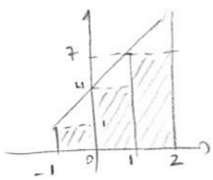
Name: Solutions

AP Calc

Chapter 5 TEST
NO Calculator

Free Response Questions

1. Let's consider the area below $y = 3x + 4$, above $y = 0$, from $x = -1$ to $x = 2$.
   a) Determine an approximation of this area using LRAM with 3 subintervals. [1]

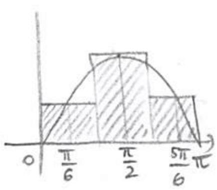


$$A \simeq 1\times 1 + 4\times 1 + 7\times 1 = 12$$

   b) Determine the exact value of this area using a geometric interpretation. [1]

$$A = \frac{3(1+10)}{2} = \frac{33}{2} = 16.5$$

2. Let's consider the area below $y = sinx$, above $y = 0$, from $x = 0$ to $x = \pi$.
   a) Determine an approximation of this area using MRAM with 3 subintervals. (Sketch the area) [2]



$$A \simeq \frac{\pi}{3}\left(\sin\frac{\pi}{6} + \sin\frac{\pi}{2} + \sin\frac{5\pi}{6}\right)$$

$$\simeq \frac{\pi}{3}\left(\frac{1}{2} + 1 + \frac{1}{2}\right)$$

$$\simeq \frac{2\pi}{3}$$

   b) Determine the exact value of this area using antiderivatives. [1]

$$A = \int_0^{\pi} \sin x\,dx = -\cos x\Big]_0^{\pi}$$

$$= -\cos\pi + \cos 0 = 2$$

3. Interpret the given limit as a definite integral: $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \ln\left(1 + \frac{2i}{n}\right)$ [1.5]

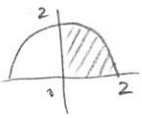
$$\int_0^2 \ln(1+x)\,dx$$

4. Rewrite the following integral as the limit of a Riemann Sum: $\int_0^{\pi} sinx.dx$ [1.5]

$$\lim_{n\to+\infty} \sum_{i=1}^{n} \frac{\pi}{n} \sin\left(\frac{\pi i}{n}\right)$$

5. Evaluate the following integrals: [9]

a) $\int_0^2 \sqrt{4-x^2}\,dx$ $= \frac{1}{4}\pi(2)^2 = \pi$



b) $\int_{1/2}^1 \frac{1}{x^2}\,dx$ $= \int_{1/2}^{1} x^{-2}\,dx = -\frac{1}{x}\Big]_{1/2}^{1} = -1+2 = 1$

c) $\int_{-1}^2 (3x^2 - 4x + 2)dx$ $= x^3 - 2x^2 + 2x\Big]_{-1}^{2} = 8 - 8 + 4 - (-1-2-2)$

$= 10$

d) $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\,dx$ $= \sin^{-1}x\Big]_0^{1/2} = \sin^{-1}\frac{1}{2} - \sin^{-1}0 = \frac{\pi}{6} - 0 = \frac{\pi}{6}$

e) $\int_4^9 \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)dx$ $= \frac{2}{3}x^{3/2} - 2\sqrt{x}\Big]_4^9 = 18 - 6 - \left(\frac{16}{3} - 4\right) = 12 - \frac{4}{3} = \frac{32}{3}$

f) $\int_{-1}^1 \frac{dx}{1+x^2}$ $= \tan^{-1}x\Big]_{-1}^{1} = \tan^{-1}1 - \tan^{-1}(-1) = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

Multiple choices **[2]**

6. Let $f(x) = \lfloor x \rfloor$ be the greatest integer function ($f(x)$ is the smallest number less than or equal to x). What is $\int_1^4 f(x)\,dx$?

