Answer Key

1. D	23. A	45. A	67. C	89. E
2. C	24. A	46. D	68. E	90. C
3. A	25. E	47. E	69. D	91. D
4. D	26. E	48. E	70. D	92. B
5. A	27. E	49. D	71. E	93. B
6. E	28. D	50. C	72. C	94. B
7. C	29. D	51. D	73. C	95. C
8. A	30. E	52. C	74. D	96. E
9. D	31. D	53. D	75. C	97. B
10. C	32. C	54. A	76. A	98. D
11. D	33. A	55. E	77. D	99. A
12. A	34. C	56. B	78. E	100. A
13. D	35. E	57. D	79. B	101. B
14. C	36. D	58. D	80. B	102. D
15. B	37. B	59. C	81. C	103. A
16. C	38. C	60. A	82. B	104. D
17. C	39. C	61. D	83. B	105. C
18. C	40. A	62. D	84. C	106. D
19. A	41. D	63. E	85. C	107. B
20. B	42. C	64. E	86. A	108. D
21. D	43. D	65. C	87. C	
22. D	44. C	66. C	88. E	

Answers Explained

Part A

1. **(D)** If $f(x) = x\sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$ then,

$$\lim_{x\to 0} f(x) = 0 = f(0);$$

thus this function is continuous at 0. In (A), $\lim_{x\to 0}\sin\frac{1}{x}$ does not exist; in (B), f has a jump discontinuity; in (C), f has a removable discontinuity; and in (E), f has an infinite discontinuity.

2. **(C)** To find the y-intercept, let $x = 0$; $y = -1$.

3. **(A)** $\lim_{x\to 1^-}[x] - \lim_{x\to 1^-}|x| = 0 - 1 = -1$.

4. **(D)** The line $x + 3y + 3 = 0$ has slope $-\frac{1}{3}$; a line perpendicular to it has slope 3.

The slope of the tangent to $y = x^2 - 2x + 3$ at any point is the derivative $2x - 2$. Set $2x - 2$ equal to 3.

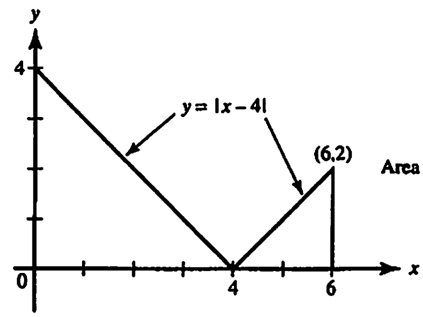
5. (A) $\lim_{x \to 1} \frac{\frac{3}{x} - 3}{x - 1}$ is $f'(1)$, where $f(x) = \frac{3}{x}$, $f'(x) = -\frac{3}{x^2}$. Or simplify the given fraction to $\frac{3-3x}{x(x-1)} = \frac{3(1-x)}{x(x-1)} = \frac{-3}{x}(x \neq 1)$.

6. (E) Because $p''(2) < 0$ and $p''(5) > 0$, p changes concavity somewhere in the interval [2,5], but we cannot be sure p'' changes sign at $x = 4$.

7. (C) $\int_0^6 |x-4|\,dx = \int_0^4 (4-x)\,dx + \int_4^6 (x-4)\,dx = \left(4x - \frac{x^2}{2}\right)\Big|_0^4 + \left(\frac{x^2}{2} - 4x\right)\Big|_4^6$

$$= 8 + [(18-24) - (8-16)] = 8 + (-6+8) = 10.$$

Save time by finding the area under $y = |x - 4|$ from a sketch!



Area $= \frac{1}{2}(4 \cdot 4) + \frac{1}{2}(2 \cdot 2) = 10.$

8. (A) Since the degrees of numerator and denominator are the same, the limit as $x \to \infty$ is the ratio of the coefficients of the terms of highest degree: $\frac{-2}{4}$.

9. (D) On the interval [1, 4], $f'(x) = 0$ only for $x = 3$. Since $f(3)$ is a relative minimum, check the endpoints to find that $f(4) = 6$ is the absolute maximum of the function.

10. (C) To find $\lim f$ as $x \to 5$ (if it exists), multiply f by $\frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}$.

$$f(x) = \frac{x-5}{(x-5)\left(\sqrt{x+4}+3\right)}$$

and if $x \neq 5$ this equals $\frac{1}{\sqrt{x+4}+3}$. So $\lim f(x)$ as $x \to 5$ is $\frac{1}{6}$. For f to be continuous at $x = 5$, $f(5)$ or c must also equal $\frac{1}{6}$.

11. (D) Evaluate $-\frac{1}{3}\cos^3 x \Big|_0^{\pi/2}$.

12. (A) $\cos x = \frac{1}{y}\frac{dy}{dx}$ and thus $\frac{dy}{dx} = y \cos x$. From the equation given, $y = e^{\sin x}$.

13. (D) If $f(x) = x \cos x$, then $f'(x) = -x \sin x + \cos x$, and

$$f'\left(\frac{\pi}{2}\right) = -\frac{\pi}{2} \cdot 1 + 0.$$

14. (C) If $y = e^x \ln x$, then $\frac{dy}{dx} = \frac{e^x}{x} + e^x \ln x$, which equals e when $x = 1$. Since also $y = 0$ when $x = 1$, the equation of the tangent is $y = e(x - 1)$.

15. (B) $v = 4(t - 2)^3$ and changes sign exactly once, when $t = 2$.

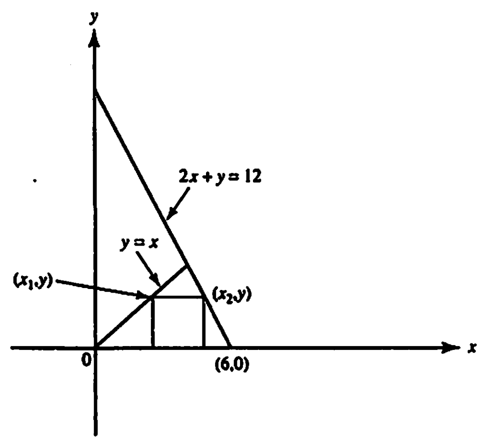
16. (C) Evaluate $-e^{-x}\Big|_{-1}^{0}$.

17. (C) $\int_{-1}^{4} f(x)\,dx = \int_{-1}^{2} x^2\,dx + \int_{2}^{4}(4x - x^2)\,dx = \frac{x^3}{3}\Big|_{-1}^{2} + \left(2x^2 - \frac{x^3}{3}\right)\Big|_{2}^{4}.$

18. (C) Since $v = 3t^2 + 3$, it is always positive, while $a = 6t$ and is positive for $t > 0$ but negative for $t < 0$. The speed therefore increases for $t > 0$ but decreases for $t < 0$.

19. (A) Note from the figure that the area, A, of a typical rectangle is

$$A = (x_2 - x_1) \cdot y = \left(\frac{12 - y}{2} - y\right) \cdot y = 6y - \frac{3y^2}{2}.$$



For $y = 2$, $\frac{dA}{dy} = 0$. Note that $\frac{d^2A}{dy^2}$ is always negative.

20. (B) If S represents the square of the distance from (3, 0) to a point (x, y) on the curve, then $S = (3 - x)^2 + y^2 = (3 - x)^2 + (x^2 - 1)$. Setting $\frac{dS}{dx} = 0$ yields the minimum distance at $x = \frac{3}{2}$.

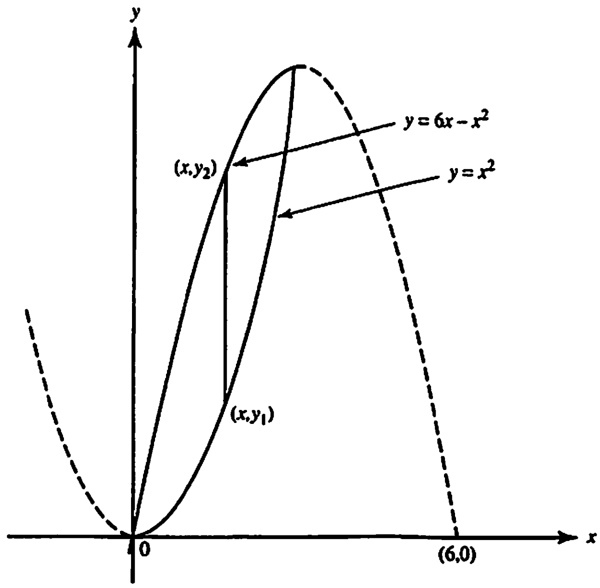
21. (D) $\frac{dy}{dx} = \frac{4}{4x+1} = 4(4x+1)^{-1}$, so $\frac{d^2y}{dx^2} = 4\left(-1(4x+1)^{-2} \cdot 4\right)$

22. (D) See the figure. Since the area, A, of the ring equals $\pi\,(y_2^{\,2} - y_1^{\,2})$,

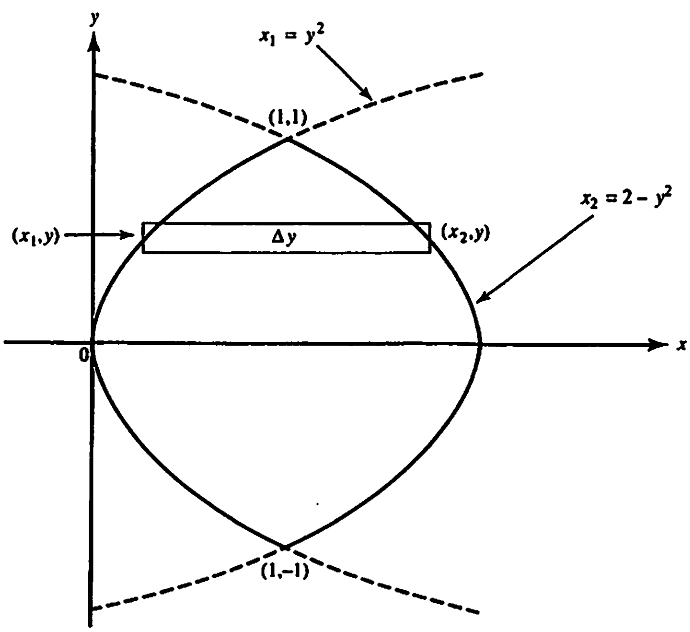
$$A = \pi\,[(6x - x^2)^2 - x^4] = \pi\,[36x^2 - 12x^3 + x^4 - x^4]$$

and $\frac{dA}{dx} = \pi\,(72x - 36x^2) = 36\pi x\,(2 - x)$.

It can be verified that $x = 2$ produces the maximum area.



23. (A) This is of type $\int \frac{du}{u}$ with $u = \ln x$: $\int \frac{\frac{1}{x}dx}{\ln x}$.



24. (A) About the y-axis; see the figure. Washer.

$$\Delta V = \pi (x_2^{\,2} - x_1^{\,2})\Delta y, \text{ so } V = 2\pi \int_0^1 \left[(2-y^2)^2 - y^4\right] dy = 2\pi \int_0^1 (4-4y^2)\, dy$$

25. (E) Separating variables, we get $y\,dy = (1 - 2x)\,dx$. Integrating gives

$$\frac{1}{2}y^2 = x - x^2 + C$$

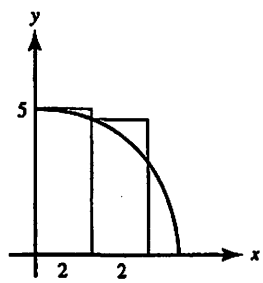
or

$$y^2 = 2x - 2x^2 + k$$

or

$$2x^2 + y^2 - 2x = k.$$

26. (E) $2(5) + 2\sqrt{21}$.



27. (E) $\frac{1}{\pi}\int_0^7 \sin \pi x\,(\pi\,dx) = \frac{-1}{\pi}\cos(\pi x)\Big|_0^7$.

28. (D) Use L'Hôpital's Rule or rewrite the expression as $\lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \frac{1}{\cos 3x} \cdot \frac{3}{2}$.

29. (D) For $f(x) = \tan x$, this is $f'\left(\frac{\pi}{4}\right) = \sec^2\left(\frac{\pi}{4}\right)$.

30. (E) The parameter k determines the amplitude of the sine curve. For $f = k \sin x$ and $g = e^x$ to have a common point of tangency, say at $x = q$, the curves must both go through (q, y) and their slopes must be equal at q. Thus, we must have

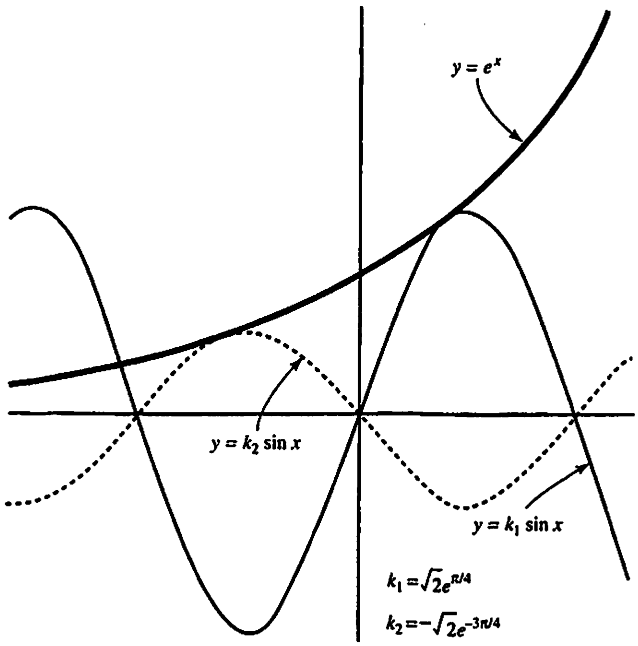
$$k \sin q = e^q \text{ and } k \cos q = e^q,$$

and therefore

$$\sin q = \cos q.$$

Thus, $q = \frac{\pi}{4} \pm n\pi$ and $k = \dfrac{e^q}{\sin\left(\frac{\pi}{4} \pm n\pi\right)}$.

The figure shows $k_1 = \sqrt{2}\, e^{\pi/4}$ and $k_2 = -\sqrt{2}\, e^{-3\pi/4}$.



31. (D) We differentiate implicitly to find the slope $\frac{dy}{dx}$:

$$2\left(x^2\frac{dy}{dx}+2xy\right)+2y\,\frac{dy}{dx}=2,$$

$$\frac{dy}{dx}=\frac{1-2xy}{x^2+y}.$$

At (3, 1), $\frac{dy}{dx}=-\frac{1}{2}$. The linearization is $y=-\frac{1}{2}(x-3)+1$.

32. (C) $\int\cos^3 x\,dx=\int\cos^2 x\cdot\cos x\,dx=\int\left(1-\sin^2 x\right)\cos x\,dx$

$$=\int\cos x\,dx-\int\sin^2 x\cdot\cos x\,dx=\sin x-\frac{\sin^3 x}{3}+C.$$

33. (A) About the x-axis. Disk.

$$\Delta V=\pi y^2\Delta x,$$

$$V=\pi\int_0^{\pi/4}\tan^2 x\,dx=\pi\int_0^{\pi/4}\left(\sec^2 x-1\right)dx=\pi\left[\tan x-x\right]_0^{\pi/4}$$

$$=\pi\left(1-\frac{\pi}{4}\right).$$

34. (C) Let $f(x)=a^x$; then $\lim_{h\to 0}\frac{a^h-1}{h}=\lim_{h\to 0}\frac{a^{0+h}-a^0}{h}=f'(0)=a^0\ln a=\ln a$.

35. (E) $\frac{dy}{dx}$ is a function of x alone; curves appear to be asymptotic to the y-axis and to increase more slowly as $|x|$ increases.

36. (D) The given limit is equivalent to

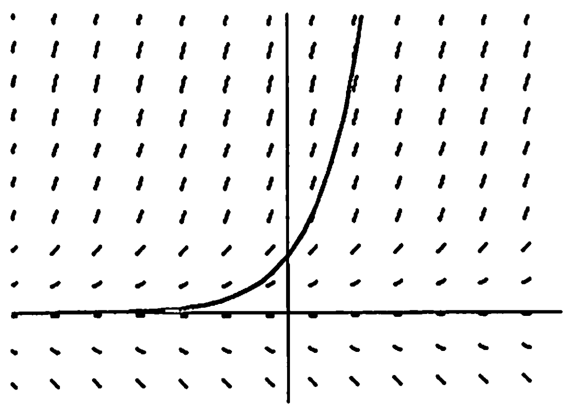
$$\lim_{h\to 0}\frac{F\left(\frac{\pi}{4}+h\right)-F\left(\frac{\pi}{4}\right)}{h}=F'\left(\frac{\pi}{4}\right),$$

where $F'(x)=\frac{\sin x}{x}$. The answer is $\frac{2\sqrt{2}}{\pi}$.

37. (B) $\int_0^{12} g(x)\,dx = \int_0^4 g(x)\,dx+\int_4^6 g(x)\,dx+\int_6^9 g(x)\,dx+\int_9^{12} g(x)\,dx$

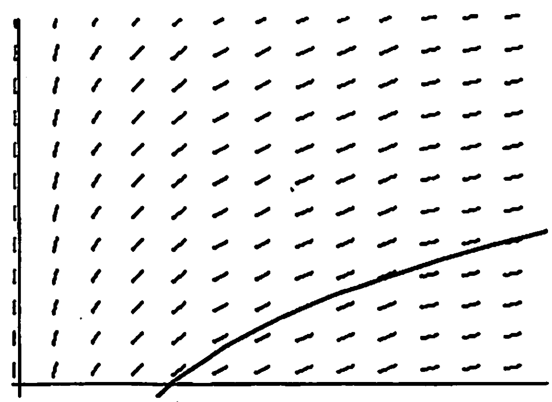
$$=4\pi - 3 - \frac{9\pi}{4}+\frac{15}{2}.$$

38. (C) In the figure, the curve for $y=e^x$ has been superimposed on the slope field.



