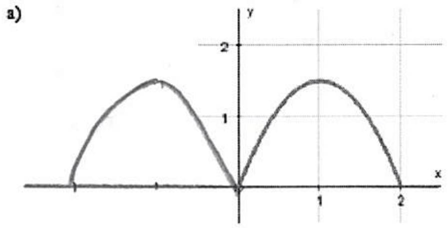
Prerequisites Worksheet

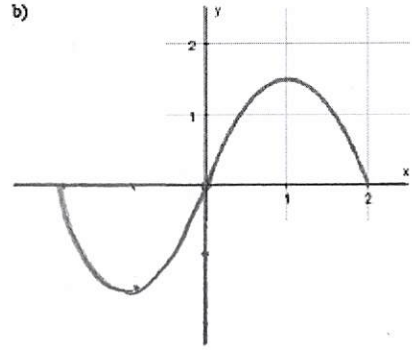
Even and Odd Functions

1. Complete each graph, assuming that a) is even and b) is odd:

a)



b)



2. Prove whether the following functions are even, odd or neither:

a) $\overset{f(x)}{y} = 3 - x^2$

$f(-x) = 3 - (-x)^2$
$= 3 - x^2$
$= f(x)$

$\Rightarrow$ even

b) $g(x) = \dfrac{x^2 - 1}{x^3}$

$g(-x) = \dfrac{(-x)^2 - 1}{(-x)^3}$
$= \dfrac{x^2 - 1}{-x^3}$
$= -\dfrac{x^2 - 1}{x^3}$
$= -g(x)$

$\Rightarrow$ odd

c) $f(x) = 2x - 5x^3$

$f(-x) = 2(-x) - 5(-x)^3$
$= -2x - 5(-x^3)$
$= -(2x - 5x^3)$
$= -f(x)$

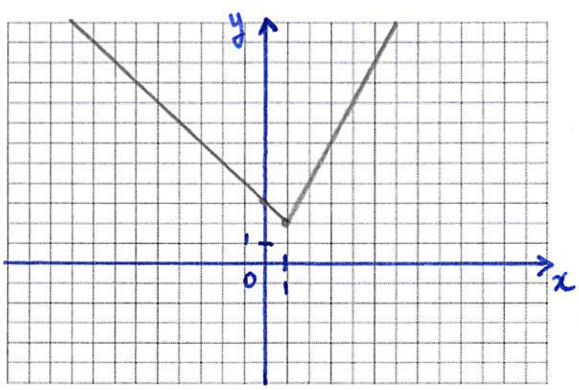
$\Rightarrow$ odd

<u>N.B</u>: most functions are neither...

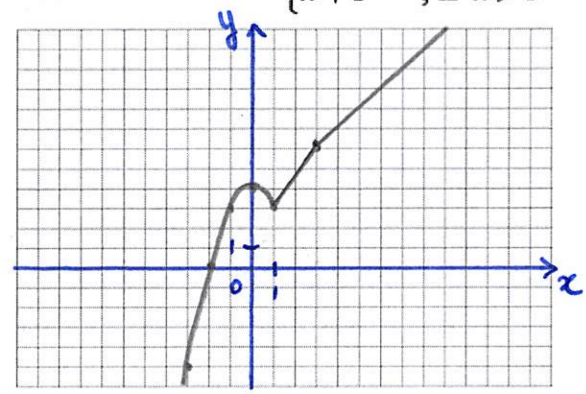
Piecewise Functions

3. Graph each of the following functions:

a) $f(x) = \begin{cases} 3 - x & , \text{if } x \le 1 \\ 2x & , \text{if } 1 < x \end{cases}$

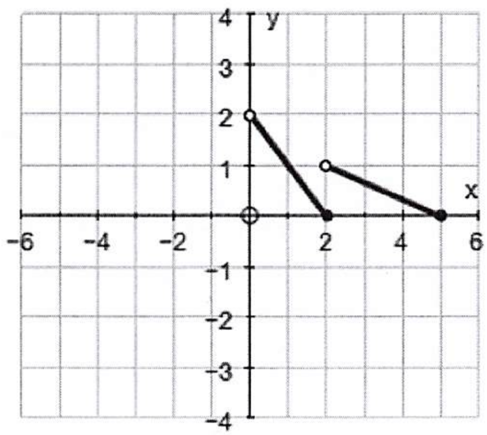


b) $f(x) = \begin{cases} 4 - x^2 & , \text{if } x < 1 \\ \frac{3}{2}x + \frac{3}{2} & , \text{if } 1 \le x \le 3 \\ x + 3 & , \text{if } x > 3 \end{cases}$

