Additional Practice – Local min or max with piecewise functions

Identify the critical points and determine the local extreme values.

a) $y = x^{\frac{2}{3}}(x+2)$

$$\frac{dy}{dx} = \frac{2}{3}x^{-1/3}(x+2) + x^{2/3} = \frac{2(x+2)+3x}{3\sqrt[3]{x}} = \frac{5x+4}{3\sqrt[3]{x}}$$

x	$-\infty$		$-4/5$		0		$+\infty$
$\frac{dy}{dx}$		$+$	0	$-$	‖	$+$	
y		↗		↘		↗	

critical points at 0 and $-4/5$

- local max : $\left(-\frac{4}{5}, \frac{6}{5}\left(\frac{4}{5}\right)^{2/3}\right)$
- local min : $(0,0)$

b) $y = \begin{cases} 3-x, & x<0 \\ 3+2x-x^2, & x \geq 0 \end{cases}$

$$\left.\frac{dy}{dx}\right|_{x<0} = -1 \qquad \left.\frac{dy}{dx}\right|_{x>0} = -2x+2$$

not diff at 0 (but continuous at 0)

x	$-\infty$		0		1		$+\infty$
$\frac{dy}{dx}$		$-$	‖	$+$	0	$-$	
y		↘		↗		↘	

critical points at 0 and 1

- local max : $(1,4)$
- local min : $(0,3)$

c) $y = x^{\frac{2}{3}}(x^2-4)$

$$\frac{dy}{dx} = \frac{2}{3}x^{-1/3}(x^2-4) + 2x \cdot x^{2/3} = \frac{2(x^2-4)+6x^2}{3\sqrt[3]{x}} = \frac{8x^2-8}{3\sqrt[3]{x}} = \frac{8(x+1)(x-1)}{3\sqrt[3]{x}}$$

x	$-\infty$		-1		0		1		$+\infty$
$\frac{dy}{dx}$		$-$	0	$+$	‖	$-$	0	$+$	
y		↘		↗		↘		↗	

critical points at -1, 0 and 1

- local max : $(0,0)$
- local min : $(-1,-3)$ and $(1,-3)$

d) $y = \begin{cases} -x^2-2x+4, & x \leq 1 \\ -x^2+6x-4, & x>1 \end{cases}$

$$\left.\frac{dy}{dx}\right|_{x<1} = -2x-2 \qquad \left.\frac{dy}{dx}\right|_{x>1} = -2x+6$$

not diff at 1 but continuous at 1

x	$-\infty$		-1		1		3		$+\infty$
$\frac{dy}{dx}$		$+$	0	$-$	‖	$+$	0	$-$	
y		↗		↘		↗		↘	

critical points at -1, 1 and 3

- local max : $(-1,5)$ and $(3,5)$
- local min : $(1,1)$

e) $y = x\sqrt{4 - x^2}$ $D = [-2, 2]$ $\frac{dy}{dx} = \sqrt{4-x^2} + \frac{x(-2x)}{2\sqrt{4-x^2}} = \frac{2(2-x^2)}{\sqrt{4-x^2}}$

x	-2		$-\sqrt{2}$		$\sqrt{2}$		2
$\frac{dy}{dx}$	$\Vert$	$-$	0	$+$	0	$-$	$\Vert$
y		$\searrow$		$\nearrow$		$\searrow$	

critical points at $-2, -\sqrt{2}, \sqrt{2}$ and 2

- local min $(-\sqrt{2}, -2)$ and $(2, 0)$
- local max $(-2, 0)$ and $(\sqrt{2}, 2)$

f) $y = \begin{cases} 4 - 2x, & x \le 1 \\ x + 1, & x > 1 \end{cases}$ $\left.\frac{dy}{dx}\right|_{x<1} = -2$ $\left.\frac{dy}{dx}\right|_{x>1} = 1$ not diff at 1 but cont.

x	$-\infty$		1		$+\infty$
$\frac{dy}{dx}$		$-$	$\Vert$	$+$	
y		$\searrow$		$\nearrow$	

critical point at 1

- no local max
- local min $(1, 2)$

g) $y = x^2\sqrt{3 - x}$ $D = (-\infty, 3]$ $\frac{dy}{dx} = 2x\sqrt{3-x} - \frac{x^2}{2\sqrt{3-x}} = \frac{4x(3-x) - x^2}{2\sqrt{3-x}}$

$= \frac{-5x^2 + 12x}{2\sqrt{3-x}}$

$= \frac{-x(5x-12)}{2\sqrt{3-x}}$

x	$-\infty$		0		$\frac{12}{5}$		3
$\frac{dy}{dx}$		$-$	0	$+$	0	$-$	$\Vert$
y		$\searrow$		$\nearrow$		$\searrow$	

critical points at 0 and $\frac{12}{5}$

- local max: $\left(\frac{12}{5}, \frac{144}{25}\sqrt{\frac{3}{5}}\right)$
- local min: $(0, 0)$ and $(3, 0)$

h) $y = \begin{cases} -\frac{1}{4}x^2 - \frac{1}{2}x + \frac{15}{4}, & x \le 1 \\ x^3 - 6x^2 + 8x, & x > 1 \end{cases}$ $\left.\frac{dy}{dx}\right|_{x<1} = -\frac{1}{2}x - \frac{1}{2}$ $\left.\frac{dy}{dx}\right|_{x>1} = 3x^2 - 12x + 8$ (zeros: $2 \pm \frac{2}{3}\sqrt{3}$)

this function is differentiable (and continuous) at 1.

x	$-\infty$		-1		$2+\frac{2}{3}\sqrt{3}$		$+\infty$
$\frac{dy}{dx}$		$+$	0	$-$	0	$+$	
y		$\nearrow$		$\searrow$		$\nearrow$	

critical points at -1 and $2 + \frac{2\sqrt{3}}{3}$

- local max: $(-1, 4)$
- local min: $\left(2 + \frac{2}{3}\sqrt{3}, -\frac{16\sqrt{3}}{9}\right)$