Solutions

Derivative of an inverse function – Extra Practice

The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

(E) ✓ $g(8) = 3$ and $g'(8) = \frac{1}{9}$

x	$f(x)$	$g(x)$	$f'(x)$
−4	0	−9	5
−2	4	−7	4
0	6	−4	2
2	7	−3	1
4	10	−2	3

The table above gives values of the differentiable functions f and g, and f', the derivative of f, at selected values of x. If $g(x) = f^{-1}(x)$, what is the value of $g'(4)$?

(A) $-\frac{1}{3}$

(B) $-\frac{1}{4}$

(C) $-\frac{3}{100}$

(D) ✓ $\frac{1}{4}$

(E) $\frac{1}{3}$

Let f be the function defined by $f(x) = x^3 + x^2 + x$. Let $g(x) = f^{-1}(x)$, where g(3) = 1. What is the value of $g'(3)$?

A 1/39

B 1/34

C 1/6

D 1/3

E 39

Let f be the function defined by $f(x) = x^3 + x^2 + x$. Let $g(x) = f^{-1}(x)$, where g(3) = 1. What is the value of $g'(3)$?

A 1/39

B 1/34

C 1/6

D 1/3

E 39

Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(-2) = 5$ and f'(5)=-1/2, then $g'(-2) =$

A 2

B 1/2

C 1/5

D $-\frac{1}{5}$

E -2

Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

A $\frac{1}{13}$

B $\frac{1}{4}$

C $\frac{7}{4}$

D 4

E 13

Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

A $-\frac{1}{2}$

$$g'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(6)} = \frac{1}{-2}$$

B $-\frac{1}{8}$

C $\frac{1}{6}$

D $\frac{1}{3}$

E The value of $g'(3)$ cannot be determined from the information given.

An increasing function f satisfies $f(10) = 5$ and $f'(10) = 8$. Which of the following statements about the inverse of f must be true?

A $(f^{-1})'(5) = 10$

$$(f^{-1})'(5) = \frac{1}{f'(10)} = \frac{1}{8}$$

B $(f^{-1})'(8) = 10$

C $(f^{-1})'(5) = 8$

D $(f^{-1})'(5) = \frac{1}{8}$

Let f and g be inverse functions that are differentiable for all x. If $f(3) = -2$ and $g'(-2) = -4$, which of the following statements must be false?

I. $f'(0) = \frac{1}{4}$
II. $f'(3) = -\frac{1}{4}$
III. $f'(5) = -\frac{1}{4}$

$f'(x) = \frac{1}{g'(f(x))} < 0$

A) I only

$f'(3) = \frac{1}{g'(f(3))} = \frac{1}{g'(-2)} = -\frac{1}{4}$

B) II only

C) III only

III could be true ex: $f(x) = \frac{1}{4}(x-3)-2$, $g(x) = -4(x+2)+3$ inverses

D) I and III only

$f'(5) = -\frac{1}{4}$

Let f and g be inverse functions that are differentiable for all x. If $f(-5) = 7$ and $g'(7) = 3$, which of the following statements must be false?

I. $f'(3) = -\frac{1}{3}$
II. $f'(-5) = \frac{1}{3}$
III. $f'(7) = \frac{1}{3}$

f' should be positive

A) I only

III could be true

B) II only

ex: $f(x) = \frac{1}{3}(x+5)+7$, $g(x) = 3(x-7)-5$ inverses

C) III only

D) I and III only

$f'(7) = \frac{1}{3}$

For which of the following decreasing functions f does $(f^{-1})'(10) = -\frac{1}{8}$?

A) $f(x) = -5x + 15$

$(f^{-1})'(10) = \frac{1}{f'(1)} = -\frac{1}{8}$

B) $f(x) = -2x^3 - 2x + 14$

C) $f(x) = -x^5 - 4x + 15$

D) $f(x) = e^{-2x} - x + 9$

A decreasing function g satisfies $g(4) = 6$ and $g'(4) = -2$. Which of the following statements about the inverse of g must be true?

A) $(g^{-1})'(6) = 4$

B) $(g^{-1})'(-2) = 4$

C) $(g^{-1})'(6) = -2$

D) $(g^{-1})'(6) = -\frac{1}{2}$

$$\frac{d}{dx} g(g^{-1}(x)) = g'(g^{-1}(x))(g^{-1})'(x)$$
$$= \frac{d}{dx} x = 1$$
$$\Rightarrow (g^{-1})'(x) = \frac{1}{g'(g^{-1}(x))}$$
$$(g^{-1})'(6) = \frac{1}{g'(g^{-1}(6))} = \frac{1}{g'(4)} = -\frac{1}{2}$$

Let $f(x) = (2x+1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

A) $-\frac{2}{27}$

B) $\frac{1}{54}$

C) $\frac{1}{27}$

D) $\frac{1}{6}$

E) 6

Let f be the function defined by $f(x) = 2x + e^x$. If $g(x) = f^{-1}(x)$ for all x and the point (0,1) is on the graph of f, what is the value of $g'(1)$?

A) $\frac{1}{2+e}$

B) $\frac{1}{3}$

C) $\frac{1}{2}$

D) 3

E) $2 + e$

x	$f(x)$	$f'(x)$
0	49	0
1	2	–8
2	–1	–80

The table above gives selected values for a differentiable and decreasing function *f* and its derivative. If f^{-1} is the inverse function of *f*, what is the value of $(f^{-1})'(2)$?

(A) -80

(B) $-\frac{1}{8}$

(C) $-\frac{1}{80}$

(D) $\frac{1}{80}$

(E) $\frac{1}{8}$

x	$f(x)$	$f'(x)$
0	1	1
1	3	4
2	11	13

The table above gives selected values for a differentiable and increasing function *f* and its derivative. If *g* is the inverse function of *f*, what is the value of g ' (3) ?

(A) $\frac{1}{13}$

(B) $\frac{1}{4}$

(C) 1

(D) 4

(E) 13

The function h is given by $h(x) = x^5 + 3x - 2$ and $h(1) = 2$. If h^{-1} is the inverse of h, what is the value of $(h^{-1})'(2)$?

A $\frac{1}{83}$

B $\frac{1}{8}$

C $\frac{1}{2}$

D 1

E 8

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	−4	4	2
4	−1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

$g(1) = 2$ so $g^{-1}(2) = 1$

$(g^{-1})'(2) = \frac{1}{g'(1)} = \frac{1}{5}$ $\boxed{y - 1 = \frac{1}{5}(x - 2)}$

x	$f(x)$	$f'(x)$
0	1	1
1	3	4
2	11	13

The table above gives selected values for a differentiable and increasing function f and its derivative. If g is the inverse function of f, what is the value of $g'(3)$?

A $\frac{1}{13}$

B $\frac{1}{4}$

C 1

D 4

E 13

$g'(3) = \frac{1}{f'(1)} = \frac{1}{4}$