Name: $\dfrac{}{38}$ AP Calc

Chapter 6 TEST
NON-CALCULATOR SECTION

Multiple Choice

1 1. $\int \frac{x}{x^2+1}dx \underset{u=x^2+1 \quad du=2x\,dx}{=} \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|x^2+1|+c = \ln\sqrt{x^2+1}+C$

A) $\ln\sqrt{x^2+1}+C$ B) $\frac{2x}{(x^2+1)^2}+C$ C) $\arctan x + C$ D) $\ln(x^2+1)+C$

Free Responses

2 2. Determine whether the function $y = 2\sin 2x$ is a solution to the differential equation $y''' - 8y = 0$

$y' = 4\cos 2x$

$y'' = -8\sin 2x$

$y''' = -16\cos 2x$

$y''' - 8y = -16\cos 2x - 16\sin 2x$

$= -16(\cos 2x + \sin 2x) \neq 0$

$\boxed{\text{NO}}$ ex: if $x=0$ $y'''-8y=-16\neq 0$

3. Solve the differential equations :

1 a) $\frac{dy}{dx} = e^{2-x}$ $\boxed{y = -e^{2-x}+c \quad , c\in\mathbb{R}}$

2 b) $y' - e^y \sin x = 0$

$y' = e^y \sin x$

$y'e^{-y} = \sin x$

$-e^{-y} = -\cos x + C$

$e^{-y} = \cos x - C$

$-y = \ln(\cos x - C)$

$\boxed{y = -\ln(\cos x - C)}$ $C\in\mathbb{R}$

3 c) $xy' - (x+2)y = 0$ with $y(1) = -1$

$xy' = (x+2)y$

$\frac{y'}{y} = \frac{x+2}{x}$

$\ln|y| = x + 2\ln|x| + C$

when $x=1$, $y=-1$

$0 = 1 + C$

$\underline{C=-1}$

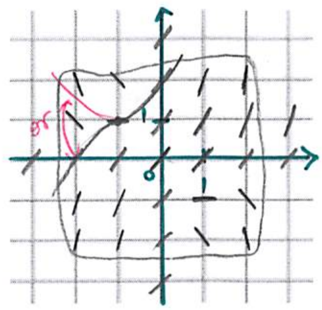
$\ln|y| = x + 2\ln|x| - 1$

$|y| = e^{x+2\ln|x|-1}$

$y = \pm e^{x+2\ln|x|-1}$

$\boxed{y = -e^{x+2\ln|x|-1}}$

4. a) Sketch the slope field for the differential equation $y' = xy+1$



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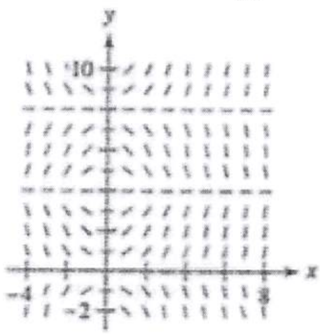
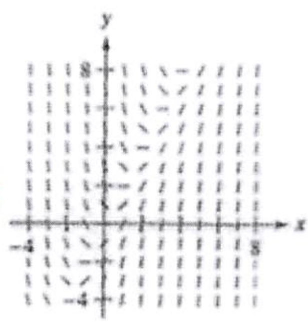
b) Use the slope field to sketch the solution that passes through $(-1,1)$.

1

5. Match each equation to its slope field :

A : $\frac{dy}{dx} = 2x - y$ B : $\frac{dy}{dx} = -\frac{4x}{y}$ C : $\frac{dy}{dx} = x\sin\frac{\pi y}{4}$ D : $\frac{dy}{dx} = 3 - 2y$

C 

A 

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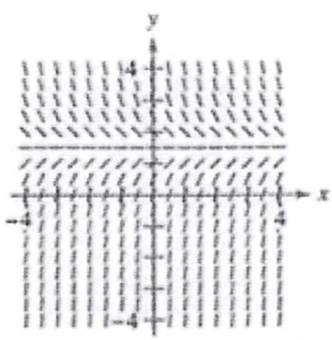
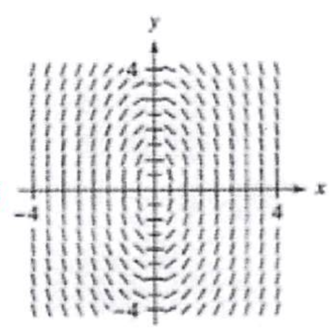
D 

B 

6. Evaluate the following integrals :

a) $\int_0^{\pi} \cos\frac{x}{2}\,dx = 2\int_0^{\pi/2} \cos u\,du = 2\sin u\Big]_0^{\pi/2} = 2$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

2

b) $\int_0^1 x^2(x^3-2)^3 dx = \frac{1}{3}\int_{-2}^{-1} u^3 du = \frac{1}{3}\frac{u^4}{4}\Big]_{-2}^{-1}$

