**7.1 – Integrals as a Net Change**



Consumption over Time:

Velocity is not the only rate in which you can integrate to get a total. In fact if you were given a function that gave the number of tickets per hour that the police wrote each day, and you wanted to find the total number of tickets in a 24-hour period, you could integrate.

Example 3: The tide removes sand from Sandy Point Beach at a rate modeled by the function R given by

.

A pumping station adds to the beach at a rate modeled by the function *S* given by

 .

Both and have units of cubic yards per hour and *t* is measured in hours for .

At time , the beach contains 2500 cubic yards of sand.

1. How much sand will the tide remove from the beach during the 6-hour period? Indicate units of measure.
2. Write an expression for , the total number of cubic yards of sand on the beach at time *t*.
3. Find the rate at which the total amount of sand on the beach is changing at time .

**7.2 – Areas in the Plane**



 

Example: Find the area of the region bounded by the graphs of , , and .

Note: It is very important to start with a rough graph in order to know which one is “above”…

Example: Find the area of the region bounded by the graphs of and .

Example: Find the area of the region bounded by the graphs of , , and .

**7.3 - Volumes**

We already know that the integral can be seen as a sum of thin strips with values *f*(*x*) and width *dx*.



The same way, it can calculate volumes if we add thin slices with a base area *A*(*x*) and width *dx*.

 

We will need to determine the area of a cross section (as a function of *x* most of the time, but sometimes *y*) and just integrate it…

In the previous example, , and you get:

Example:

The base of a solid is the region in the first quadrant enclosed by the parabola , the line , and the *x*-axis. Each plane section of the solid perpendicular to the *x*-axis is a square. Determine the volume of the solid.

Example:

The base of a solid is the region enclosed by the parabola , the line , and the *y*-axis.

Each plane section of the solid perpendicular to the *x*-axis is a square. Determine the volume of the solid.

Example:

The base of a solid is a region in the first quadrant bounded by the *x*-axis and the line .

If the cross-section of the solid perpendicular to the *x*-axis are semicircles. What is the volume of the solid to the nearest hundredth?

Example: The base of a solid is the region in the first quadrant enclosed by the graph of and the coordinate axes. If every cross section of the solid perpendicular to the *y*-axis is a square, what integral gives you the volume of the solid?

**Solids of Revolution:**



Example: What is the volume of the cylinder formed by revolving the graph of around the *x*-axis on the interval [0;4]?

Note: when we rotate a curve around an axis, the cross-section is always a circle. Therefore, .

*f*(*x*) is then the radius of the cross-section, and we usually note it *R*(*x*).



Example: a) Determine the volume of the solid formed by revolving the following region about the *x*-axis.



 b) Same question about the *y*-axis.

Your turn: Same questions for the following region:



Example: Find the volume of the solid generated by revolving the region bounded by the graphs of the equations , , and about the indicated lines.

1. The line
2. The line

**The Washer method:**





Examples: Determine the integral that represents the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated lines:

