AP Calculus
7.1 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Fill in the blanks.

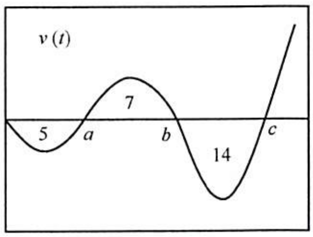
a) Integrating velocity gives displacement

b) Integrating the absolute value of velocity gives total distance.

c) New position = initial position + displacement.

d) In general, integrating a rate (say of clowns per car) gives you the net change (# of clowns)

2. A particle moves along the x-axis (units in cm). Its initial position at $t = 0$ sec is $x(0) = 15$. The figure shows the graph of the particle's velocity $v(t)$. The numbers are the *areas* of the enclosed regions.



a) Describe the motion of the particle (include the position and when the particle moving right, left, and stopped)

	0		a		b		c	
$v(t)$		−	0	+	0	−	0	+
$x(t)$	15	↘	10	↗	17	↘	3	↗

Initial position: $x(0)=15$, then moves to the left for a sec. Position $x(a)=10$. then moves to the right to position $x(b)=17$. Then left $x(c)=3$ the right...

b) What is the particle's displacement between $t = 0$ and $t = c$?

$\int_0^c v(t)dt = x(c) - x(0) = 3 - 15 = -12$ or $\int_0^c v(t)dt = \int_0^a v(t)dt + \int_a^b v(t)dt + \int_b^c v(t)dt$
$= -5 + 7 - 14 = -12$

c) What is the total distance traveled by the particle in the same time period?

$\int_0^c |v(t)|dt = 5+7+14 = 26$ (cm)

d) Give the positions of the particle at times a, b, and c.

$x(a) = 10$
$x(b) = 17$
$x(c) = 3$

e) Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, b]$?

around a

3. [Calculator] Pollution is being removed from a lake at a rate modeled by the function $y = 20e^{-0.5t}$ tons/yr, where t is the number of years since 1995. How much pollution was removed from the lake between 1995 and 2005.

$$\int_0^{10} 20e^{-0.5t}\,dt = 20\,\frac{e^{-0.5t}}{-0.5}\Big]_0^{10} = -40\left(e^{-5} - e^{0}\right) \simeq 39.7 \text{ tons.}$$

4. [Calculator] The rate at which customers arrive at a counter to be served is modeled by the function F defined by

$$F(t) = 12 + 6\cos\left(\frac{t}{\pi}\right)$$

for $0 \le t \le 60$, where $F(t)$ is measured in customers per minute and t is measured in minutes.

a) To the nearest whole number, how many customers arrive at the counter over the 60-minute period?

$$\int_0^{60} F(t)\,dt = \int_0^{60}\left(12 + 6\cos\left(\frac{t}{\pi}\right)\right)dt = \left[12t + 6\pi\sin\left(\frac{t}{\pi}\right)\right]_0^{60} \simeq 725 \text{ customers}$$

b) What is the average number of customers served each minute over the 60-minute period?

$$\frac{1}{60}\int_0^{60} F(t)\,dt = \frac{12\times 60 + 6\pi\sin\left(\frac{60}{\pi}\right)}{60} \simeq 12.1 \text{ customers/min}$$

5. [Calculator] A particle moves along the x – axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

$a(t) = v'(t) = -\sin\left(\frac{t^2}{2}\right) - t(t+1)\cos\left(\frac{t^2}{2}\right)$

$a(2) \simeq 1.59$

$\left.\begin{array}{l} a(2) > 0 \\ v(2) < 0 \end{array}\right\} \Rightarrow$ decreasing

b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

$v(t) = 0$ $\quad -\underbrace{(t+1)}_{\neq 0 \text{ on } (0,3)}\sin\left(\frac{t^2}{2}\right) = 0$

$\frac{t^2}{2} = 0 + 2\pi n$ or $\frac{t^2}{2} = \pi + 2\pi n$

$t^2 = 0 + 4\pi n$ $\quad t^2 = 2\pi + 4\pi n$

$t = \sqrt{4\pi n}$ or $t = \sqrt{2\pi + 4\pi n}$, $n \in \mathbb{Z}$

$\Rightarrow \boxed{t \simeq 2.5}$ the only one in the domain

c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

$$\int_0^3 |v(t)|\,dt = -\int_0^{\sqrt{2\pi}} v(t)\,dt + \int_{\sqrt{2\pi}}^3 v(t)\,dt \simeq 4.3$$

d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

t	0		$\sqrt{2\pi}$		3
$v(t)$	0	−	0	+	
$x(t)$	1	↘	−2.27	↗	−1.20

$x(\sqrt{2\pi}) = x(0) + \int_0^{\sqrt{2\pi}} v(t)\,dt \approx -2.27$

$x(3) = x(0) + \int_0^3 v(t)\,dt \approx -1.20$

Greatest distance: −2.27 (approx.)

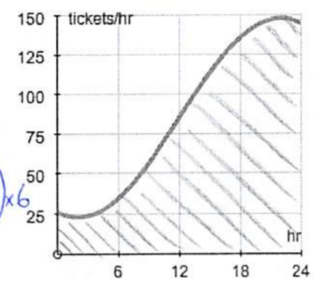
6. [Calculator] The rate of consumption of oil in the United States during the 1980s (in billions of barrels per year) is modeled by the function $C = 27.08e^{t/25}$, where t is the number of years after January 1, 1980. Find the total consumption of oil in the United States from January 1, 1980 to January 1, 1990.

$$\int_0^{10} 27.08e^{t/25}\,dt = \left[27.08 \times 25\, e^{t/25}\right]_0^{10}$$

$$= 677e^{10/25} - 677$$

$$\simeq 332.965 \text{ billions of barrels.}$$

7. Tickets to a concert were sold out in 24 hours. The graph below shows the rate at which people bought tickets to the concert during the 24-hour period. Before the tickets went on sale, 100 tickets had been set aside for VIPs.



a) How does the graph indicate the total number of tickets purchased in the 24 hours.

Area under the curve.

b) *Approximate* the total tickets purchased in the last 24 hours. $A \simeq 1950$

$A \simeq 25 \times 6 + \frac{1}{2}(50+75)\times 6 + \frac{1}{2}(75+125)\times 6 + \frac{1}{2}(130+140)\times 6$

c) If all ticket holders show up to the concert, how many people will attend?

$N = \int_0^{24} f(x)\,dx + 100$ $\quad N \simeq 2050$ people.

8. [Calculator] The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{t^2 - 24t + 160}.$$

The rate at which people leave the same amusement park on the same day is modeled by function L defined by

$$L(t) = \frac{9890}{t^2 - 38t + 370}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight.
These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

a) How many people have entered the park by 5:00 pm ($t = 17$)? Round your answer to the nearest whole number.

$N = \int_9^{17} E(t)\,dt \approx 6004$ people

b) The price of admission to the park is \$15 until 5:00 pm ($t = 17$). After 5:00 pm, the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest dollar.

$M = 15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt \approx \$104048$

(w int: \$104041)

c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725.

Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

$H'(17) = E(17) - L(17) \simeq 380.49 - 760.77 \simeq -380$

$H(17)$ represents the number people in the park at 5 pm.

