Chapter 3 Practice TEST

NO CALCULATOR

1. Find the derivative of $y = 2e^x - 3x^e + \sqrt{e}$

$$\frac{dy}{dx} = 2e^x - 3ex^{e-1}$$

2. Find the derivative of $y = \frac{2x}{1-3x^2}$

$$y' = \frac{2(1-3x^2) + 6x \times 2x}{(1-3x^2)^2}$$

$$y' = \frac{6x^2 + 2}{(1-3x^2)^2}$$

3. (multiple choice) If $y = (3x^2+5)^5(x+2)^4$, then $\frac{dy}{dx} =$ $5(3x^2+5)^4 \cdot 6x(x+2)^4 + 4(x+2)^3(3x^2+5)^5$

$= 2(3x^2+5)^4(x+2)^3\left[15x(x+2) + 2(3x^2+5)\right]$

$= 2(3x^2+5)^4(x+3)^3(21x^2+30x+10)$

A) $2(x+2)^3(3x^2+5)^4$

B) $2(21x^2+30x+10)(x+2)^3(3x^2+5)^4$

C) $(x+2)^3(3x^2+5)(21x^2+30x+10)$

D) $24(x+2)^3(3x^2+5)^4(21x^2+30x+10)$

E) $12(x+2)^3(3x^2+5)^4(21x+30)$

4. Determine $\frac{d}{dx}2^{cosx}$ $= 2^{\cos x} \cdot \ln 2 \cdot (-\sin x)$

$= -\ln 2 \cdot \sin x \cdot 2^{\cos x}$

5. If $y = \tan^{-1}(e^{2x})$, then $\frac{dy}{dx} =$ $\frac{1}{1+e^{4x}} \cdot 2e^{2x}$

$$= \frac{2e^{2x}}{1+e^{4x}}$$

6. If $y = (sinx)^x$, then $\frac{dy}{dx} =$

$\ln y = x \ln(\sin x)$

$\frac{y'}{y} = \ln(\sin x) + x \cdot \frac{1}{\sin x} \cdot \cos x$

$y' = (\ln(\sin x) + x \cdot \cot x)\, y$

$\boxed{y' = (\ln(\sin x) + x\cot x)(\sin x)^x}$

7. (multiple choice) If $f(x) = \frac{x}{tanx}$, then $f'\left(\frac{\pi}{4}\right)=$

$f'(x) = \frac{\overset{1}{\tan x} - x\overset{2}{\sec^2 x}}{\underset{1}{\tan^2 x}}$

A) 2 B) $\frac{1}{2}$ C) $1 + \frac{\pi}{2}$ D) $\frac{\pi}{2} - 1$ E) $1 - \frac{\pi}{2}$

8. (multiple choice) If $y^2 - 2xy = 16$, then $\frac{dy}{dx} =$

$2yy' - 2y - 2xy' = 0 \quad 2y'(y-x) = 2y$

A) $\frac{x}{y-x}$ B) $\frac{y}{x-y}$ C) $\frac{y}{y-x}$ D) $\frac{y}{2y-x}$ E) $\frac{2y}{x-y}$

9. Determine an equation in standard form of the line normal to the graph of $y = x^3 + 3x^2 + 7x - 1$ at the point where $x = -1$.

$y' = 3x^2 + 6x + 7$

$y'|_{x=-1} = 3 - 6 + 7 = 4$

slope of the normal line : $-\frac{1}{4}$

Point : $(-1, -6)$

$\Rightarrow \quad y + 6 = -\frac{1}{4}(x+1)$

$4y + 24 = -x - 1$

$\boxed{x + 4y = -25}$

10. Evaluate the following limits:

a) $\lim_{h\to 0} \frac{\ln(3+h) - ln3}{h}$ let $f(x) = \ln x$

$= \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = f'(3) = \boxed{\frac{1}{3}}$

b) $\lim_{x\to 0} \frac{\cos\left(x+\frac{\pi}{6}\right) - \frac{\sqrt{3}}{2}}{x}$ let $f(x) = \cos x$

$= \lim_{x\to 0} \frac{f(\pi/6 + x) - f(\pi/6)}{x} = f'\left(\frac{\pi}{6}\right) = \boxed{-\frac{1}{2}}$

c) $\lim_{h\to 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h}$ let $f(x) = \sqrt[3]{x}$