39. (C) The general solution is $y = 3\ln|x^2 - 4| + C$. The differential equation $\frac{dy}{dx}=\frac{6x}{x^2-4}$ reveals that the derivative does not exist for $x = \pm 2$. The particular solution must be differentiable in an interval containing the initial value $x = -1$, so the domain is $-2 < x < 2$.

40. (A) The solution curve shown is $y = \ln x$, so the differential equation is $y' = \frac{1}{x}$.



41. (D) $\sqrt{1+\tan^2\theta} = \sec\theta$; $dx = \sec^2\theta$; $0 \leq x \leq 1$; so $0 \leq \theta \leq \frac{\pi}{4}$.

42. **(C)** The equations may be rewritten as $\frac{x}{2} = \sin u$ and $y = 1 - 2\sin^2 u$,

giving $y = 1 - 2 \cdot \frac{x^2}{2}$.

43. **(D)** Use the formula for area in polar coordinates,

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta;$$

then the required area is given by

$$4 \cdot \frac{1}{2}\int_0^{\pi/4} 2\cos 2\theta .$$

(See polar graph 63 in the Appendix.)

44. **(C)** $\int_{-\infty}^{\infty} \frac{dx}{x^2+1} = \lim_{b\to\infty} \tan^{-1} x \Big|_{-b}^{b} = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi.$

45. **(A)** The first three derivatives of $\frac{1}{1-2x}$ are $\frac{2}{(1-2x)^2}$, $\frac{8}{(1-2x)^3}$, and $\frac{48}{(1-2x)^4}$.

The first four terms of the Maclaurin series (about $x = 0$) are 1, $+2x$,

$+\frac{8x^2}{2!}$, and $+\frac{48x^3}{3!}$.

Note also that $\frac{1}{1-2x}$ represents the sum of an infinite geometric series with first term 1 and common ratio $2x$. Hence,

$$\frac{1}{1-2x} = 1 + 2x + (2x)^2 + (2x)^3 + \cdots$$

46. **(D)** We use parts, first letting $u = x^2$, $dv = e^{-x}dx$; then $du = 2x\,dx$, $v = -e^{-x}$ and

$$\int x^2 e^{-x} dx = -x^2 e^{-x} + 2\int x e^{-x} dx$$

Now we use parts again, letting $u = x$, $dv = e^{-x}dx$; then $du = dx$, $v = -e^{-x}$ and

$$-x^2e^{-x} + 2\int xe^{-x}dx = -x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}dx\right)$$

Alternatively, we could use the Tic-Tac-Toe Method (See page 226):

u		dv
x^2	+	e^{-x}
$2x$	−	$-e^{-x}$
2	+	e^{-x}
0		$-e^{-x}$

Then $\int x^2 e^{-x} dx = x^2(-e^{-x}) - (2x)e^{-x} + 2(-e^{-x}) + C$

47. **(E)** Use formula (20) in the Appendix to rewrite the integral as

$$\frac{1}{2}\int_0^{\pi/2}(1-\cos 2x)\,dx = \frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)\Big|_0^{\pi/2} = \frac{1}{2}\left(\frac{\pi}{2}\right).$$

48. **(E)** The area, A, is represented by $\int_0^{2\pi}(1-\cos t) = 2\pi$.

49. **(D)** $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2a\sin\theta\cos\theta}{-a\csc^2\theta} = -2\sin^3\theta\cos\theta.$

50. **(C)** Check to verify that each of the other improper integrals converges.

51. **(D)** Note that the integral is improper.

$$\lim_{k\to 4^-}\int_2^k \frac{du}{\sqrt{16-u^2}} = \lim_{k\to 4^-}\frac{1}{4}\int_2^k \frac{du}{\sqrt{1-\frac{u^2}{16}}} = \lim_{k\to 4^-}\frac{1}{4}\cdot 4\int_2^k \frac{\frac{1}{4}du}{\sqrt{1-\frac{u^2}{16}}} = \lim_{k\to 4^-}\sin^{-1}\frac{u}{4}\Big|_2^k = \frac{\pi}{3}$$

See Example 26, page 312.

52. **(C)** Let $y = \left(\frac{1}{x}\right)^x$. Then $\ln y = -x\ln x$ and

$$\lim_{x\to 0^+}\ln y = \lim_{x\to 0^+}\frac{-\ln x}{1/x}.$$

Now apply L'Hôpital's Rule:

$$\lim_{x\to 0^+}\ln y = \lim_{x\to 0^+}\frac{-1/x}{-1/x^2} = 0.$$

So, if $\lim_{x\to 0^+}\ln y = 0$, then $\lim_{x\to 0^+} y = 1$.

53. **(D)** The speed, $|\mathbf{v}|$, equals $\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$, and since $x = 3y - y^2$,

$$\frac{dx}{dt} = (3-2y)\,\frac{dy}{dt} = (3-2y)\cdot 3.$$

Then $|\mathbf{v}|$ is evaluated, using $y = 1$, and equals $\sqrt{(3)^2+(3)^2}$.

54. **(A)** This is an indeterminate form of type $\frac{\infty}{\infty}$; use L'Hôpital's Rule:

$$\lim_{x\to 0^+}\frac{\cot x}{\ln x} = \lim_{x\to 0^+}\frac{-\csc^2 x}{1/x} = \lim_{x\to 0^+}\frac{x}{\sin x}\cdot\frac{-1}{\sin x} = -\infty$$

55. (E) We find A and B such that $\dfrac{3x+2}{(x+3)(x-4)} = \dfrac{A}{x+3}+\dfrac{B}{x-4}$.