$H'(17)$ represents the rate at which the people are leaving at 5 pm.
(380 ppl/hr at 5 pm) because it's negative

d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

The number will be maximum when $H'(t)$ goes from positive to negative.

$H'(t) = E(t) - L(t)$

On the graphing calculator, by graphing $H'(t)$, we get:

t	9	15.8	23
$H'(t)$	+	0	−

around: 3:48 pm

10. [Calculator] Population density measures the number of people per square mile inhabiting a given living area. The population of a local town decreases as you move away from the city center can be approximated by the function 10,000(3 – r) at a distance r miles from the city center.

a) If the population density approaches zero at the edge of the city, what is the city's radius?

b) A thin ring around the center of the city has thickness Δr and radius r.
(*Hint*: Let r = distance from center of city to "middle of ring")
Draw a picture of this and determine the area of the thin ring.

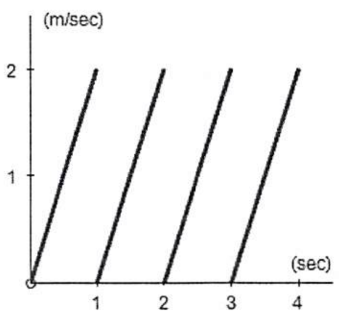
c) Explain why the population of the ring in part b is approximately $10000(3-r)(2\pi r)\Delta r$.

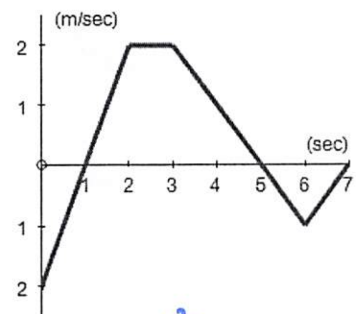
d) Estimate the total population of the town by setting up and evaluating a definite integral.

11. Each graph shown below shows the velocity (in m/sec) of a particle moving on the x-axis. Each particle starts at $x = 2$ when $t = 0$.

a) Find where each particle is at the end of the trip.

b) Find the total distance traveled by each particle.





a) $p = 2 + 4\int_0^1 2t\,dt = 6$

b) $D = 4\int_0^1 |2t|\,dt = 4$

a) $p = 2 + \int_0^2 (2t-2)\,dt + \int_2^3 2\,dt + \int_3^6 (-t+5)\,dt + \int_6^7 (t-7)\,dt$
$= 5$

b) $D = \int_0^1 (-2t+2)\,dt + \int_1^2 (2t-2)\,dt + \int_2^3 2\,dt + \int_3^5 (-t+5)\,dt + \int_5^6 (t-5)\,dt + \int_6^7 (-t+7)\,dt$
$= 7$

12. [True or False] If the velocity of a particle moving along the x-axis is always positive, then the displacement and the total distance traveled are equal.

True.

AP Calculus
7.2 Worksheet

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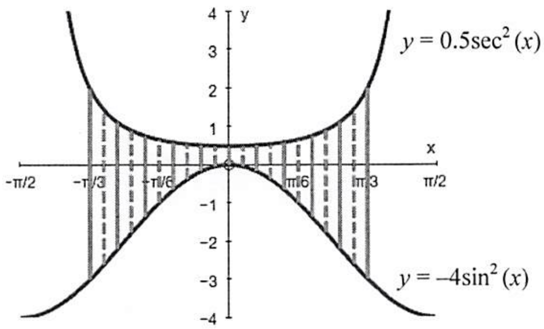
1. Find the area of the shaded region analytically (without a calculator).

a)



$$A = \int_{-\pi/3}^{\pi/3} (0.5\sec^2 x + 4\sin^2 x)\,dx$$

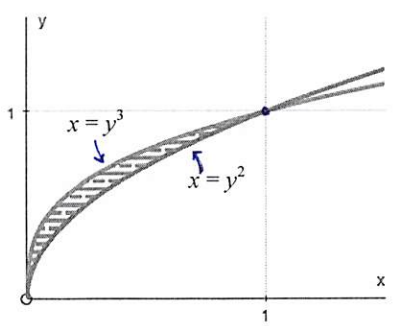
$$= 0.5\tan x\Big]_{-\pi/3}^{\pi/3} + 4\int_{-\pi/3}^{\pi/3} \frac{1-\cos 2x}{2}\,dx$$

$$= 0.5(\sqrt{3}+\sqrt{3}) + 4\left(\frac{1}{2}x - \frac{\sin 2x}{4}\right)\Big]_{-\pi/3}^{\pi/3}$$

$$= \sqrt{3} + 4\frac{\pi}{3} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = \boxed{\frac{4\pi}{3}}$$

b)

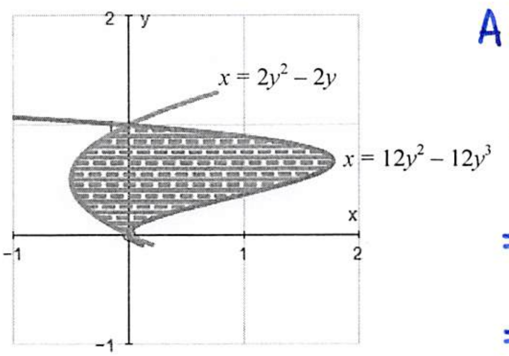
intersection:
$y^2 = y^3$
$y^2(y-1) = 0$
$\begin{cases} y=0 \\ x=0 \end{cases}$ $\begin{cases} y=1 \\ x=1 \end{cases}$



$$A = \int_0^1 (y^2 - y^3)\,dy = \frac{y^3}{3} - \frac{y^4}{4}\Big]_0^1$$

$$= \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12}}$$

c)



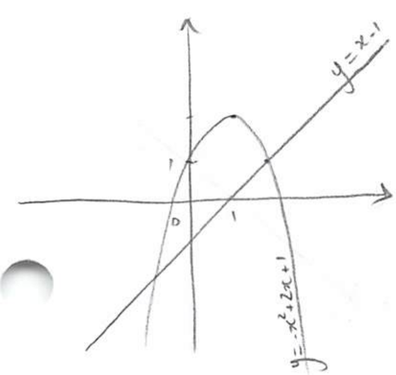
$$A = \int_0^1 \left(12y^2 - 12y^3 - (2y^2 - 2y)\right)dy$$

$$= \int_0^1 (-12y^3 + 10y^2 + 2y)\,dy$$

$$= -3y^4 + \frac{10}{3}y^3 + y^2\Big]_0^1$$

$$= -3 + \frac{10}{3} + 1 = \boxed{\frac{4}{3}}$$

2. [No Calculator] Find the area of the region bounded by the graphs of $f(x) = 1 + 2x - x^2$ and $g(x) = x - 1$.



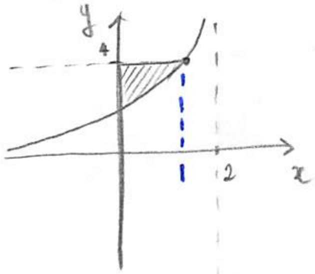
Intersections: $-x^2 + 2x + 1 = x - 1$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$\begin{cases} x=2 \\ y=1 \end{cases}$ $\begin{cases} x=-1 \\ y=-2 \end{cases}$

$$A = \int_{-1}^{2} \left(-x^2 + 2x + 1 - (x-1)\right)dx = -\frac{x^3}{3} + \frac{x^2}{2} + 2x\Big]_{-1}^{2}$$

$$= \int_{-1}^{2} (-x^2 + x + 2)\,dx \qquad = -\frac{8}{3} + \frac{4}{2} + 4 - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$

$$= \boxed{\frac{9}{2}}$$

3. [No Calculator] Find the area of the region bounded by the graphs of $g(x)=\dfrac{4}{2-x}$, $y=4$, and $x=0$.



