 0$ even though $f(1) = 1$,

$\lim_{x \to 1^+} f(x) = 1$,

f has no limit as $x \to 1$. (The right- and left-hand limits at 1 are not equal, so $\lim_{x \to 1} f(x)$ does not exist.)

At $x = 2$: $\lim_{x \to 2^-} f(x) = 1$,

$\lim_{x \to 2^+} f(x) = 1$,

$\lim_{x \to 2} f(x) = 1$ even though $f(2) = 2$.

At $x = 3$: $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 2 = f(3) = \lim_{x \to 3} f(x)$.

At $x = 4$: $\lim_{x \to 4^-} f(x) = 1$.

At noninteger values of c between 0 and 4, f has a limit as $x \to c$.

Now try Exercise 37.

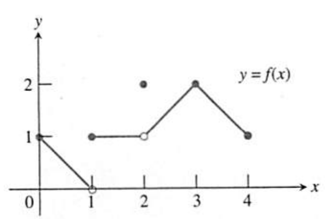


Figure 2.6 The graph of the function

$$f(x) = \begin{cases} -x + 1, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \\ 2, & x = 2 \\ x - 1, & 2 < x \le 3 \\ -x + 5, & 3 \le x \le 4. \end{cases}$$

(Example 8)

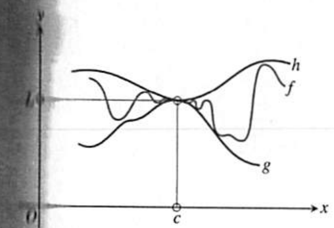


Figure 2.7 Sandwiching f between g and h forces the limiting value of f to be between the limiting values of g and h.

Sandwich Theorem

If we cannot find a limit directly, we may be able to find it indirectly with the Sandwich Theorem. The theorem refers to a function f whose values are sandwiched between the values of two other functions, g and h. If g and h have the same limit as $x \to c$, then f has that limit too, as suggested by Figure 2.7.

THEOREM 4 The Sandwich Theorem

If $g(x) \le f(x) \le h(x)$ for all $x \ne c$ in some interval about c, and

$$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L,$$

then

$$\lim_{x \to c} f(x) = L.$$

EXAMPLE 9 Using the Sandwich Theorem

Show that $\lim_{x \to 0} [x^2 \sin(1/x)] = 0$.

SOLUTION

We know that the values of the sine function lie between −1 and 1. So, it follows that

$$\left|x^2 \sin \frac{1}{x}\right| = |x^2| \cdot \left|\sin \frac{1}{x}\right| \le |x^2| \cdot 1 = x^2$$

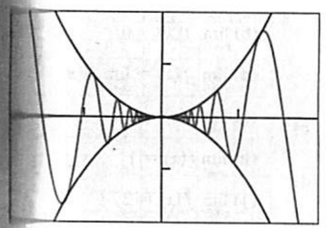
and

$$-x^2 \le x^2 \sin \frac{1}{x} \le x^2.$$

Because $\lim_{x \to 0} (-x^2) = \lim_{x \to 0} x^2 = 0$, the Sandwich Theorem gives

$$\lim_{x \to 0} \left(x^2 \sin \frac{1}{x}\right) = 0.$$

The graphs in Figure 2.8 support this result.



[−0.2, 0.2] by [−0.02, 0.02]

Figure 2.8 The graphs of $y_1 = x^2$, $y_2 = x^2 \sin(1/x)$, and $y_3 = -x^2$. Notice that $y_3 \le y_2 \le y_1$. (Example 9)

Quick Review 2.1 *(For help, go to Section 1.2.)*

In Exercises 1–4, find $f(2)$.

1. $f(x) = 2x^3 - 5x^2 + 4$
2. $f(x) = \dfrac{4x^2 - 5}{x^3 + 4}$
3. $f(x) = \sin\left(\pi \dfrac{x}{2}\right)$
4. $f(x) = \begin{cases} 3x - 1, & x < 2 \\ \dfrac{1}{x^2 - 1}, & x \ge 2 \end{cases}$

In Exercises 5–8, write the inequality in the form $a < x < b$.

5. $|x| < 4$
6. $|x| < c^2$
7. $|x - 2| < 3$
8. $|x - c| < d^2$

In Exercises 9 and 10, write the fraction in reduced form.

9. $\dfrac{x^2 - 3x - 18}{x + 3}$
10. $\dfrac{2x^2 - x}{2x^2 + x - 1}$