After multiplying by the common denominator, we have

$$3x+2 = A(x-4)+B(x+3).$$

Substituting $x=-3$ yields $A=1$, and $x=4$ yields $B=2$; hence,

$$\frac{3x+2}{x^2-x-12} = \frac{1}{x+3}+\frac{2}{x-4}.$$

56. (B) Since $\cos x \approx 1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}$, $\dfrac{1-\cos x}{x} \approx \dfrac{x}{2}-\dfrac{x^3}{4!}$.

Then $\displaystyle\int_0^1 \frac{1-\cos x}{x}\,dx \approx \left(\frac{x^2}{4}-\frac{x^4}{96}\right)\Bigg|_0^1 = \frac{1}{4}-\frac{1}{96}$.

Note that $\displaystyle\lim_{x\to 0^+} \frac{1-\cos x}{x} = 0$, so the integral is proper.

57. (D) We represent the spiral as $P(\theta) = (\theta\cos\theta, \theta\sin\theta)$. So

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{\theta\cos\theta+\sin\theta}{-\theta\sin\theta+\cos\theta}=\frac{\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}}{-\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}}=\frac{\pi/4+1}{-\pi/4+1}.$$

Part B

58. (D) Since h is increasing, $h' \geq 0$. The graph of h is concave downward for $x<2$ and upward for $x>2$, so h'' changes sign at $x=2$, where it appears that $h'=0$ also.

59. (C) I is false since, for example, $f'(-2)=f'(1)=0$ but neither $g(-2)$ nor $g(1)$ equals zero.

II is true. Note that $f=0$ where g has relative extrema, and f is positive, negative, then positive on intervals where g increases, decreases, then increases.

III is also true. Check the concavity of g: when the curve is concave down, $h<0$; when up, $h>0$.

60. (A) If $y=\displaystyle\int_3^x \frac{1}{\sqrt{3+2t}}\,dt$, then $\dfrac{dy}{dx}=\dfrac{1}{\sqrt{3+2x}}$, so $\dfrac{d^2y}{dx^2} = -\dfrac{1}{2}(3+2x)^{-3/2}(2)$.

61. (D) $\displaystyle\int_{-4}^{3} f(x+1)\,dx$ represents the area of the same region as $\displaystyle\int_{-3}^{4} f(x)\,dx$, translated one unit to the left.

62. **(D)** According to the Mean Value Theorem, there exists a number c in the interval $[1,1.5]$ such that $f'(c)=\frac{f(1.5)-f(1)}{1.5-1}$. Use your calculator to solve the equation $\cos c=\frac{\sin 1.5-\sin 1}{0.5}$ for c (in radians).

63. **(E)** Here are the relevant sign lines:

$$\underset{0\qquad 1}{-\ |\ -\ |\ +} \text{ signs of } f'(x) \qquad \underset{0\qquad \frac{2}{3}}{+\ |\ -\ |\ +} \text{ signs of } f''(x)$$

We see that f' and f'' are both positive only if $x>1$.

64. **(E)** Note from the sign lines in Question 63 that f changes from decreasing to increasing at $x=1$, so f has a local minimum there.

Also, the graph of f changes from concave up to concave down at $x=0$, then back to concave up at $x=\frac{2}{3}$; hence f has two points of inflection.

65. **(C)** The derivatives of $\ln(x+1)$ are $\frac{1}{x+1}, \frac{-1}{(x+1)^2}, \frac{+2!}{(x+1)^3}, \frac{-(3!)}{(x+1)^4}, \ldots$

The nth derivative at $x=2$ is $\frac{(-1)^{n-1}(n-1)!}{3^n}$.

66. **(C)** The absolute-value function $f(x)=|x|$ is continuous at $x=0$, but $f'(0)$ does not exist.

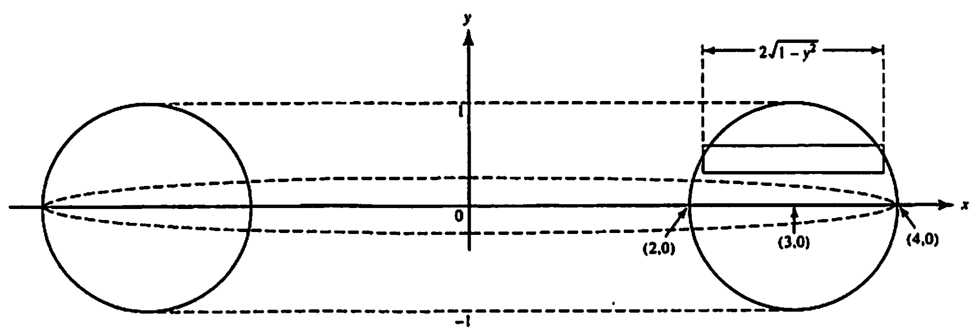
67. **(C)** Let $F'(x)=f(x)$; then $F'(x+k)=f(x+k)$;

$$\int_0^3 f(x+k)\,dx = F(3+k)-F(k);$$

$$\int_k^{3+k} f(x)\,dx = F(3+k)-F(k).$$

Or let $u=x+k$. Then $dx=du$; when $x=0$, $u=k$; when $x=3$, $u=3+k$.

68. **(E)** See the figure. The equation of the generating circle is $(x-3)^2+y^2=1$, which yields $x=3\pm\sqrt{1-y^2}$.



About the y-axis: $\Delta V = 2\pi \cdot 3 \cdot 2\sqrt{1-y^2}\,\Delta y$.

Thus, $V = 2\int_0^1 12\pi\sqrt{1-y^2}\,dy$.

$= 24\pi$ times the area of a quarter of a unit circle $= 6\pi^2$.