Intersection: $\dfrac{4}{2-x}=4$

$2-x=1$

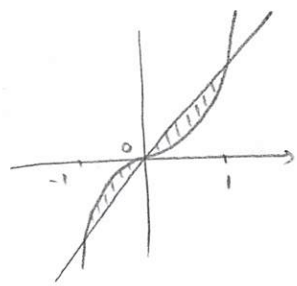
$x=1$

$A=\int_0^1\left(4-\dfrac{4}{2-x}\right)dx$

$=\left(4x+4\ln|2-x|\right]_0^1$

$=4-4\ln 2$

4. [No Calculator] Consider the area of the region bounded by the graphs of $y=x^3$ and $y=x$.



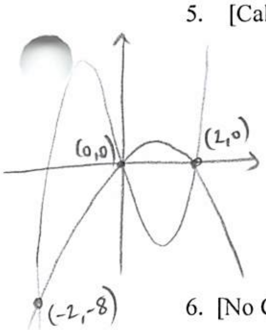
a) Explain why the area ***cannot*** be found by the single integral $\int_{-1}^{1}\left(x^3-x\right)dx$.

$y=x^3$ is not always "above" $y=x$ on $[-1,1]$

b) Write an expression involving a ***single*** integral that DOES represent the area of the region?

$\int_{-1}^{1}|x^3-x|\,dx$

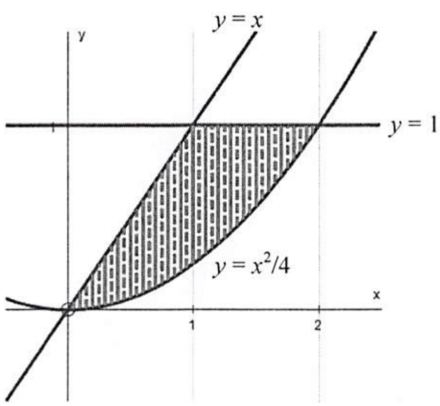
5. [Calculator] Find the area of the region between the graphs of $f(x)=3x^3-x^2-10x$ and $g(x)=-x^2+2x$.



$A=\int_{-2}^{0}\left(3x^3-x^2-10x-(-x^2+2x)\right)dx+\int_0^2\left(-x^2+2x-(3x^3-x^2-10x)\right)dx$

$=\int_{-2}^{0}\left(3x^3-12x\right)dx+\int_0^2\left(-3x^3+12x\right)dx=\boxed{24}$

6. [No Calculator] Find the area of the shaded region analytically.



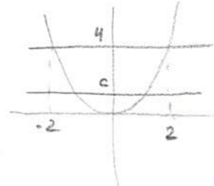
$A=\int_0^1\left(x-\dfrac{x^2}{4}\right)dx+\int_1^2\left(1-\dfrac{x^2}{4}\right)dx$

$=\dfrac{x^2}{2}-\dfrac{x^3}{12}\Big]_0^1+x-\dfrac{x^3}{12}\Big]_1^2$

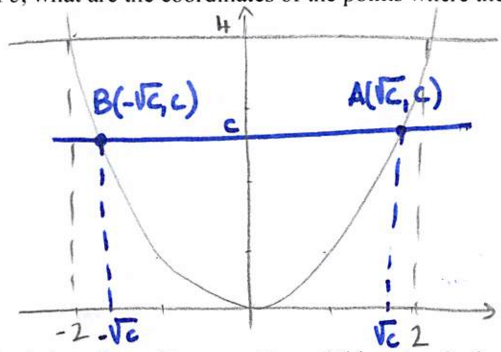
$=\dfrac{1}{2}-\cancel{\dfrac{1}{12}}+2-\dfrac{8}{12}-1+\cancel{\dfrac{1}{12}}$

$=\boxed{\dfrac{5}{6}}$

7. [No Calculator] The region bounded below by the parabola $y = x^2$ and above by the line $y = 4$ is to be partitioned into two subsections of equal area by cutting across it with the horizontal line $y = c$.



a) Sketch the region and draw a line $y = c$ across it that looks about right.
In terms of c, what are the coordinates of the points where the line and the parabola intersect?



b) Find c by integrating with respect to y. (This puts c in the limits of integration.)

$y = x^2$

$x = \pm\sqrt{y}$

$$\int_0^c \sqrt{y}\,dy = \frac{1}{2}\int_0^4 \sqrt{y}\,dy$$

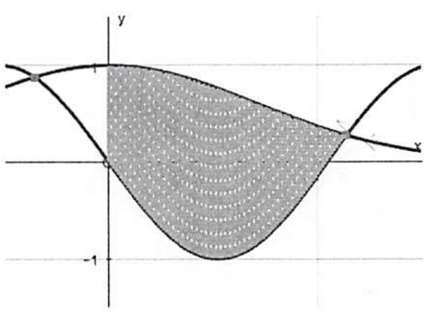
$$\frac{2}{3}y^{3/2}\Big]_0^c = \frac{1}{2}\cdot\frac{2}{3}y^{3/2}\Big]_0^4$$

$$\frac{2}{3}c^{3/2} = \frac{1}{3}4^{3/2}$$

$$c^{3/2} = 4$$

$$\boxed{c = 4^{2/3}}$$

8. [Calculator] Let R be the shaded region enclosed by the graphs of $y = e^{-x^2}$, $y = -\sin(3x)$, and the y-axis as shown in the figure below. Find the area of R.



intersection: $(\overset{x_I}{1.14}\,;\,0.27)$

$$R = \int_0^{x_I}\left(e^{-x^2} + \sin(3x)\right)dx$$

$\simeq 1.45$

T (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

[Calculator] A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon ($t = 0$) and 8 P.M. ($t = 8$). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

$$E'(6) \approx \frac{E(7)-E(5)}{7-5} = \frac{21-13}{2} = 4$$ approx 400 entries per hour.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

$$\frac{1}{8}\int_0^8 E(t)\,dt = \left[\frac{1}{2}(0+4)\times 2 + \frac{1}{2}(4+13)\times 3 + \frac{1}{2}(13+21)\times 2 + \frac{1}{2}(21+23)\times 1\right]\times\frac{1}{8} \approx 10.6875$$

↑ hundred of entries over the 8 hours

It's the average number of entries over the 8 hours.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight ($t = 12$) ?

$$E(8) - \int_8^{12}(t^3 - 30t^2 + 298t - 976)\,dt = 23 - \left[\frac{t^4}{4} - 10t^3 + 149t^2 - 976t\right]_8^{12} = 7$$

700 entries had yet to be processed at midnight.

