Name: Solutions

$\frac{}{32}$

AP Calc

Chapter 4 TEST

Show your work on multiple choices for partial marks.

1. If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$

(A) 3 (B) 1 (C) −1 (D) −3 (E) −5

$f'(x) = -3x^2 + 1 - \frac{1}{x^2}$

$f'(-1) = -3 + 1 - 1 = -3$

2. The graph of $y = x^4 - 6x^3 - 24x^2 + 37$ is concave down for

(A) $x < 0$ (B) $x > 0$ (C) $x < -1$ or $x > 4$ (D) $x < -4$ or $x > 1$ (E) $-1 < x < 4$

$y' = 4x^3 - 18x^2 - 48x$

$y'' = 12x^2 - 36x - 48$

$= 12(x^2 - 3x - 4)$

$\Delta = 9 - 4(1)(-4)$

$= 25$

$x = \frac{+3 \pm 5}{2}$ → -1, 4

x	$-\infty$		-1		4		$+\infty$
y''		$+$	0	$-$	0	$+$	

2

3. $\dfrac{d}{dx}\cos^2(x^3) =$

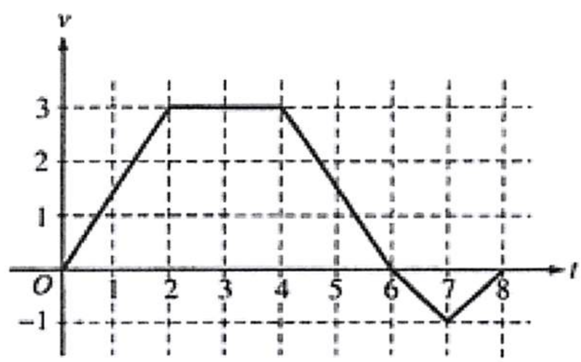
(A) $6x^2\sin(x^3)\cos(x^3)$ (B) $6x^2\cos(x^3)$ (C) $\sin^2(x^3)$

(D) $-6x^2\sin(x^3)\cos(x^3)$ (E) $-2\sin(x^3)\cos(x^3)$

$$\frac{d}{dx}\cos^2(x^3) = -2\cos(x^3)\cdot\sin(x^3)\cdot 3x^2$$

$$= -6x^2\cos x^3 \cdot \sin x^3$$

4 A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.At what value of t does the bug change direction?



(A) 2 (B) 4 (C) 6
(D) 7 (E) 8

2

5. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \dfrac{\pi}{4}$ is

(A) $y - 1 = -\left(x - \dfrac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \dfrac{\pi}{4}\right)$

(C) $y = 2\left(x - \dfrac{\pi}{4}\right)$

(D) $y = -\left(x - \dfrac{\pi}{4}\right)$

(E) $y = -2\left(x - \dfrac{\pi}{4}\right)$

$y' = -2\sin(2x)$

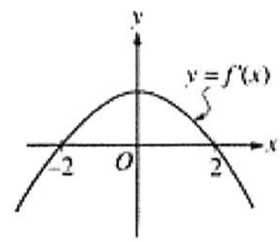
point: $\left(\frac{\pi}{4} ; 0\right)$

$y'\big|_{x=\frac{\pi}{4}} = -2\sin\frac{\pi}{2}$

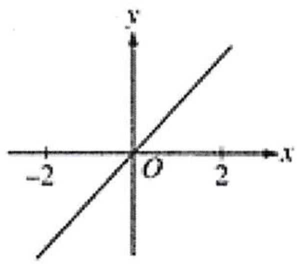
$= -2$

$\Rightarrow y - 0 = -2\left(x - \frac{\pi}{4}\right)$

7. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?



