Name: AP Calc

Chapter 4 TEST
NO Calculator Section

1. a) Explain why the MVT applies to $f(x) = \frac{1}{x}$, on $[1,2]$ and what it states.

f is continuous on $[1,2]$
f is differentiable on $(1,2)$ } Therefore MVT applies, and states

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that there exists at least one point $c \in (1,2)$ such that:

$$f'(c) = \frac{f(2) - f(1)}{2-1}$$

b) Find any points in the open interval $(1,2)$ where the tangent line to $y = \frac{1}{x}$ is parallel to the chord line joining $(1, f(1))$ and $(2, f(2))$.

$f'(x) = -\frac{1}{x^2}$ chord: $m = \frac{f(2) - f(1)}{2-1} = \frac{\frac{1}{2} - 1}{1} = -\frac{1}{2}$

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$-\frac{1}{x^2} = -\frac{1}{2}$ $\Rightarrow$ 1 point on $(1,2)$: $\boxed{\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)}$

$x^2 = 2$

$x = \pm\sqrt{2}$

2. Determine whether the given function has any local or absolute extreme values, and find those values if possible.

a) $f(x) = \frac{1}{x^2+1}$ $D = \mathbb{R}$ $f'(x) = \frac{-2x}{(x^2+1)^2}$

critical point at $x = 0$

x	$-\infty$		0		$+\infty$
$f'(x)$		$+$	0	$-$	
$f(x)$	0	$\nearrow$	1	$\searrow$	0

$\Rightarrow$ no local or absolute min

local (& absolute) max: $(0,1)$

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b) $g(x) = (x-2)^{1/3}$ $\quad D = \mathbb{R} \quad g'(x) = \frac{1}{3}(x-2)^{-2/3} = \frac{1}{3\sqrt[3]{x-2}^{\,2}}$

critical point at $x = 2$

x	$-\infty$		2		$+\infty$
$g'(x)$		$+$	$\Vert$	$+$	
$g(x)$	$-\infty$	$\nearrow$	0	$\nearrow$	$+\infty$

$\Rightarrow$ $\boxed{\text{no}}$ absolute or local min or max

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c) $h(x) = x\sqrt{2-x^2}$

restrictions: $2 - x^2 \geq 0$

$x^2 \leq 2$

$-\sqrt{2} \leq x \leq \sqrt{2}$

$D = [-\sqrt{2}, \sqrt{2}]$

$h'(x) = \sqrt{2-x^2} + \frac{x(-2x)}{2\sqrt{2-x^2}}$

$= \frac{2 - x^2 - x^2}{\sqrt{2-x^2}}$

$= \frac{2(1-x^2)}{\sqrt{2-x^2}}$

critical points at $\pm\sqrt{2}$ and at ± 1

x	$-\sqrt{2}$		-1		1		$\sqrt{2}$
$h'(x)$	$\Vert$	$-$	0	$+$	0	$-$	$\Vert$
$h(x)$	0	$\searrow$	-1	$\nearrow$	1	$\searrow$	0

absolute max at $(1, 1)$
absolute min at $(-1, -1)$
local max at $(-\sqrt{2}, 0)$
local min at $(\sqrt{2}, 0)$

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3. Evaluate the following limits:

a) $\lim_{x\to 2} \frac{\ln(2x-3)}{x^2-4} \underset{\text{H.R}}{=} \lim_{x\to 2} \frac{\frac{2}{2x-3}}{2x} = \frac{1}{2}$

$\lim_{x\to 2} \ln(2x-3) = 0$

$\lim_{x\to 2} (x^2 - 4) = 0$

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b) $\lim_{x\to 0} \frac{\sin^{-1} x}{\tan^{-1} x} \underset{H.R}{=} \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{\frac{1}{1+x^2}} = 1$

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$\lim_{x\to 0} \sin^{-1} x = 0$

$\lim_{x\to 0} \tan^{-1} x = 0$

4. The volume of a right circular cylinder is increasing at a rate of 2 cm^3/min. When the volume of the cylinder is 60 cm^3, the radius is 5cm and is increasing at 1cm/min. How fast is the height of the cylinder changing at that time?