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

$P'(t) = 3t^2 - 60t + 298$ $\quad \Delta = 24 \quad t = \frac{60 \pm \sqrt{24}}{6} = \frac{30 \pm \sqrt{6}}{3}$

t	8		$\frac{30-\sqrt{6}}{3}$		$\frac{30+\sqrt{6}}{3}$		12
$P'(t)$		+	0	−	0	+	
$P(t)$	0	↗		↘		↗	8

$P\left(\frac{30-\sqrt{6}}{3}\right) = 4 + \frac{4\sqrt{6}}{9} \approx 5.1$

P_{max} = 8 hundred entries per hour (at midnight)

AP Calculus
7.3 Worksheet (Day 1)

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. [No Calculator] The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x – axis are squares, then its volume is

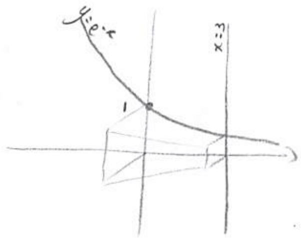
A) $\dfrac{1-e^{-6}}{2}$

B) $\dfrac{1}{2}e^{-6}$

C) e^{-6}

D) e^{-3}

E) $1-e^{-3}$



$$V = \int_0^3 (e^{-x})^2\,dx$$

$$= \int_0^3 e^{-2x}\,dx$$

$$= -\frac{1}{2}\int_0^{-6} e^u\,du$$

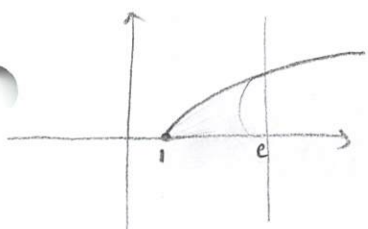
$u = -2x$
$du = -2dx$

$$= -\frac{1}{2}\left[e^u\right]_0^{-6}$$

$$= -\frac{1}{2}(e^{-6} - e^0)$$

$$= \frac{1-e^{-6}}{2}$$

2. [Calculator] The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x – axis are semicircles, then the volume of S is

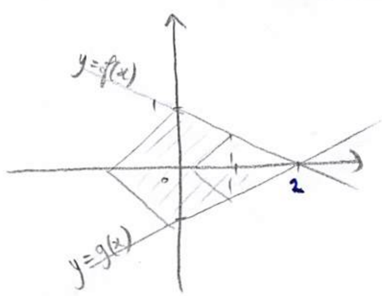


$$V = \int_1^e \frac{1}{2}\pi\left(\frac{\sqrt{\ln x}}{2}\right)^2 dx$$

$$= \frac{\pi}{8}\underbrace{\int_1^e \ln x\,dx}_{=1\ (\text{calc})}$$

$$V = \frac{\pi}{8}$$

3. The base of a solid is the region bounded by the lines $f(x) = 1 - \frac{x}{2}$, $g(x) = -1 + \frac{x}{2}$, and $x = 0$. If the cross sections perpendicular to the x-axis are equilateral triangles, find the volume of the solid.



$$V = \frac{1}{2}\int_0^2 (2-x)\sqrt{\tfrac{3}{4}x^2 - 3x + 3}\,dx$$

$$= -\frac{1}{3}\int_3^0 \sqrt{u}\,du$$

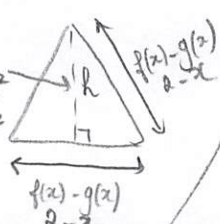
$$= -\frac{1}{3}\cdot\frac{2}{3}\left[u^{3/2}\right]_3^0 = \frac{2}{9}\cdot 3^{3/2}$$

$$= \frac{2}{\sqrt{3}} = \boxed{\frac{2}{3}\sqrt{3}}$$

$u = \frac{3}{4}x^2 - 3x + 3$
$du = \left(\frac{3}{2}x - 3\right)dx$
$du = \frac{3}{2}(x-2)\,dx$
$\frac{2}{3}du = (x-2)\,dx$

Cross section:

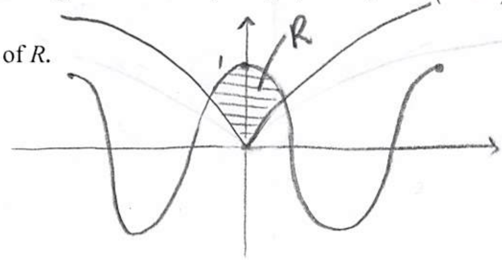
Pyth.
$(2-x)^2 = (1-\frac{x}{2})^2 + h^2$
$4 - 4x + x^2 = 1 - x + \frac{x^2}{4} + h^2$
$h^2 = \frac{3}{4}x^2 - 3x + 3$



$f(x) - g(x) = 1 - \frac{x}{2} - (-1 + \frac{x}{2}) = 2 - x$

4. [Calculator] Let R be the region enclosed by the graphs of $y = \ln(x^2+1)$ and $y = \cos x$. $\ln(4\pi^2+1) \simeq 3.7$

a) Find the area of R.



$\ln(x^2+1) = \cos x$

$x_I \simeq 0.916$

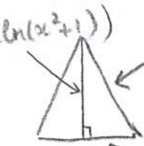
$$R = 2\int_0^{x_I} \left(\cos x - \ln(x^2+1)\right)dx$$

$$R \simeq 1.168$$

b) The base of a solid is the region R. Each cross section of the solid perpendicular to the x-axis is an equilateral triangle. Write an expression involving one or more integrals that gives the volume of the solid. *Do not evaluate.*

$$V = 2\int_0^{x_I} \frac{\sqrt{3}}{4}\left(\cos x - \ln(x^2+1)\right)^2 dx$$

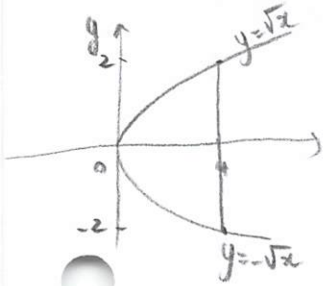
$\frac{\sqrt{3}}{2}(\cos x - \ln(x^2+1))$



$\cos x - \ln(x^2+1)$

$\frac{1}{2}(\cos x - \ln(x^2+1))$

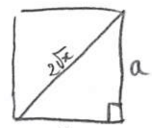
5. The solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 4$. The cross sections perpendicular to the axis on the interval $0 \le x \le 4$ are squares whose diagonals run from $y = \sqrt{x}$ to $y = -\sqrt{x}$. Find the volume of the solid.



$$V = \int_0^4 2x\,dx$$

$$= x^2\Big]_0^4$$

$$\boxed{V = 16}$$

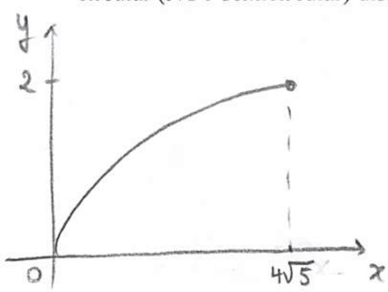


$2a^2 = 4x$

$a^2 = 2x$

$(a = \sqrt{2x})$

6. The solid lies between planes perpendicular to the y-axis at $y = 0$ and $y = 2$. The cross sections perpendicular to the y-axis are circular (NOT semicircular) disks with diameters running from the y-axis to the parabola $x = \sqrt{5}y^2$. Find the volume of the solid.



