

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO

Percent of Total Grade:  
00%

# Practice Test 1

## CALCULUS AB

## SECTION I, Part A

Time—60 Minutes

Number of questions—30

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

**Directions:** Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

**In this test:** Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

1. If  $f(x) = 3x^2 - 10x + \ln(x - 3)$ , then  $f'(4) =$

- (A) 8
- (B) 9
- (C) 14
- (D) 15

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2.  $\lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 1}{4x^2 + 2x + 5}$  is

- (A) 0
- (B)  $\frac{4}{5}$
- (C)  $\frac{5}{4}$
- (D)  $\infty$

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3. If  $f(x) = \frac{3x^2 + x}{3x^2 - x}$ , then  $f'(x)$  is

(A) 1

(B)  $\frac{6x^2 + 1}{6x^2 - 1}$

(C)  $\frac{-6}{(3x - 1)^2}$

(D)  $\frac{-2x^2}{(x^2 - x)^2}$

4.  $\lim_{x \rightarrow 0} \frac{3\sin(2x)}{x} =$

(A) 0

(B) 3

(C) 6

(D) The limit does not exist.

GO ON TO THE NEXT PAGE.

Section I

5. If  $y = \sin(xy)$ , find  $\frac{dy}{dx}$ .

- (A)  $\cos(xy)$
- (B)  $\frac{y\cos(xy)}{1 - x\cos(xy)}$
- (C)  $x\cos(xy)$
- (D)  $\frac{y\cos(xy)}{1 - x}$

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6. Using a right Riemann sum with four subintervals, approximate  $\int_0^4 3x^2 + 1 \, dx$ .

- (A) 45
- (B) 46
- (C) 94
- (D) 95

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7. If  $h(x) = f(x)g(4x)$ , then  $h'(2) =$

- (A)  $f'(2)g'(8)$
- (B)  $f'(2)g(8) + 4f(2)g'(8)$
- (C)  $f'(2)g(8) + f(2)g'(8)$
- (D)  $4f'(2)g'(8)$

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8. An equation of the line normal to the graph of  $y = \sqrt{(3x^2 + 2x)}$  at  $(2, 4)$  is

- (A)  $4x + 7y = 20$
  - (B)  $-7x + 4y = 2$
  - (C)  $7x + 4y = 30$
  - (D)  $4x + 7y = 36$
- 

9.  $4 \int \csc^2 x \, dx =$

- (A)  $-4\cot x + C$
  - (B)  $-\frac{1}{4}\cot x + C$
  - (C)  $\frac{4\csc^3 x}{3} + C$
  - (D)  $-\frac{4\csc^3 x}{3} + C$
- 

10. If  $f(x) = \cos^2 x$ , then  $f''(\pi) =$

- (A)  $-2$
  - (B)  $0$
  - (C)  $1$
  - (D)  $2$
- 

GO ON TO THE NEXT PAGE.

**Section I**

11. If  $f(x) = \frac{5}{x^2 + 1}$  and  $g(x) = 3x$ , then  $g(f(2)) =$

- (A)  $\frac{5}{37}$
- (B) 3
- (C) 5
- (D)  $\frac{37}{5}$

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12.  $\int x\sqrt{1+x^2} dx =$

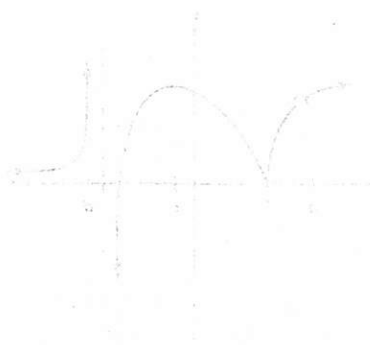
- (A)  $\frac{1}{3}(1+x^2)^{\frac{3}{2}} + C$
- (B)  $\frac{4}{3}(1+x^2)^{\frac{3}{2}} + C$
- (C)  $-(1+x^2)^{\frac{1}{2}} + C$
- (D)  $-\frac{1}{4}(1+x^2)^{\frac{1}{2}} + C$

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13. The slope of the line tangent to the graph of  $3x^2 + 5 \ln y = 12$  at  $(2, 1)$  is

- (A)  $-\frac{12}{5}$   
(B)  $\frac{12}{5}$   
(C)  $\frac{5}{12}$   
(D)  $-7$



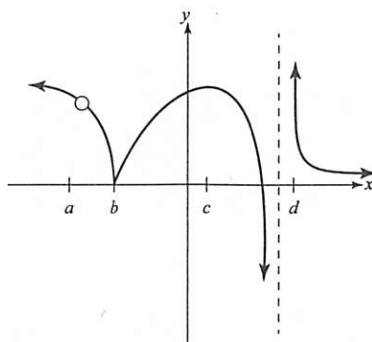
14. The equation  $y = 2 - 3 \sin \frac{\pi}{4} (x - 1)$  has a fundamental period of

- (A)  $\frac{1}{8}$   
(B)  $\frac{4}{\pi}$   
(C) 8  
(D)  $2\pi$

GO ON TO THE NEXT PAGE.