(A)



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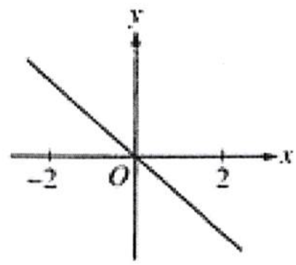
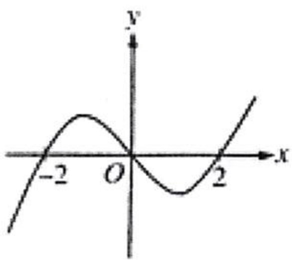
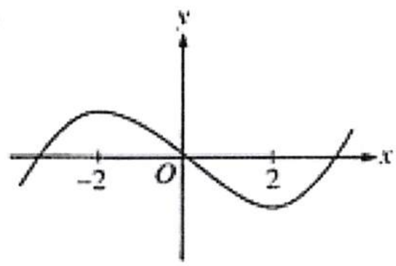


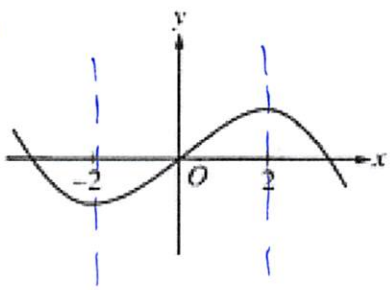
(C)



(D)



(E)



x	$-\infty$		-2		2		$+\infty$
y'		$-$	0	$+$	0	$-$	
y		$\searrow$		$\nearrow$		$\searrow$	

6. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$? (L)

(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2,2)$

(L): $y = \frac{1}{2}x - \frac{3}{4} \Rightarrow$ slope : $\frac{1}{2}$

$y = \frac{1}{2}x^2 \Rightarrow y' = x$

$y' = \frac{1}{2} \Leftrightarrow x = \frac{1}{2}$

point : $\left(\frac{1}{2} ; \frac{1}{8}\right)$

2

8. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

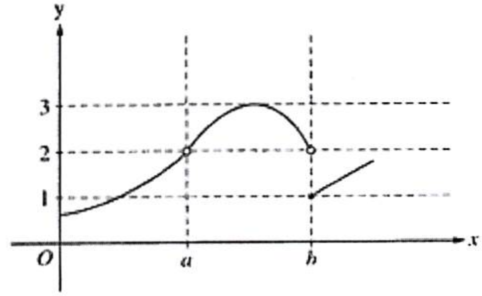
(A) $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$ DNE ×

(B) $\lim_{x\to a} f(x) = 2$ ✓

(C) $\lim_{x\to b} f(x) = 2$ × DNE

(D) $\lim_{x\to b} f(x) = 1$ × DNE

(E) $\lim_{x\to a} f(x)$ does not exist. ×



9. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{y} \longrightarrow$ when $x=4$ & $y=3$ $\frac{dy}{dx} = -\frac{4}{3}$

$\frac{d^2y}{dx^2} = \frac{-y + x\frac{dy}{dx}}{y^2} \longrightarrow \left.\frac{d^2y}{dx^2}\right|_{\substack{x=4\\y=3}} = \frac{-3+4\times\left(-\frac{4}{3}\right)}{9}$

$= -\frac{25}{27}$

2

10. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of x.

(B) $x < -1$ and $x > 3$

(C) $-3 < x < 1$

(D) $-1 < x < 3$

(E) All values of x

$$f'(x) = 2x \cdot e^{-x} - (x^2-3)e^{-x}$$

$$= \underbrace{e^{-x}}_{>0}\underbrace{(-x^2 + 2x + 3)}$$

$\Delta = 4 - 4(-1)(3)$

$= 16$

$x = -\frac{2 \pm 4}{-2} \nearrow 3, \searrow -1$

x	$-\infty$		-1		3		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	
$f(x)$		$\searrow$		$\nearrow$		$\searrow$	

1

11. Let $f(x) = x^3 - x^2 - 2x$. Does the Mean Value Theorem apply on the interval [-1, 1]? If no, explain why. If yes, apply it and give the value of c.

f is continuous on $[-1, 1]$ }
f is differentiable on $(-1, 1)$ } yes it applies.

There exists a value of $c \in (-1, 1)$ such that $f'(c) = \underbrace{\frac{f(1) - f(-1)}{1 - (-1)}}$ $= \frac{-2-0}{2} = -1$

$f'(x) = 3x^2 - 2x - 2$

$f'(x) = -1 \Leftrightarrow 3x^2 - 2x - 1 = 0$

$\Delta = 4 - 4(3)(-1) = 16$

$x = \frac{2 \pm 4}{6} \nearrow -1/3, \searrow 1 \notin (-1,1)$

$\Rightarrow \boxed{c = -\frac{1}{3}}$

3

12. a) Find the local linear approximation for $f(x) = x^3$ at 1.