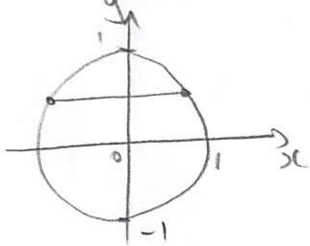
$$V = \pi\int_0^2 \left(\frac{\sqrt{5}}{2}y^2\right)^2 dy$$

$$\boxed{V = 8\pi}$$

$$= \frac{5\pi}{4}\int_0^2 y^4\,dy$$

$$= \frac{5\pi}{4}\cdot\frac{y^5}{5}\Big]_0^2$$

7. The base of the solid is the disk $x^2 + y^2 \le 1$. The cross sections are perpendicular to the y-axis between $y = 1$ and $y = -1$ are isosceles right triangles with one leg in the disk. Find the volume of the solid.

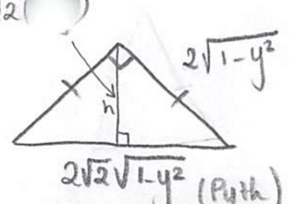


$$V = \int_{-1}^{1} 2(1-y^2)\,dy$$

$$= 2y - \frac{2}{3}y^3\Big]_{-1}^{1}$$

$$= 2 - \frac{2}{3} + 2 - \frac{2}{3}$$

$\sqrt{2}($

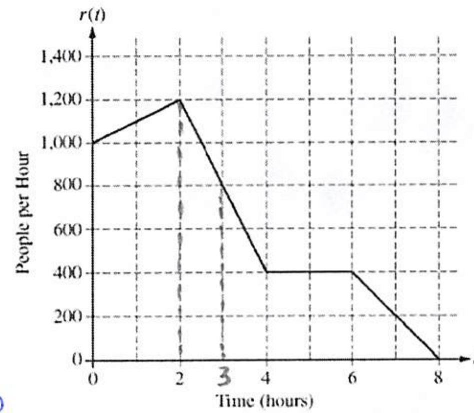


$h^2 = 4(1-y^2) - 2(1-y^2)$

$$\boxed{V = \frac{8}{3}}$$

A little AP Preparation ....

[No Calculator] There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.



a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

$$\int_0^3 r(t)\,dt = \int_0^2 r(t)\,dt + \int_2^3 r(t)\,dt$$

$$= \frac{1}{2}\times 2(1000 + 1200) + \frac{1}{2}(1200 + 800)$$

$= 3200$ people

b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

between $t = 2$ and $t = 3$, $r(t) - 800 \geq 0$

Therefore, the number of people waiting in line is increasing.

c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

The number of people in line is increasing as long as $r(t) \geq 800$.

Therefore, the line is the longest at $t = 3$.

$$\int_0^3 (r(t) - 800)\,dt + 700 = 3200 - 2400 + 700 = 1500 \text{ people}.$$

d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

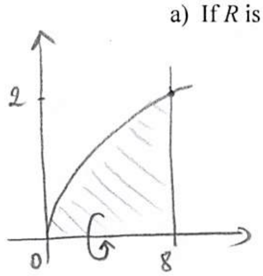
$$\int_0^t (r(x) - 800)\,dx + 700 = 0$$

AP Calculus
7.3 Worksheet (Day 2)

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Suppose region R is in the first quadrant bounded by the graphs of $y = \sqrt[3]{x}$ and $x = 8$.

   a) If R is rotated about the x-axis find the resulting volume.



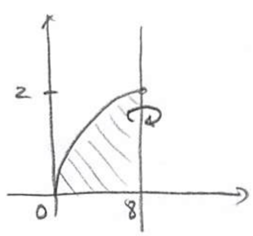
$$V = \pi \int_0^8 \sqrt[3]{x}^{\,2}\, dx$$

$$= \pi \cdot \frac{3}{5} \cdot x^{5/3} \Big]_0^8$$

$$= \frac{3\pi}{5} \times 32$$

$$\boxed{V = \frac{96\pi}{5}}$$

   b) Find the volume if R is rotated around the line $x = 8$.



$$V = \pi \int_0^2 (8 - y^3)^2\, dy$$

$y = \sqrt[3]{x}$

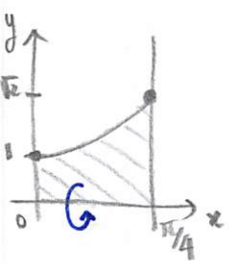
$y^3 = x$

$$= \pi \int_0^2 (y^6 - 16y^3 + 64)\, dy$$

$$= \pi \left[\frac{1}{7}y^7 - 4y^4 + 64y\right]_0^2 = \boxed{\frac{576}{7}\pi}$$

2. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

A) $\frac{\pi^2}{4}$ B) $\pi - 1$ C) π D) 2π E) $\frac{8\pi}{3}$



$$V = \pi \int_0^{\pi/4} \sec^2 x \, dx$$

$$= \pi \tan x \Big]_0^{\pi/4}$$

$$V = \pi$$

3. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis is

A) 2π B) 4π C) 6π D) 9π E) 12π

$$y^2 = -\frac{1}{9}x^2 + 1$$

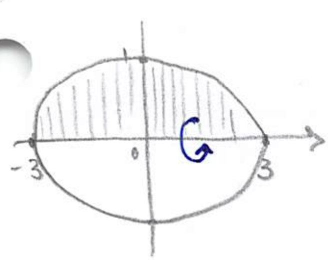
$$y = \pm\sqrt{1 - \frac{x^2}{9}}$$

$$V = \pi \int_{-3}^{3} \left(1 - \frac{x^2}{9}\right) dx \quad \text{or} \quad V = 2\pi \int_0^3 \left(1 - \frac{x^2}{9}\right) dx$$

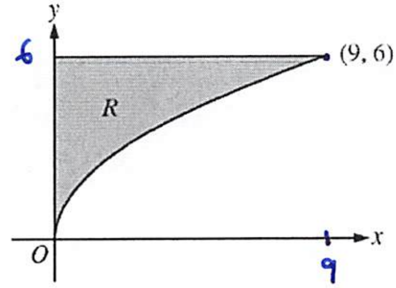
$$= 2\pi \left[x - \frac{x^3}{27}\right]_0^3$$

$$= 2\pi \times 2$$

$$= 4\pi$$



4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.



a) Find the area of R.

$$A = \int_0^9 (6 - 2\sqrt{x})\,dx$$

$$= \left[6x - \frac{4}{3}x^{3/2}\right]_0^9$$

$$A = 54 - \frac{4}{3} \times 27$$

$$\boxed{A = 18}$$

b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

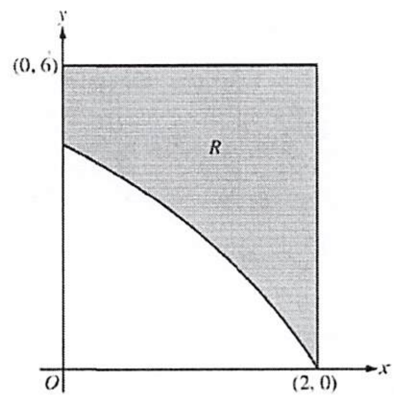
$$V = \pi \int_0^9 (6 - 2\sqrt{x})^2\,dx$$

c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$$x = \left(\frac{y}{2}\right)^2$$

$$V = 3\int_0^6 \left(\frac{y}{2}\right)^4 dy$$

5. [Calculator] In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.



a) Find the area of R.