A) 3 B) 4 C) 5 D) 6 E) DNE (f is not continuous)

7. What is $\lim_{h\to 0} \frac{1}{h}\int_x^{x+h} f(t)dt$?

$\int_x^{x+h} f(t)dt = F(x+h) - F(x)$

A) 0 B) 1 C) $f'(x)$ D) $f(x)$ E) ∞

Free Response Questions

8. Given that $\int_0^1 f(x)dx = a$ and $\int_0^2 f(x)dx = b$, evaluate the following integrals in terms of a and b. [2]

a) $\int_0^2 (3f(x)+1)dx = 3\int_0^2 f(x)dx + \int_0^2 1dx = \boxed{3b+2}$

b) $\int_1^2 f(x)dx = \int_0^2 f(x)dx - \int_0^1 f(x)dx = \boxed{b-a}$

9. Find the average value of $f(t) = 1 + \cos t$ over $[-\pi, \pi]$ [2]

$$Av = \frac{1}{2\pi}\int_{-\pi}^{\pi}(1+\cos t)dt$$

$$= \frac{1}{2\pi}\left[t + \sin t\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi}(\pi + 0 - (-\pi + 0))$$

$\boxed{Av = 1}$

10. Find the indicated derivatives: [3]

a) $\frac{d}{dx}\int_2^x \frac{\sin t}{t}dt = \frac{\sin x}{x}$ (FTC)

b) $\frac{d}{dx}\int_x^4 \frac{\cos t}{1+t^2}dt = -\frac{\cos x}{1+x^2}$

c) $\frac{d}{d\theta}\int_{\sin\theta}^{\cos\theta} \frac{1}{1-x^2}dx = \frac{1}{1-\cos^2\theta}\cdot(-\sin\theta) - \frac{1}{1-\sin^2\theta}\cdot\cos\theta$

$$= -\frac{\sin\theta}{1-\cos^2\theta} - \frac{\cos\theta}{1-\sin^2\theta}$$

11. Evaluate the following indefinite integrals: [2]

a) $\int (x^2+3)^2 dx = \int(x^4 + 6x^2 + 9)dx = \frac{1}{5}x^5 + 2x^3 + 9x + C, \quad C \in \mathbb{R}$

b) $\int (e^x - 2^x)dx = e^x - \frac{1}{\ln 2}\cdot 2^x + C, \quad C \in \mathbb{R}$

Name: AP Calc

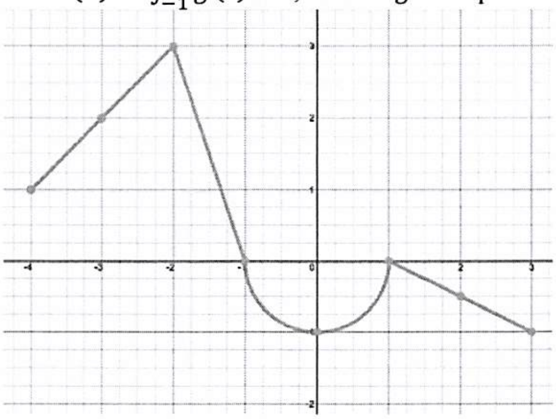
12. Evaluate the following limit: $\lim_{n\to\infty} \frac{1}{n}\left(\frac{1}{1+\frac{1}{n}} + \frac{1}{1+\frac{2}{n}} + \cdots + \frac{1}{1+\frac{n}{n}}\right)$ $= \int_0^1 \frac{1}{1+x}\,dx$ [2]

$= \ln(1+x)\Big]_0^1$

$= \ln 2 - \ln 1$

$= \boxed{\ln 2}$

13. Let $h(x) = \int_{-1}^{x} g(t)\,dt$, where g is represented below. [6]



a) Evaluate $h(0)$ $= \int_{-1}^{0} g(t)\,dt = -\frac{1}{4}\pi(1)^2 = -\frac{\pi}{4}$

b) Evaluate $h(-4)$ $= \int_{-1}^{-4} g(t)\,dt = -\int_{-4}^{-1} g(t)\,dt = -\left(\frac{2(1+3)}{2} + \frac{3\times 1}{2}\right) = -\frac{11}{2}$

c) On which interval(s) is g negative? No explanation required.

$(-1,1)$ and $(1,3]$

d) On which interval(s) is h negative? No explanation required.

$[-4,3]$

e) On which interval(s) is h decreasing? Justify

FTC : $h'(x) = g(x)$

$\Rightarrow [-1,3]$

f) On which interval(s) is h concave down? Justify

FTC : $h''(x) = g'(x)$ h is concave down when g is decreasing

$\Rightarrow$ on $[-2,0]$ and on $[1,3]$