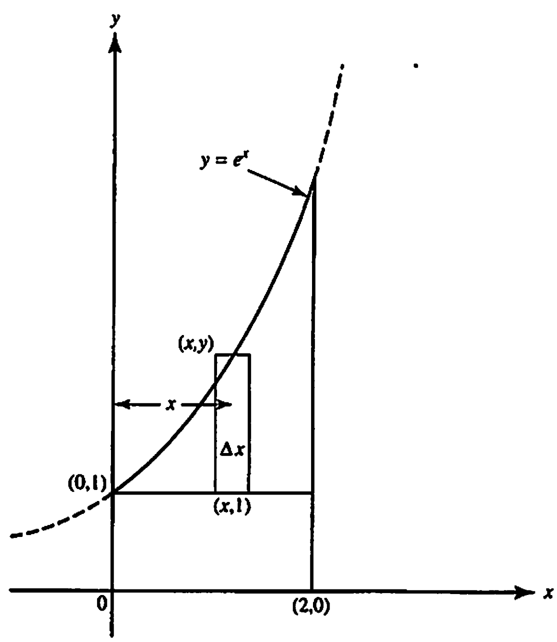
69. (D) Note that $f(g(u)) = \tan^{-1}(e^{2u})$; then the derivative is $\frac{1}{1+(e^{2u})^2}(2e^{2u})$.

70. (D) Let $y' = \frac{dy}{dx}$. Then $\cos(xy)[xy' + y] = y'$. Solve for y'.

71. (E) $\frac{d^2}{dx^2}f(x^2) = \frac{d}{dx}\left[\frac{d}{dx}f(x^2)\right] = \frac{d}{dx}\left[\frac{d}{dx}f(x^2)\cdot\frac{dx^2}{dx}\right] = \frac{d}{dx}\left[g(x^2)\cdot 2x\right]$

$$= g(x^2)\frac{d}{dx}(2x) + 2x\frac{d}{dx}g(x^2) = g(x^2)\cdot 2 + 2x\frac{d}{dx^2}g(x^2)\frac{dx^2}{dx}$$

$$= 2g(x^2) + 2x\cdot f\left(\sqrt{x^2}\right)\cdot 2x = 2g(x^2) + 4x^2\cdot f(x).$$



72. (C) About the x-axis; see the figure. Washer.

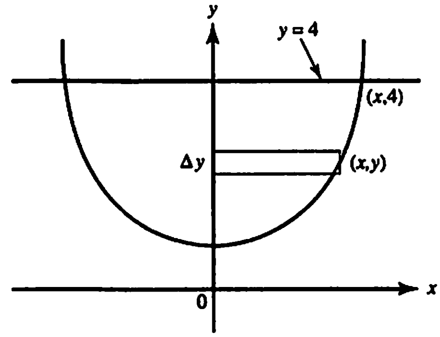
$\Delta V = \pi\left(y^2 - 1^2\right)\Delta x$,

$V = \pi\int_0^2\left(e^{2x} - 1\right)dx$.

73. (C) By the Mean Value Theorem, there is a number c in $[1, 2]$ such that

$$f'(c) = \frac{f(2) - f(1)}{2-1} = -3.$$

74. (D) The enclosed region, S, is bounded by $y = \sec x$, the y-axis, and $y = 4$. It is to be rotated about the y-axis.



Use disks; then $\Delta V = \pi R^2 H = \pi(\text{arc sec } y)^2\,\Delta y$. Using the calculator, we find that

$$\pi\int_1^4 \left(\arccos\left(\frac{1}{y}\right)\right)^2 dx \approx 11.385.$$

75. (C) If Q is the amount at time t, then $Q = 40e^{-kt}$. Since $Q = 20$ when $t = 2$, $k = -0.3466$. Now find Q when $t = 3$, from $Q = 40e^{-(0.3466)3}$, getting $Q = 14$ to the nearest gram.

76. (A) The velocity $v(t)$ is an antiderivative of $a(t)$, where $a(t) = \pi t + \frac{2}{1+t^2}$. So

$v(t) = \frac{\pi t^2}{2} + 2 \arctan t + C$. Since $v(1) = 0$, $C = -\pi$.

$$\text{Required average velocity } = \frac{1}{2-0}\int_0^2 v(t)\,dt$$

$$= \frac{1}{2}\int_0^2\left(\frac{\pi t^2}{2} + 2\arctan t - \pi\right)dt \approx 0.362.$$

77. (D) Graph $y = \tan x$ and $y = 2 - x$ in $[-1, 3] \times [-1, 3]$ as shown on page 485. Note that

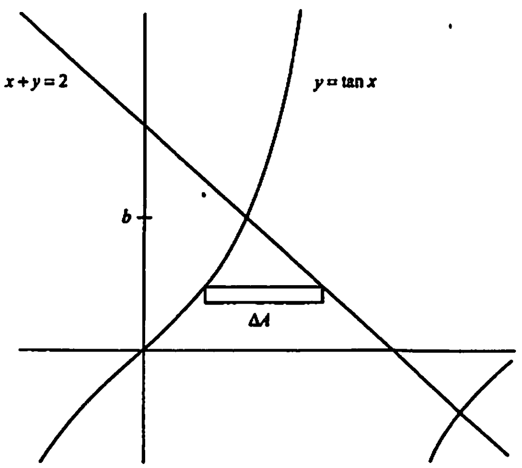
$$\Delta A = (x_{\text{line}} - x_{\text{curve}})\,\Delta y$$
$$= (2 - y - \arctan y)\,\Delta y.$$

The limits are $y = 0$ and $y = b$, where b is the ordinate of the intersection of the curve and the line. Using the calculator, solve

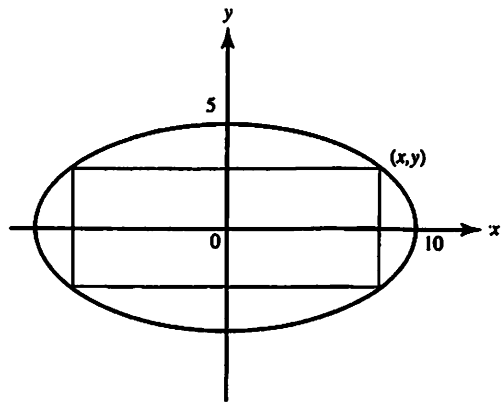
$$\arctan y = 2 - y$$

and store the answer in memory as B. Evaluate the desired area:

$$\int_0^B (2 - y - \arctan y)\,dy \approx 1.077.$$



78. **(E)** Center the ellipse at the origin and let (x, y) be the coordinates of the vertex of the inscribed rectangle in the first quadrant, as shown in the figure.