$$A = \int_0^2 (6 - 4\ln(3-x))\,dx \simeq 6.817$$

b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 6$.

$$V = \pi \int_0^2 (6 - 4\ln(3-x))^2\,dx \simeq 82.519$$

c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

$$V = \int_0^2 (6 - 4\ln(3-x))^2\,dx \simeq 26.267$$

[Calculator] The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

a) How many people are in the auditorium when the concert begins?

$$\int_0^2 R(t)\,dt = \left[\frac{1380}{3}t^3 - \frac{675}{4}t^4\right]_0^2$$

$$= 980 \text{ people.}$$

b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

$$R'(t) = 2760t - 2025t^2$$

$$= 15t(184 - 135t)$$

t	0		184/135		2
$R'(t)$	0	+	0	−	
$R(t)$	0	↗		↘	

when $t = \frac{184}{135}$, $R'(t)$ changes sign from positive to negative. Therefore, $R(t)$ is maximum.

c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t. The derivative of w is given by $w'(t) = (2-t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

$$w(2) - w(1) = \int_1^2 w'(t)\,dt$$

$$= \int_1^2 (2-t)(1380t^2 - 675t^3)\,dt$$

$$w(2) - w(1) = \int_1^2 (675t^4 - 2730t^3 + 2760t^2)\,dt$$

$$= \left[135t^5 - \frac{1365}{2}t^4 + 920t^3\right]_1^2$$

$$= 387.5 \text{ hours}$$

d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part *c*.

$$\frac{w(2) - w(0)}{980} = \frac{1}{980}\int_0^2 (675t^4 - 2730t^3 + 2760t^2)\,dt$$

$$= \frac{1}{980}\left[135t^5 - \frac{1365}{2}t^4 + 920t^3\right]_0^2$$

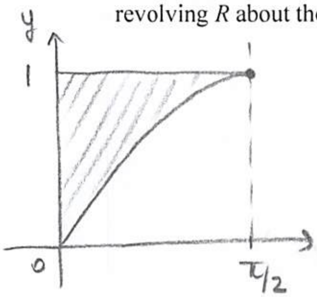
$$= \frac{38}{49}$$

average wait time: 0.776 h

AP Calculus
7.3 Worksheet (day 3)

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Let R be the region between the graphs of $y = 1$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. What is the volume of the solid obtained by revolving R about the x-axis?



$$V = \pi \int_0^{\pi/2} (1 - \sin^2 x)\,dx$$

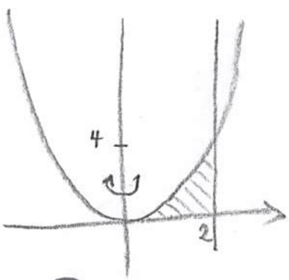
$$= \pi \int_0^{\pi/2} \cos^2 x\,dx$$

$$= \pi \int_0^{\pi/2} \frac{\cos 2x + 1}{2}\,dx$$

$$V = \frac{\pi}{2}\left[\frac{\sin 2x}{2} + x\right]_0^{\pi/2}$$

$$\boxed{V = \frac{\pi^2}{4}}$$

2. The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. What is the volume of the solid generated?



$x = \sqrt{y}$ in quadrant I

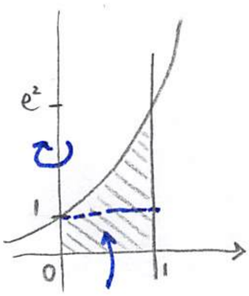
$$V = \pi \int_0^4 (4 - y)\,dy$$

$$= \pi \left[4y - \frac{y^2}{2}\right]_0^4$$

$$= 16\pi - 8\pi$$

$$\boxed{V = 8\pi}$$

3. [Calculator] A region in the first quadrant is enclosed by the graphs of $y = e^{2x}$, $x = 1$, and the coordinate axes. If the region is rotated about the y-axis, what is the volume of the solid generated?



$y = e^{2x}$ $(y > 0)$

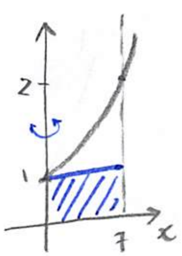
$\ln y = 2x$

$x = \frac{1}{2}\ln y$

$$V = \pi\left(1 + \int_1^{e^2} \left(1 - \left(\tfrac{1}{2}\ln y\right)^2\right)dy\right)$$

$$V \approx 13.178$$

4. [Calculator] Let R be the region in the first quadrant enclosed by the graph of $y = (x+1)^{1/3}$, the line $x = 7$, the x-axis, and the y-axis. What is the volume of the solid generated when R is revolved about the y-axis?



$y^3 = x + 1$

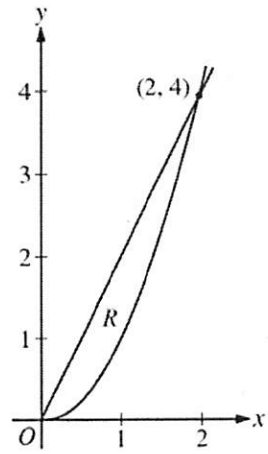
$x = y^3 - 1$

$$V = \pi\left(\int_0^1 7^2\,dy + \int_1^2 \left(7^2 - (y^3 - 1)^2\right)dy\right)$$

$$V \approx 271.299$$

$\frac{1209\pi}{14}$

5. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown below.



a) Find the area of R.

$$R = \int_0^2 (2x - x^2)\,dx$$
$$= x^2 - \frac{1}{3}x^3 \Big]_0^2 = \boxed{\frac{4}{3}}$$

b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

$$V = \int_0^2 A(x)\,dx$$
$$= \int_0^2 \sin\left(\frac{\pi}{2}x\right)dx$$

$$V = -\cos\left(\frac{\pi}{2}x\right)\cdot\frac{2}{\pi}\Big]_0^2$$
$$= \frac{2}{\pi} + \frac{2}{\pi}$$
$$\boxed{V = \frac{4}{\pi}}$$

c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

$y = 2x \Leftrightarrow x = \frac{1}{2}y$ $\quad y = x^2 \Leftrightarrow x = \sqrt{y}$ because $x \geq 0$

$$V = \int_0^4 \left(\sqrt{y} - \frac{1}{2}y\right)^2 dy$$

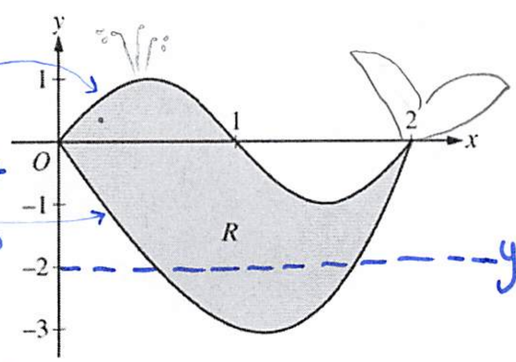
d) Write but do not evaluate, the integral which gives the volume of the solid formed by rotating R around the line $y = 5$.

$$V = \pi\int_0^2 \left((5 - x^2)^2 - (5 - 2x)^2\right)dx$$

e) Write but do not evaluate, the integral which gives the volume of the solid formed by rotating R around the line $x = -1$.

$$V = \pi\int_0^4 \left((\sqrt{y} + 1)^2 - \left(\frac{1}{2}y + 1\right)^2\right)dy$$