$\frac{dV}{dt} = 2$

$r\Big|_{V=60} = 5$

$\frac{dr}{dt}\Big|_{\substack{V=60 \\ r=5}} = 1$

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$\frac{dh}{dt}\Big|_{\substack{V=60 \\ r=5}} = ?$

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$

when V=60:

$2 = \pi\left(10 \cdot \frac{12}{5\pi} + 25\frac{dh}{dt}\right)$

$2 = 24 + 25\pi\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = -\frac{22}{25\pi} \text{ cm/min}}$

when V = 60, then

$V = \pi r^2 h$

$60 = \pi(25)h$

$h = \frac{60}{25\pi}$

$h = \frac{12}{5\pi}$

5. Let $f(x) = x^2$ if $x \geq 0$ and $f(x) = -x^2$ if $x < 0$.

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$$

a) Is 0 a critical point of *f*?

if $x>0$ $f'(x) = 2x$ $\lim_{x\to 0^-} f'(x) = \lim_{x \to 0^+} f'(x) = 0 \Rightarrow f'(0) = 0$

if $x<0$ $f'(x) = -2x$ <u>yes</u>

b) Does *f* have an inflection point at 0?

$$f'(x) = \begin{cases} 2x & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases}$$

f'' is not defined at 0 $\Rightarrow$ <u>yes</u>
+ change of sign

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$$f''(x) = \begin{cases} 2 & \text{if } x > 0 \\ -2 & \text{if } x < 0 \end{cases}$$

c) Is $f''(0) = 0$?

$f''(0)$ is undefined $\Rightarrow$ <u>no.</u>

6. Determine the intervals of constant concavity of $g(x) = e^{-x^2}$ and locate any inflection points.

$D = \mathbb{R}$ $g'(x) = -2xe^{-x^2}$

$$g''(x) = -2e^{-x^2} - 2x(-2x)e^{-x^2}$$
$$= e^{-x^2}(-2+4x^2)$$
$$= 2e^{-x^2}(2x^2-1)$$

	$-\infty$		$-1/\sqrt{2}$		$1/\sqrt{2}$		$+\infty$
$g''(x)$		+	0	−	0	+	
concavity of g		up	I	down	I	up	

points of inflection: $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{e}}\right)$

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$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{e}}\right)$

7. Use a suitable linearization to approximate $\sqrt[4]{85}$.

Let $f(x) = \sqrt[4]{x}$ $f(81) = 3$ $f'(x) = \frac{1}{4}x^{-3/4}$ $f'(81) = \frac{1}{4} \cdot \frac{1}{\sqrt[4]{81}^3} = \frac{1}{108}$

tangent line at $x=81$:

$y - 3 = \frac{1}{108}(x-81) \Rightarrow L(x) = \frac{1}{108}(x-81)+3$

$f(85) \simeq L(85) = \frac{1}{108}(85-81)+3$

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$= \frac{4}{108} + 3 = \boxed{\frac{82}{27}}$

$= \frac{1}{27} + 3 = \frac{1+81}{27}$

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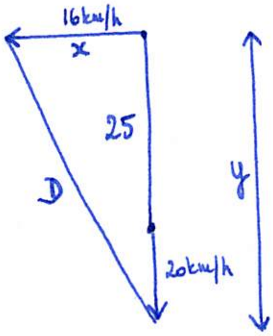
Chapter 4 TEST
Graphing Calculator Section

1. At 1:00 pm, ship A is 25km due north of ship B.
If ship A is sailing west at a rate of 16 km/h and ship B is sailing south at 20 km/h, find the rate at which the distance between the two ships is changing at 1:30 pm.

$D|_{t=0} = 25 \quad \frac{dx}{dt} = 16 \quad \frac{dy}{dt} = 20 \quad \frac{dD}{dt}\Big|_{t=\frac{1}{2}} = ?$



$$D^2 = x^2 + y^2$$

$$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

when $t = \frac{1}{2}$, we get :

$$2\sqrt{1289}\,\frac{dD}{dt} = 16 \times 16 + 70 \times 20$$

$$\frac{dD}{dt} = \frac{1656}{2\sqrt{1289}}$$

$$\boxed{\frac{dD}{dt}\Big|_{t=\frac{1}{2}} \simeq 23\text{km/h}}$$

When $t = \frac{1}{2}$:

$x = 8$

$y = 35$

$D = \sqrt{1289}$

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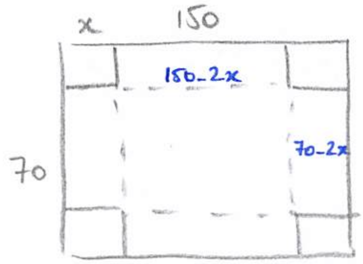
2. A box is to be made from a rectangular sheet of cardboard 70 cm by 150 cm by cutting equal squares out of the four corners and bending up the resulting 4 flaps to make the sides of the box. The box has no top. What is the largest possible volume of the box?



$D = [0, 35]$

$V = x(70-2x)(150-2x)$

$= 4x^3 - 440x^2 + 10500x$

$\frac{dV}{dt} = 12x^2 - 880x + 10500$

$\Delta = 270400$

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x	0		15		35
V'		+	0	−	
V		↗		↘	

$V_{max} = V\Big|_{x=15} = 15(70-30)(150-30)$

$= \boxed{72\,000 \text{ cm}^3}$