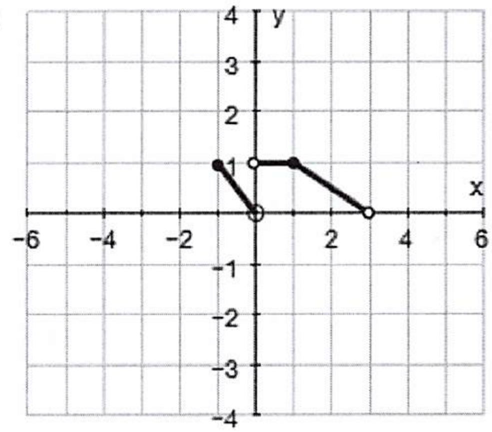


4. Determine a piecewise function for each of the following graphs:

a)



b)



$$y = \begin{cases} -x+2 & \text{if } 0 < x \le 2 \\ -\frac{1}{3}x+\frac{5}{3} & \text{if } 2 < x \le 5 \end{cases}$$

$$y = \begin{cases} -x & \text{if } -1 \le x \le 0 \\ 1 & \text{if } 0 < x \le 1 \\ -\frac{1}{2}x+\frac{3}{2} & \text{if } 1 < x < 3 \end{cases}$$

5. An earthquake that occurred at 9:17AM cracked a water tower in a small town. Water began leaking out of the tower at rate 12 cm^3/min for the first 25 minutes, then the rate increased to 24 cm^3/min for the next 30 minutes. It took 45 minutes before the leak was fixed, and in that final 45 minutes, the water was leaking at a rate of 20 cm^3/min. Write a piecewise function for the amount of water leaking out of the tower as a function of time.

$t=0$ 9:17AM

300 cm^3 have leaked

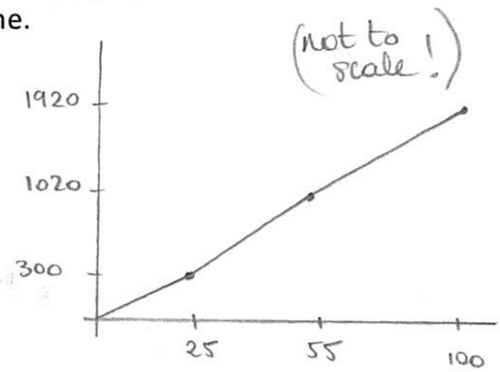
$t=25$ 9:42 AM

720 cm^3 have leaked

$t=55$ 10:12 AM

900 cm^3 have leaked

$t=100$ 10:57 AM



$$y = \begin{cases} 12t & \text{if } 0 \le t \le 25 \\ 24t-300 & \text{if } 25 < t \le 55 \\ 20t-80 & \text{if } 55 < t \le 100 \end{cases}$$

One to one Functions

6. Explain in your own words what a one-to-one function is.

It's a function for which you don't get twice the same value.

Each value of "x" is associated to a different value of "y".

Composite Functions and Inverse Functions

7. What are the 3 domain issues you must remember for this course?

- denominators $\neq 0$
  $\frac{B}{A} \rightarrow A \neq 0$
- radicand ≥ 0 for even index radical
  $\sqrt{A} \rightarrow A \geq 0$
- arg of logs > 0
  $\ln A \rightarrow A > 0$

8. What are the domains of the following functions?

a) $g(x) = \frac{\log_2(x-5)}{x-8}$

- $x - 5 > 0$ and $x - 8 \neq 0$
  $x > 5$ $\quad x \neq 8$

$D = (5; 8) \cup (8; +\infty)$

b) $f(x) = \frac{\sqrt{9-x^2}}{x}$

- $9 - x^2 \geq 0$ and $x \neq 0$
  $x^2 \leq 9$
  $-3 \leq x \leq 3$

$D = [-3; 0) \cup (0; +3]$

9. Suppose $f(x) = x + 5$ and $g(x) = x^2 - 3$. Find each of the following

a) $f(g(x))$

$= x^2 - 3 + 5$

$= x^2 + 2$

b) $g(f(x))$

$= (x+5)^2 - 3$

$= x^2 + 10x + 22$

c) $f(g(0))$

$= f(-3)$

$= 2$

d) $g(f(0))$

$= g(5)$

$= 22$

e) $g(g(-2))$

$= g(1)$

$= -2$

f) $f(f(x))$

$= x + 5 + 5$

$= x + 10$

10. Complete the following table:

$g(x)$	$f(x)$	$f(g(x))$
x^2	$\sqrt{x-5}$	$\sqrt{x^2-5}$
$\frac{1}{x-1}$	$1+\frac{1}{x}$	x
$\frac{1}{x}$	$\frac{1}{x}$	x
$\sqrt{x}$	x^2	x, $x \geq 0$

11. Find $f(g(x))$ and determine their domain.

a) $f(x) = x^2 + 7$ and $g(x) = \sqrt{x+4}$

$f(g(x)) = \sqrt{x+4}^2 + 7$ $\quad D = [-4; +\infty)$

$= x + 11$

b) $f(x) = \frac{4x+1}{x-3}$ and $g(x) = \frac{3x+1}{x-4}$

$f(g(x)) = \dfrac{4\left(\frac{3x+1}{x-4}\right)+1}{\frac{3x+1}{x-4} - 3} = \dfrac{\frac{4(3x+1)+x-4}{x-4}}{\frac{3x+1-3(x-4)}{x-4}} = \frac{13x}{13} = x$ $\quad D = \mathbb{R} \setminus \{4\}$

12. Explain how to find an inverse of a given function algebraically.

- switch x and y.
- Solve for the "new" y.