$f'(x) = 3x^2$

$f'(1) = 3$

point $(1, 1)$

$L(x) = 3(x-1) + 1$

$\boxed{L(x) = 3x - 2}$

1

b) Use the linear approximation to calculate 1.1^3 to the nearest tenth.

$L(1.1) = 3(1.1) - 2$

$= 3.3 - 2$

$= 1.3$

$\boxed{1.1^3 \simeq 1.3}$

1

Section II (calculator permitted)

13. If $f(x) = \dfrac{e^{2x}}{2x}$, then $f'(x) =$

(A) 1 (B) $\dfrac{e^{2x}(1-2x)}{2x^2}$ (C) e^{2x} (D) $\dfrac{e^{2x}(2x+1)}{x^2}$ (E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

$$f'(x) = \frac{2e^{2x}(2x) - 2e^{2x}}{4x^2}$$

$$= \frac{2e^{2x}(2x-1)}{4x^2}$$

$$= \frac{e^{2x}(2x-1)}{2x^2}$$

14. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) −1.58 (B) −1.63 (C) −1.67 (D) −1.89 (E) −2.33

$y' = 3x^2 + 12x + 7 + 2\sin x$

$y'' = 6x + 12 + 2\cos x$

$y'' = 0 \Leftrightarrow 6x + 12 + 2\cos x = 0$

↳ graphing calc.

15. Let f be a function such that $\lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{h} = 5$ i.e $f'(2) = 5$

I. f is continuous at $x = 2$. ✓

II. f is differentiable at $x = 2$. ✓ if II is true, I must be true

III. The derivative of f is continuous at $x = 2$. ?

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

3

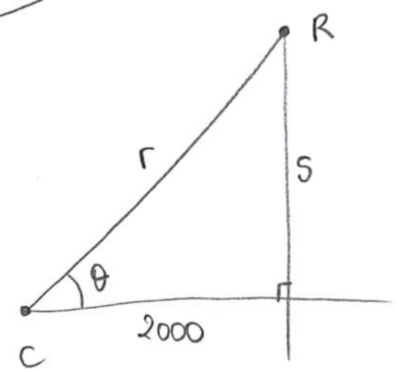
X. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3 ?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

Long Answer

16. A television camera at ground level is filming the lift off of a rocket that is rising vertically according to the position equation $s = 50t^2$, where s is measured in feet and t is measured in seconds. The camera is 2000 feet from the launch pad. Find the rate of change in the angle of elevation of the camera at 10 seconds after lift off.

3



$\left.\frac{d\theta}{dt}\right|_{t=10}$

$\tan\theta = \frac{s}{2000}$

$\tan\theta = \frac{50t^2}{2000}$

$\tan\theta = \frac{1}{40}t^2$

* when $t = 10$:

- $s = 5000$ ft
- $r^2 = s^2 + 2000^2$

  $r = \sqrt{29\,000\,000}$

  $r = 1000\sqrt{29}$
- $\cos\theta = \frac{2000}{1000\sqrt{29}}$

  $\cos\theta = \frac{2}{\sqrt{29}}$

$\longrightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{40} \times 2t$

$\frac{d\theta}{dt} = \frac{\cos^2\theta}{20} t$

$\left.\frac{d\theta}{dt}\right|_{t=10} = \frac{\frac{4}{29}}{20} \times 10$

$= \boxed{\frac{2}{29} \text{ rad/s}}$

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17. Let $f(x) = ln(x^2 + 2x + 3)$. Determine the variations, the extrema points, and the concavity on its domain and finally, sketch a rough graph.