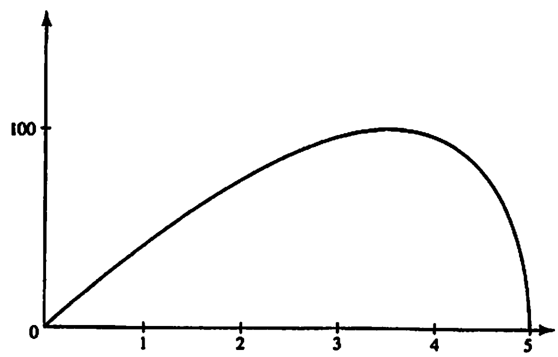
$$\frac{x^2}{100}+\frac{y^2}{25}=1$$

To maximize the rectangle's area $A = 4xy$, solve the equation of the ellipse, getting

$$x=\sqrt{100-4y^2}\ =\ 2\sqrt{25-y^2}\,.$$

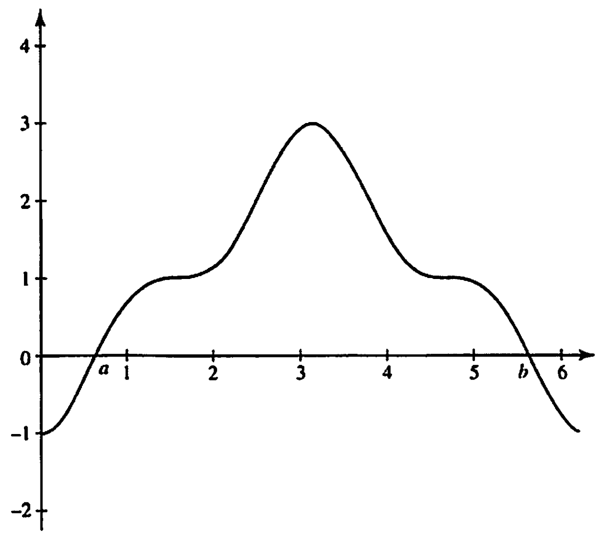
So $A = 8y\sqrt{25-y^2}$. Graph $y = 8x\ \sqrt{(25-x^2)}$ in the window $[0,5] \times [0,150]$.

The calculator shows that the maximum area (the y-coordinate) equals 100.



79. (B) $\dfrac{\int_1^e x \ln x \, dx}{e-1} \approx 1.221.$

80. (B) When f' is positive, f increases. By the Fundamental Theorem of Calculus, $f'(x) = 1 - 2(\cos x)^3$. Graph f' in $[0, 2\pi] \times [-2, 4]$. It is clear that $f' > 0$ on the interval $a < x < b$. Using the calculator to solve $1 - 2(\cos x)^3 = 0$ yields $a = 0.654$ and $b = 5.629$.



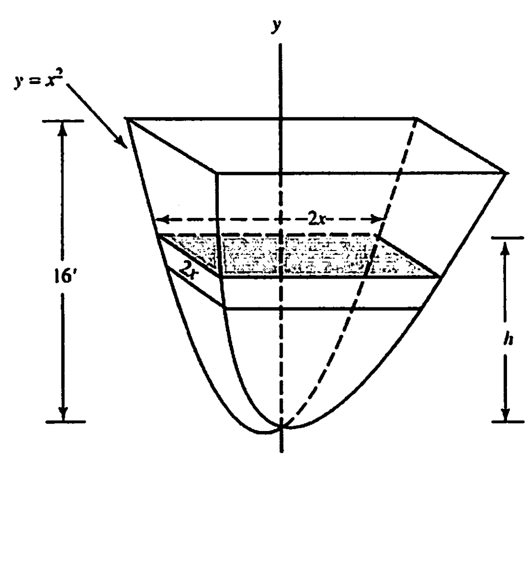
81. (C) $a(1.5) \approx \dfrac{v(2)-v(1)}{2-1} = \dfrac{12-22}{1}.$

82. (B) The volume is composed of elements of the form $\Delta V = (2x)^2 \, \Delta y$. If h is the depth, in feet, then, after t hr,

$$V(h) = 4\int_0^h y \, dy \text{ and } \frac{dV}{dt} = 4h\frac{dh}{dt}.$$

Thus, $12 = 4(9)\dfrac{dh}{dt}$

and $\dfrac{dh}{dt} = \dfrac{1}{3}$ ft/hr.



83. **(B)** Separating variables yields

$$\frac{dP}{1000-P}=k\,dt,$$

$$-\ln(1000-P)=kt+C,$$

$$1000-P=ce^{-kt}.$$

Then

$$P(t)=1000-ce^{-kt}.$$

$P(0)=300$ gives $c=700$. $P(5)=500$ yields $500=1000-700e^{-5k}$, so $k\simeq+0.0673$. Now $P(10)=1000-700e^{-0.673}\simeq 643$.

84. **(C)** $H(1)=\int_0^1\frac{4}{x^2+1}\,dx=4\arctan 1=\pi.\ H'(1)=f(1)=2.$

The equation of the tangent line is $y-\pi=2(x-1)$.