13. Explain how to verify/prove that two functions are inverses of each other.

Prove that $f(g(x)) = x$ and $g(f(x)) = x$

14. Find the inverse of each following functions and determine their domain and ranges:

a) $f(x) = 2x + 3$ $\quad D=\mathbb{R}$ $R=\mathbb{R}$

↳ $x = 2y + 3$

$2y = x - 3$

$y = \frac{1}{2}(x-3)$ $\quad D=\mathbb{R}$ $R=\mathbb{R}$

b) $g(x) = \sqrt[4]{2x+7} - 5$ $\quad D = \left[-\frac{7}{2}, +\infty\right)$ $R = [-5; +\infty)$

↳ $x = \sqrt[4]{2y+7} - 5$

$\sqrt[4]{2y+7} = x + 5$

$2y + 7 = (x+5)^4$

$y = \frac{1}{2}(x+5)^4 - \frac{7}{2}$ $\quad D = [-5; +\infty)$ $R = \left[-\frac{7}{2}, +\infty\right)$

c) $h(x) = \frac{2x+1}{x+3}$ $\quad D=\mathbb{R}\setminus\{-3\}$ $R = \mathbb{R}\setminus\{2\}$

↳ $x = \frac{2y+1}{y+3}$

$x(y+3) = 2y+1$

$xy + 3x = 2y + 1$

$y(x-2) = 1 - 3x$

$y = \frac{1-3x}{x-2}$

$D = \mathbb{R}\setminus\{2\}$ $R = \mathbb{R}\setminus\{-3\}$

d) $k(x) = 2\sin(x) + 1$ $\quad D_R = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $R = [-1, 3]$

$x = 2\sin y + 1$

$\frac{x-1}{2} = \sin y$

$y = \sin^{-1}\left(\frac{x-1}{2}\right)$ $\quad D = [-1, 3]$ $R = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

e) $j(x) = 5\cos^{-1}x + 2$ $\quad D = [-1, 1]$ $R = [2; 5\pi+2]$

↳ $x = 5\cos^{-1}y + 2$

$x - 2 = 5\cos^{-1}y$

$\frac{x-2}{5} = \cos^{-1}y$

$y = \cos\left(\frac{x-2}{5}\right)$ $\quad D = [2; 5\pi+2]$ $R = [-1, 1]$

f) $l(x) = 5\ln(2x-1) + 3$ $\quad D = \left(\frac{1}{2}, +\infty\right)$ $R = \mathbb{R}$

↳ $x = 5\ln(2y-1) + 3$

$\frac{x-3}{5} = \ln(2y-1)$

$2y - 1 = e^{\frac{x-3}{5}}$

$y = \frac{1}{2}e^{\frac{x-3}{5}} + \frac{1}{2}$ $\quad D = \mathbb{R}$ $R = \left(\frac{1}{2}, +\infty\right)$

15. Verify that the function $k(x) = \sqrt{1 - x^2}$ for $0 < x < 1$ is its own inverse.

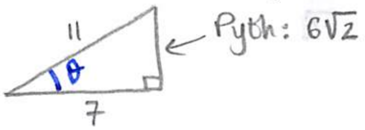
$$k(k(x)) = \sqrt{1 - \sqrt{1-x^2}^{\,2}}$$

$$= \sqrt{1 - (1 - x^2)}$$

$$= \sqrt{x^2} = x \text{ if } 0 < x < 1$$

16. Without using a calculator, evaluate the following expressions:

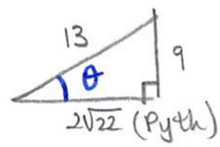
a) $sin\left(cos^{-1}\left(\frac{7}{11}\right)\right) = \sin\theta$



$= \frac{6\sqrt{2}}{11}$

$\cos^{-1}\left(\frac{7}{11}\right) = \theta$

b) $tan\left(sin^{-1}\left(\frac{9}{13}\right)\right) = \tan\theta$



$= \frac{9}{2\sqrt{22}}$

$= \frac{9\sqrt{22}}{44}$

$\sin^{-1}\left(\frac{9}{13}\right) = \theta$

Greatest Integer Function

17. Determine the following values for $f(x) = Int\ (x)$

$f(1.3) =$ 1 $\qquad f(0.9) =$ 0 $\qquad f(-1.6) =$ -2

$f(5.8) =$ 5 $\qquad f(-4.2) =$ -5 $\qquad f(3) =$ 3

18. Make sure you know how to draw the Greatest Integer function precisely.