6. [Calculator] Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.



a) Find the area of R.

$$R = \int_0^2 \left(\sin(\pi x) - (x^3 - 4x)\right)dx = -\frac{\cos \pi x}{\pi} - \frac{x^4}{4} + 2x^2\Big]_0^2$$
$$= -\frac{1}{\pi} - 4 + 8 - \left(-\frac{1}{\pi}\right) = \boxed{4}$$

b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

Intersections : $x^3 - 4x = -2$
$x^3 - 4x + 2 = 0$
$x \simeq 1.675 \quad x \simeq 0.539$

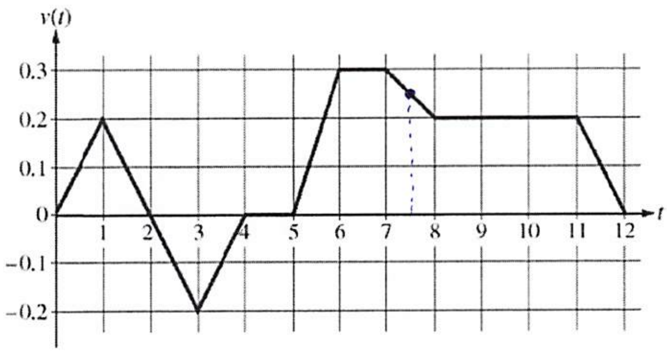
$$\int_{0.539}^{1.675} (-2 - x^3 + 4x)\,dx$$

c) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

$$V = \int_0^2 (\sin \pi x - x^3 + 4x)(3 - x)\,dx$$

$$\simeq 8.370$$

7. [Calculator] Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity, $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown at the right.



a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

$a(7.5) = v'(7.5) = -\frac{0.1}{1}$

$a(7.5) = -0.1 \text{ mi/min}^2$

b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\,dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\,dt$.

$\int_0^{12} |v(t)|\,dt$ is the total distance travelled between Caren's home to school in miles.

$\int_0^{12} |v(t)|\,dt = \frac{1}{2} \times 0.2 \times 2 + \frac{1}{2} \times 0.2 \times 2 + \frac{1}{2} \times 0.3 \times 1 + 0.3 \times 1 + \frac{1}{2}(0.2+0.3) + 0.2 \times 3 + \frac{1}{2}(0.2)$

$= 1.8$ mi

c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

That's not the question :

$\int_0^t v(x)\,dx$ is the displacement between for the t first minutes.

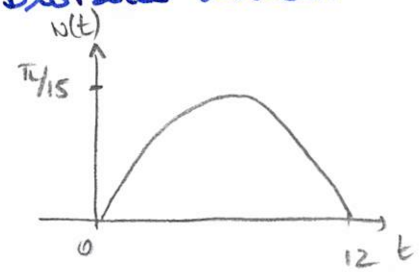
By symmetry, $\int_0^4 v(t) = 0$ therefore: she's back at home after 4 min.

→ She turned around at $t = 2$ (when velocity becomes negative)

d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

Distance between Caren's home and school: $\int_4^{12} |v(t)|\,dt = 1.4$ mi

Distance between Larry's home and school: $\int_0^{12} |w(t)|\,dt = \frac{\pi}{15}\int_0^{12} \sin\left(\frac{\pi}{12}t\right) dt$



$= \frac{\pi}{15}\left(-\cos\frac{\pi}{12}t\right) \cdot \frac{12}{\pi}\Big]_0^{12}$

$= \frac{4}{5} + \frac{4}{5}$

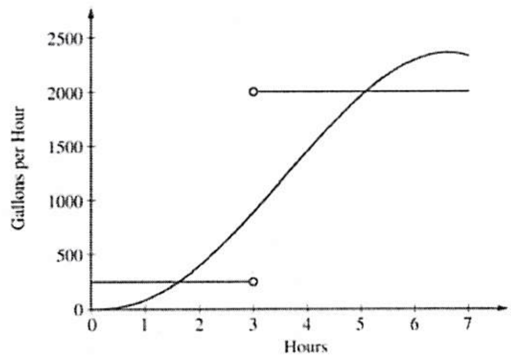
$= \frac{8}{5}$

$= 1.6$ mi

Caren lives closer to school than Larry.

A little AP Preparation ...

[Calculator] The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:



i) The rate at which water enters the tank is
$f(t) = 100t^2 \sin\left(\sqrt{t}\right)$ gallons per hour for $0 \le t \le 7$.

ii) The rate at which water leaves the tank is
$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$

The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure to the right. At time $t = 0$, the amount of water in the tank is 5000 gallons.

a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

$100\int_0^7 t^2 \sin(\sqrt{t})\,dt \simeq 8264$ gal.

b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for your answer.

The amount of water in the tank is decreasing when $g(t) > f(t)$.
(i.e when the graph of g is above the graph of f)

$\Rightarrow$ when $t \in [0, 1.617]$ and $t \in [3, 5.076]$

c) For $0 \le t \le 7$, at what time t is the amount of water in the tank the greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

	0	1.617	3	5.076	7
Amount	5000 ↘	↗	↘	↗	

$A(t) = 5000 + \int_0^t (f(x) - g(x))\,dx$

when the derivative $(f-g)$ goes from positive to negative

$A_{max} = \underbrace{A(0) \text{ or } A(3) \text{ or } A(7)}$

$A(3) = 5000 + \int_0^3 (100x^2 \sin\sqrt{x} - 250)\,dx \simeq 5127$

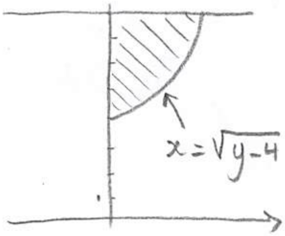
$\boxed{A_{max} \simeq 5127 \text{ gal}}$

$A(7) = 5000 + \int_0^3 (100x^2 \sin\sqrt{x} - 250)\,dx + \int_3^7 (100x^2 \sin\sqrt{x} - 2000)\,dx \simeq 4514$

AP Calculus
7.3 Worksheet (day 4)

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Let R be the region bounded by the graphs of $y = x^2 + 4$, $y = 8$, and $x = 0$, set up and evaluate the integral that gives the volume of the solid generated by revolving R about the y – axis.

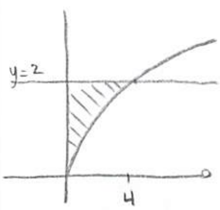


$$V = \pi \int_4^8 (y-4)\,dy$$

$$= \pi\left[\frac{y^2}{2} - 4y\right]_4^8 = \pi(32 - 32 - 8 + 16) \qquad \boxed{V = 8\pi}$$

2. Set an integral and use your calculator to find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about … (I would draw a different picture each time)

a) … th x-axis.



$$V = \pi \int_0^4 (4 - x)\,dx$$

b) … the y-axis.