$u = x^3 - 2$
$du = 3x^2 dx$

$= \frac{1}{12}(1-16) = -\frac{15}{12} = -\frac{5}{4}$

2

c) $\int_1^e \frac{1-lnx}{x} dx = \int_1^e \frac{1}{x} dx - \int_1^e \frac{\ln x}{x} dx$

$= \ln|x|\Big]_1^e - \int_0^1 u\,du$

with $u = \ln x$
$du = \frac{1}{x} dx$

2

$= 1 - \frac{u^2}{2}\Big]_0^1$

$= 1 - \frac{1}{2} = \frac{1}{2}$

d) $\int_1^2 x^2\sqrt{x-1}dx$

let $u = x-1$ $\quad x = u+1$
$du = dx$

2

$= \int_0^1 (u+1)^2\sqrt{u}\,du$

$= \int_0^1 \left(u^{5/2} + 2u^{3/2} + u^{1/2}\right) du$

$= \frac{2}{7}u^{7/2} + 2.\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\Big]_0^1$

$= \frac{2}{7} + \frac{4}{5} + \frac{2}{3} = \frac{184}{105}$

7. Find the indefinite integral :

a) $\int \sin^3 x \cos x\, dx = \int u^3 du = \frac{u^4}{4} + C$, $C \in \mathbb{R}$

$u = \sin x$
$du = \cos x\, dx$

$= \boxed{\frac{1}{4}\sin^4 x + C}$

2

b) $\int \frac{\cos\theta}{\sqrt{1-\sin\theta}}\,d\theta = -\int u^{-1/2}\,du = -2u^{1/2} + C$, $C \in \mathbb{R}$

$u = 1 - \sin\theta$

$du = -\cos\theta\, d\theta$

$= \boxed{-2\sqrt{1-\sin\theta} + C}$

2

c) $\int \frac{e^{2x}}{e^{2x}+1}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \boxed{\frac{1}{2}\ln\left|e^{2x}+1\right| + C}$, $C \in \mathbb{R}$

$u = e^{2x} + 1$

$du = 2e^{2x}\,dx$

2

d) $\int (5x-3)^4\,dx = \frac{1}{5}\int u^4\,du = \frac{1}{5}\frac{u^5}{5} + C$, $C \in \mathbb{R}$

$u = 5x - 3$

$du = 5dx$

$= \boxed{\frac{1}{25}(5x-3)^5 + C}$

2

e) $\int \cos^3 x\,dx = \int \cos^2 x . \cos x\,dx$

$= \int (1 - \sin^2 x)\cos x\,dx$

$= \int \cos x\,dx - \int \sin^2 x . \cos x\,dx$

2

$= \sin x - \int u^2\,du$ with $u = \sin x$, $du = \cos x\,dx$

$= \boxed{\sin x - \frac{\sin^3 x}{3} + C}$ $C \in \mathbb{R}$

Chapter 6 TEST
CALCULATOR SECTION

Multiple Choice

8. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of a pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

$\frac{dT}{dt} = -110e^{-0.4t}$

$T = 275e^{-0.4t} + C$ 75

A) 112°F B) 119°F C) 147°F D) 238°F E) 335°F

$R' = cR$

$R = R_0 e^{ct}$

9. If Radium decomposes at a rate proportional to the amount present, then the amount R left after t years, if R_o is present initially and c is the negative constant of proportionality, is given by

A) $R = R_o ct$ B) $R = R_o e^{ct}$ C) $R = R_o + \frac{1}{2}ct^2$ D) $R = e^{R_o ct}$ E) $R = e^{R_o + ct}$

$\frac{dT}{dt} = -k(T-10)$

10. According to newton's law of cooling, the temperature of an object decreases at a rate proportional to the difference between its temperature and that of the surrounding air. Suppose a corpse at a temperature of 32°C arrives at a mortuary where the temperature is kept at 10°C. Then the differential equation satisfied by the temperature T of the corpse t hours later is

A) $\frac{dT}{dt} = -k(T-10)$ B) $\frac{dT}{dt} = -k(T-32)$ C) $\frac{dT}{dt} = -kT(T-10)$ D) $\frac{dT}{dt} = 32e^{-kt}$

3

Free Responses

11. A population grows continuously at a rate of 1.85%. How long will it take the population to double?

$\frac{dP}{dt} = 0.0185P$ $2 = e^{0.0185t}$

$P = P_0 e^{0.0185t}$ $\frac{\ln 2}{0.0185} = t$ $\boxed{t \simeq 37.467 \text{ years}}$

2

12. Find the balance in an account when \$1000 is deposited for 8 years at an interest rate of 4% compounded continuously.

$\frac{dB}{dt} = 0.04B$

$B = 1000e^{0.04t}$

$B = 1000e^{0.32}$

$\boxed{B = \$1377.13}$

2