$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$

$= \boxed{\frac{1}{3} x^{-2/3}}$

d) $\lim_{x\to 0} \frac{e^{x+3} - e^3}{x}$ let $f(x) = e^x$

$= \lim_{x\to 0} \frac{f(x+3) - f(3)}{x} = f'(3) = \boxed{e^3}$

11. If $\frac{dy}{dx} = \sqrt{1-y^2}$, then find an expression for $\frac{d^2y}{dx^2}$ in terms of y.

$$\frac{d^2y}{dx^2} = \frac{1}{2\sqrt{1-y^2}} \cdot (-2yy')$$

$$= \frac{1}{2\sqrt{1-y^2}}\left(-2y \cdot \sqrt{1-y^2}\right)$$

$$= \boxed{-y}$$

12. Given the following table of values, find the value of $\frac{d}{dx}([f(x)]^2 - 3g(x^2))$ when $x = 1$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	3	-2	-1

$$\frac{d}{dx}\left([f(x)]^2 - 3g(x^2)\right) = 2f(x)\cdot f'(x) - 3g'(x^2)\cdot 2x$$

$$\frac{d}{dx}\left([f(x)]^2 - 3g(x^2)\right)\Bigg|_{x=1} = 2\underbrace{f(1)}_{1}\cdot \underbrace{f'(1)}_{3} - 3\underbrace{g'(1^2)}_{-1}\cdot 2(1) = \boxed{12}$$

13. Find all values of x for which the tangent line to $y = (x+2)^2$ passes through the origin.

Let (x, y) the point of tangency.

slope: $\underbrace{m = 2(x+2)}_{\text{derivative at } x.}$ but also: $m = \frac{(x+2)^2 - 0}{x - 0}$

pt of tangency

origin

cf grade 10

$$2(x+2) = \frac{(x+2)^2}{x}$$

$$2x(x+2) = (x+2)^2$$

$$2x^2 + 4x = x^2 + 4x + 4$$

$$x^2 = 4$$

$$\boxed{x = \pm 2}$$

14. Let f and g be differentiable functions, and let g be the inverse function of f. If $g(-2) = 5$ and $f'(5) = -\frac{1}{2}$, then determine the value of $g'(-2)$.

$g = f^{-1}$

$$g'(-2) = \frac{1}{f'(f^{-1}(-2))}$$

$$\boxed{g'(-2) = -2}$$

$$= \frac{1}{f'(5)}$$

$$= \frac{1}{-\frac{1}{2}}$$

15. A particle has position function $s(t) = t^3 - 7t^2 + 8t + 5$, for $t \geq 0$.
a) Where is the particle at the start?
b) When is the particle speeding up? Slowing down?
c) Determine the total distance travelled in the first 5 seconds.

a) $s(0) = 5$

b) It is speeding up (slowing down) when velocity and accelerat° have the same sign (opposite signs)

$v(t) = 3t^2 - 14t + 8 = (3t-2)(t-4)$

$a(t) = 6t - 14$

t	0		$2/3$		$7/3$		4		$+\infty$
$v(t)$		+	0	−		−	0	+	
$a(t)$		−		−	0	+		+	

speeding up when $t \in (2/3, 7/3)$ and when $t \in (4, +\infty)$
slowing down when $t \in (0, 2/3)$ and $t \in (7/3, 4)$

c)

t	0		$2/3$		4		5
$v(t)$		+	0	−	0	+	
$s(t)$	5	↗	$\frac{203}{27}$	↘	-11	↗	-5

$$D = \left|\frac{203}{27} - 5\right| + \left|\frac{203}{27} + 11\right| + \left|-5 + 11\right|$$

$$= \frac{730}{27}$$

16. Find the equation of the tangent line to $y = \frac{3}{x^2-1}$ at the point where $x = 2$.

$$y' = \frac{-6x}{(x^2-1)^2}$$

$$y'\big|_{x=2} = \frac{-12}{9} = -\frac{4}{3}$$

Point: $(2, 1)$

$$\Bigg\}\quad y - 1 = -\frac{4}{3}(x-2)$$