$y = \sqrt{x} \quad x = y^2$

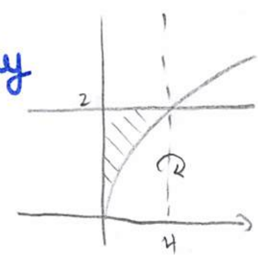
$$V = \pi \int_0^2 y^4\,dy$$

c) … the lin $y = 2$

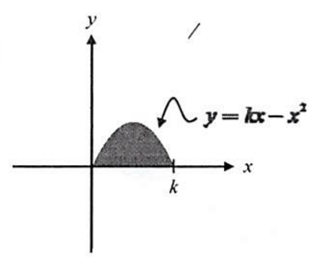
$$V = \pi \int_0^4 (2 - \sqrt{x})^2\,dx$$

d) … the li $x = 4$.

$$V = \pi \int_0^2 \left(4^2 - (4 - y^2)^2\right) dy$$



3. The shaded region R, shown in the figure below, is rotated about the y-axis to form a solid who volume is 10 cubic inches. Of the following, which best approximates k?



A) 1.51

B) 2.09

C) 2.49

D) 4.18

E) 4.77

vertex form: $y = -\left(x - \frac{k}{2}\right)^2 + \frac{k^2}{4}$

to solve for x

$$\left(x - \frac{k}{2}\right)^2 = \frac{k^2}{4} - y$$

$$x = \frac{k}{2} \pm \sqrt{\frac{k^2}{4} - y}$$

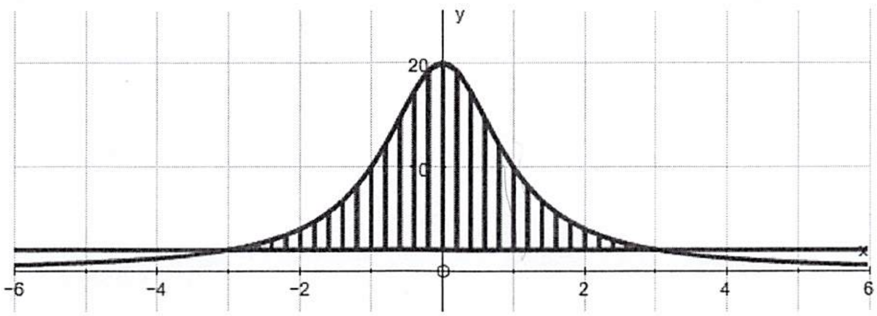
$$V = \pi \int_0^{k^2/4} \left(\left(\frac{k}{2} + \sqrt{\frac{k^2}{4} - y}\right)^2 - \left(\frac{k}{2} - \sqrt{\frac{k^2}{4} - y}\right)^2 \right) dy$$

expand

$$= \pi \int_0^{k^2/4} 2k\sqrt{\frac{k^2}{4} - y}\,dy = 2\pi k\left(\frac{k^2}{4} - y\right)^{3/2} \cdot \frac{2}{3}\Bigg]_0^{k^2/4} = \frac{2\pi}{12}k^4 = 10$$

i.e. $k = \sqrt[4]{\frac{60}{\pi}} \simeq 2.09$

4. [Calculator] Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.



a) Find the area of R.

Intersection: $\dfrac{20}{1+x^2} = 2$

$20 = 2(1+x^2)$

$x^2 = 9$

$x = \pm 3$

$$A = 2\int_0^3 \left(\frac{20}{1+x^2} - 2\right)dx = 40 \text{ Arctan} x - 4x\Big]_0^3 = \boxed{40\tan^{-1}3 - 12}$$

b) Find the volume of the solid generated when R is rotated about the x-axis.

$$V_1 = 2\pi\int_0^3 \left(\frac{400}{(1+x^2)^2} - 4\right)dx \simeq 1871.19$$

c) Find the volume of the solid generated when R is rotated about the line $y = 25$.

$$V_2 = 2\pi\int_0^3 \left(23^2 - \left(25 - \frac{20}{1+x^2}\right)^2\right)dx \simeq 4091.84$$

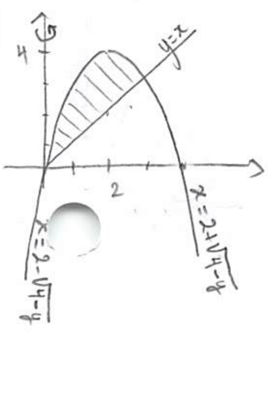
d) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

$$V_3 = 2 \times \frac{\pi}{2}\int_0^3 \frac{1}{4}\left(\frac{20}{1+x^2} - 2\right)^2 dx = \frac{\pi}{4}\int_0^3 \left(\frac{20}{1+x^2} - 2\right)^2 dx \simeq 174.268$$

5. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x$ and $y = 4x - x^2$ about the y-axis.

$y = -(x-2)^2 + 4$

$x = 2 \pm \sqrt{4-y}$



$$V = \pi\int_0^3 \left(y^2 - (2-\sqrt{4-y})^2\right)dy + \pi\int_3^4 \left((2+\sqrt{4-y})^2 - (2-\sqrt{4-y})^2\right)dy$$

$$= \pi\int_0^3 \left(y^2 - 4 + 4\sqrt{4-y} - 4 + y\right)dy + \pi\int_3^4 8\sqrt{4-y}\,dy$$

$$= \pi\left[\frac{y^3}{3} + \frac{y^2}{2} - 8y - \frac{8}{3}(4-y)^{3/2}\right]_0^3 + \pi\left[-\frac{16}{3}(4-y)^{3/2}\right]_3^4 = \boxed{\frac{27\pi}{2}}$$

Intersections:

$x = 4x - x^2$

$0 = 3x - x^2$

$-x(x-3) = 0$

$x = 0 \quad x = 3$

$(0,0)$ & $(3,3)$

A little AP Preparation ...

[Calculator] The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

i) The rate at which water enters the tank is
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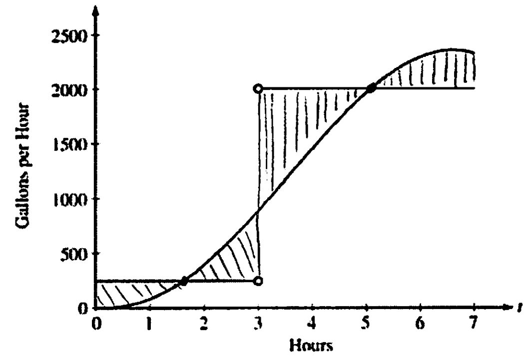
$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases}$ gallons per hour.



The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure to the right. At time $t = 0$, the amount of water in the tank is 5000 gallons.

a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

$$E = \int_0^7 100t^2 \sin\sqrt{t}\,dt$$

≈ 8264 gallons.

b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for your answer.

The amount of water is decreasing when $f(t) < g(t)$.

i.e when $t \in [0, 1.617]$ and when $t \in (3, 5.076]$

c) For $0 \le t \le 7$, at what time t is the amount of water in the tank the greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

t	0		1.617		3		5.076		7
$f(t)-g(t)$		$-$	0	$+$	‖	$-$	0	$+$	
amount of water	5000	↘		↗		↘		↗	

The amount of water in the tank could be the greatest at $t=0$, $t=3$ or $t=7$ (local max because the rates change from positive to negative)

$$W(3) = 5000 + \int_0^3 (100t^2 \sin\sqrt{t} - 250)\,dt \approx 5127 \text{ gallons}$$

$$W(7) = W(3) + \int_3^7 (100t^2 \sin\sqrt{t} - 2000)\,dt \simeq 4514 \text{ gallons}$$

The amount of water in the tank is the greatest when $t = 3$ and $W(3) \simeq 5127$ gal.