- **Domain:** $\Delta = 4 - 4(1)(3) = -8 \Rightarrow \boxed{D = \mathbb{R}}$ 0.5

- $f'(x) = \dfrac{2x+2}{x^2+2x+3}$ 1

critical point : when $x = -1$ 0.5

x	$-\infty$		-1		$+\infty$
$f'(x)$		$-$	0	$+$	
$f(x)$	$+\infty$	↘	$\ln 2$	↗	$+\infty$

$f(-1) = \ln 2$

1

$\boxed{\begin{array}{l} f \text{ is decreasing on } (-\infty; -1) \\ f \text{ is increasing on } (-1; +\infty) \\ \text{MIN}: (-1; \ln 2) \\ \text{no max} \end{array}}$

- concavity : $f''(x) = \dfrac{2(x^2+2x+3) - (2x+2)(2x+2)}{(x^2+2x+3)^2}$

7

$= \dfrac{2x^2 + 4x + 6 - 4x^2 - 8x - 4}{(x^2+2x+3)^2}$

2

$= \dfrac{-2x^2 - 4x + 2}{(x^2+2x+3)^2}$

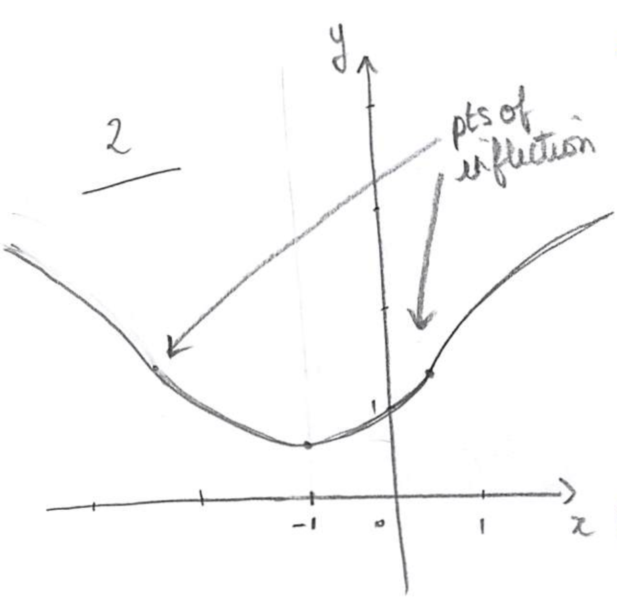
$\Delta = 4 - 4(1)(-1) = 8$

$x = \dfrac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

$= \dfrac{-2(x^2+2x-1)}{(x^2+2x+3)^2}$

$\oplus$

2

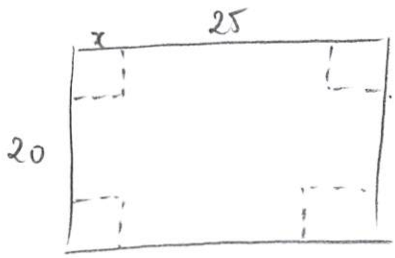


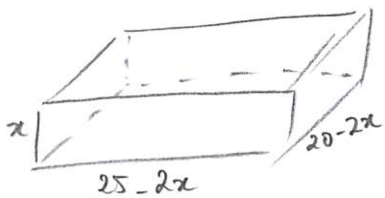
x	$-\infty$		$-1-\sqrt{2}$		-1	$-1+\sqrt{2}$		$+\infty$
$f''(x)$		$-$	0	$+$		0	$-$	
$f(x)$		concave down		concave up			concave down	

pts of infection:

$\boxed{\begin{array}{l} f \text{ is concave down on } (-\infty; -1-\sqrt{2}) \\ \quad \text{and on } (-1+\sqrt{2}, +\infty) \\ f \text{ is concave up on } (-1-\sqrt{2}, -1+\sqrt{2}) \end{array}}$

18. An open-top box is to be made by cutting congruent squares of side length x from the corners of a 20- by 25- inch sheet of tin and bending up the sides. How large should the squares be to make the box hold as much as possible? What is the resulting maximum volume?





$V = x(20-2x)(25-2x)$ $\quad D = [0, 10]$

$= x(500 - 40x - 50x + 4x^2)$

$= 4x^3 - 90x^2 + 500x$ $\quad \frac{1}{}$

$V' = 12x^2 - 180x + 500$

4

$= 4(3x^2 - 45x + 125)$

critical points:

$\Delta = 525$

$x = \dfrac{45 \pm \sqrt{525}}{6}$ → $11.3 \notin D$; → 3.7 $\quad \frac{1}{}$

x	0	$\frac{45-\sqrt{525}}{6}$	10
V'	$+$	0	$-$
V	0 ↗	820.53	↘ 0

$\frac{1}{}$

squares: 3.7 in. edge approx

V_{max}: 820.53 in^3 approx

$\frac{1}{}$