85. **(C)** Using midpoint diameters to determine cylinders, estimate the volume to be

$$V\simeq\pi\cdot 8^2\cdot 25+\pi\cdot 6^2\cdot 25+\pi\cdot 4^2\cdot 25+\pi\cdot 3^2\cdot 25.$$

86. **(A)** $\left(\frac{f}{g}\right)'(3)=\frac{g(3)\cdot f'(3)-f(3)\cdot g'(3)}{(g(3))^2}=\frac{2(2)-4(3)}{2^2}.$

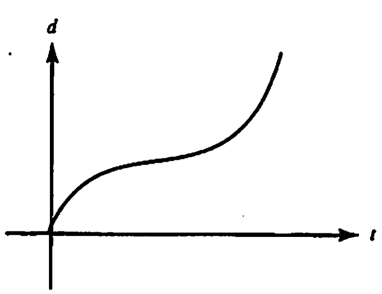
87. **(C)** $H'(3)=f'(g(3))\cdot g'(3)=f'(2)\cdot g'(3).$

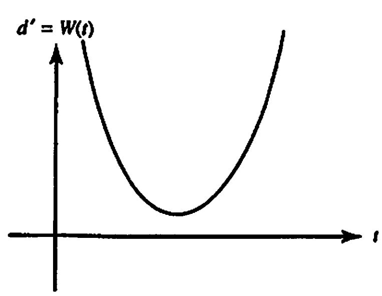
88. **(E)** $M'(3)=f(3)\cdot g'(3)+g(3)\cdot f'(3)=4\cdot 3+2\cdot 2.$

89. **(E)** $K'(3)=\frac{1}{g'(K(3))}=\frac{1}{g'(g^{-1}(3))}=\frac{1}{g'(4)}=\frac{1}{\frac{1}{2}}.$

90. **(C)** $R'(3)=\frac{1}{2}(f(3))^{-1/2}\cdot f'(3).$

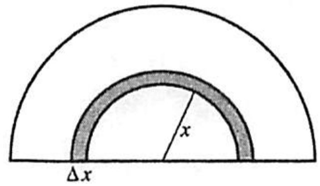
91. **(D)** Here are the pertinent curves, with d denoting the depth of the water:

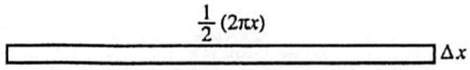




92. **(B)** Use areas; then $\int_1^7 f'=-3+10=7$. Thus, $f(7)-f(1)=7$.

93. **(B)** The region x units from the stage can be approximated by the semicircular ring shown; its area is then the product of its circumference and its width.





The number of people standing in the region is the product of the area and the density:

$$\Delta P = (\pi x \ \Delta x)\left(\frac{20}{2\sqrt{x}+1}\right).$$

To find the total number of people, evaluate

$$20\pi \int_0^{100} \frac{x}{2\sqrt{x}+1}\, dx$$

94. **(B)** $\frac{dy}{dt}$ is positive, but decreasing; hence $\frac{dy^2}{dt^2} < 0$.

95. **(C)** Average speed $= \frac{\text{total distance}}{\text{elapsed time}} = \frac{\text{total area}}{8} = \frac{8}{8}$.

96. **(E)** On $2 \leqslant t \leqslant 5$, the object moved $3\frac{1}{2}$ ft to the right; then on $5 \leqslant t \leqslant 8$, it moved only $2\frac{1}{2}$ ft to the left.

97. **(B)** Average acceleration $= \frac{\Delta v}{\Delta t} = \frac{v(8)-v(0)}{8-0} = \frac{-1-1}{8}$.

98. **(D)** Evaluate $\int_0^3 \frac{72}{2t+3}\, dt = 36 \ln\,(2t+3)\Big|_0^3 = 36 \ln 3$.

99. **(A)** $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ and $\frac{dy}{dt} = -\frac{3}{4}\frac{dx}{dt}$ at the point (3, 4).

Use, also, the facts that the speed is given by $|\mathbf{v}| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ and that the point moves counterclockwise; then $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 4$, yielding $\frac{dx}{dt} = -\frac{8}{5}$ and $\frac{dy}{dt} = +\frac{6}{5}$ at the given point. The velocity vector, **v**, at (3, 4) must therefore be $\left\langle -\frac{8}{5}, \frac{6}{5} \right\rangle$.

100. **(A)** $\mathbf{v} = \langle -ak \sin kt, ak \cos kt \rangle$, and

$\mathbf{a} = \langle -ak^2 \cos kt, ak^2 \sin kt \rangle = -k^2\mathbf{R}$.

101. **(B)** The formula for length of arc is

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

Since $y = 2^x$, we find

$$L = \int_0^2 \sqrt{1 + \left(2^x \ln 2\right)^2}\, dx \approx 3.664.$$

102. **(D)** $\mathbf{a}(t) = (0, e^t)$; the acceleration is always upward.

103. **(A)** At (0, 1), $\frac{dy}{dx} = 4$, so Euler's method yields $(0.1, 1 + 0.1(4)) = (0.1, 1.4)$.

$\frac{dy}{dx} = 4y$ has particular solution $y = e^{4x}$; the error is $e^{4(0.1)} - 1.4$.

104. **(D)** $1 - \frac{1}{2} + \frac{1}{5} = 0.7$. Note that the series converges by the Alternating Series Test. Since the first term dropped in the estimate is $-\frac{1}{10}$, the estimate is too high, but within 0.1 of the true sum.

105. **(C)** $\sum_{n=1}^{\infty} \frac{2}{3n} = \frac{2}{3}\sum_{n=1}^{\infty} \frac{1}{n}$, which equals a constant times the harmonic series.

106. **(D)** We seek x such that

$$\lim_{n\to\infty} \left| \frac{(n+1)! \cdot x^{n+1}}{(n+1)^{n+1}} \cdot \frac{n^n}{n! \cdot x^n} \right| < 1$$

or such that $|x| \cdot \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n < 1$

or such that $|x| < \dfrac{1}{\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n}$.

The fraction equals $\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$.

Then $|x| < e$ and the radius of convergence is e.

107. (B) The error is less than the maximum value of $\frac{e^c}{3!}x^3$ for $0 \le x \le \frac{1}{2}$.

This maximum occurs at $c = x = \frac{1}{2}$.

108. (D) Distance $= \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$

$$= \int_0^1 \sqrt{\left(\sec^2 t\right)^2 + (-2\sin 2t)^2}\, dt.$$

Note that the curve is traced exactly once by the parametric equations from $t = 0$ to $t = 1$.