Section I

15. The graph of  $f'$  is given below. At which point is  $f$  continuous but not differentiable?



- (A)  $a$
- (B)  $b$
- (C)  $c$
- (D)  $d$

16. For what value of  $x$  does the function  $f(x) = x^3 - 9x^2 - 120x + 6$  have a local minimum?

- (A) 10
- (B) 4
- (C) -4
- (D) -10

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17. The acceleration of a particle moving along the  $x$ -axis at time  $t$  is given by  $a(t) = 4t - 12$ . If the velocity is 10 when  $t = 0$  and the position is 4 when  $t = 0$ , then the particle is changing direction at

(A)  $t = 1$   
(B)  $t = 3$   
(C)  $t = 5$   
(D)  $t = 1$  and  $t = 5$

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18. What is the area of the region between  $y = 2x^2$  and  $y = 12 - x^2$ ?

(A) 0  
(B) 16  
(C) 32  
(D) 48

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19.  $\int (e^{3\ln x} + e^{3x}) dx =$

(A)  $3 + \frac{e^{3x}}{3} + C$   
(B)  $\frac{e^{x^4}}{4} + 3e^{3x} + C$   
(C)  $\frac{e^{x^4}}{4} + \frac{e^{3x}}{3} + C$   
(D)  $\frac{x^4}{4} + \frac{e^{3x}}{3} + C$

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GO ON TO THE NEXT PAGE.

**Section I**

20. If  $f(x) = (x^2 + x + 11)\sqrt{x^3 + 5x + 121}$ , then  $f'(0) =$

- (A)  $\frac{5}{2}$
  - (B)  $\frac{27}{2}$
  - (C) 22
  - (D)  $\frac{247}{2}$
- 

21. If  $f(x) = 5^{3x}$ , then  $f'(x) =$

- (A)  $5^{3x}(\ln 125)$
  - (B)  $\frac{5^{3x}}{3 \ln 5}$
  - (C)  $3(5^{2x})$
  - (D)  $3(5^{3x})$
- 

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22. A solid is generated when the region in the first quadrant enclosed by the graph of  $y = (x^2 + 1)^3$ , the line  $x = 1$ , the  $x$ -axis, and the  $y$ -axis is revolved about the  $x$ -axis. Its volume is found by evaluating which of the following integrals?

(A)  $\pi \int_1^8 (x^2 + 1)^3 dx$

(B)  $\pi \int_1^8 (x^2 + 1)^6 dx$

(C)  $\pi \int_0^1 (x^2 + 1)^3 dx$

(D)  $\pi \int_0^1 (x^2 + 1)^6 dx$

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23.  $\lim_{x \rightarrow 0} \frac{\sin\left(\frac{\pi}{2} + x\right) - 1}{x} =$

(A)  $-1$

(B)  $0$

(C)  $1$

(D) The limit does not exist.

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24. If  $\frac{dy}{dx} = \frac{(3x^2 + 2)}{y}$  and  $y = 4$  when  $x = 2$ , then when  $x = 3$ ,  $y =$

- (A) 18
  - (B) 58
  - (C)  $\pm\sqrt{74}$
  - (D)  $\pm\sqrt{58}$
- 

25.  $\int \frac{dx}{9+x^2} =$

- (A)  $3 \tan^{-1}\left(\frac{x}{3}\right) + C$
  - (B)  $\frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$
  - (C)  $\frac{1}{3} \tan^{-1}(x) + C$
  - (D)  $\frac{1}{9} \tan^{-1}(x) + C$
- 

26. If  $f(x) = \cos^3(x + 1)$ , then  $f'(\pi) =$

- (A)  $-3 \cos^2(\pi + 1) \sin(\pi + 1)$
  - (B)  $3 \cos^2(\pi + 1)$
  - (C)  $3 \cos^2(\pi + 1) \sin(\pi + 1)$
  - (D) 0
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27.  $\int x\sqrt{x+3} \, dx =$

(A)  $\frac{2(x+3)^{\frac{3}{2}}}{3} + C$

(B)  $\frac{2}{5}(x+3)^{\frac{5}{2}} - 2(x+3)^{\frac{3}{2}} + C$

(C)  $\frac{3(x+3)^{\frac{3}{2}}}{2} + C$

(D)  $\frac{4x^2(x+3)^{\frac{3}{2}}}{3} + C$

28. If  $f(x) = \ln(\ln(1-x))$ , then  $f'(x) =$

(A)  $-\frac{1}{\ln(1-x)}$

(B)  $\frac{1}{(1-x)\ln(1-x)}$

(C)  $\frac{1}{(1-x)^2}$

(D)  $-\frac{1}{(1-x)\ln(1-x)}$

GO ON TO THE NEXT PAGE.

**Section I**

29.  $\lim_{x \rightarrow 0} \frac{xe^x}{1 - e^x} =$

- (A)  $-\infty$
- (B)  $-1$
- (C)  $1$
- (D)  $\infty$

30.  $\int \tan^6 x \sec^2 x \, dx =$

- (A)  $\frac{\tan^7 x}{7} + C$
- (B)  $\frac{\tan^7 x}{7} + \frac{\sec^3 x}{3} + C$
- (C)  $\frac{\tan^7 x \sec^3 x}{21} + C$
- (D)  $7 \tan^7 x + C$

**END OF PART A, SECTION I**

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.**

**DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.**

## CALCULUS AB

## SECTION I, Part B

Time—45 Minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

**Directions:** Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

**In this test:**

1. The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

31.  $\int_0^{\frac{\pi}{4}} \sin x \, dx + \int_{-\frac{\pi}{4}}^0 \cos x \, dx =$

- (A)  $-1$   
(B)  $0$   
(C)  $1$   
(D)  $\sqrt{2}$

GO ON TO THE NEXT PAGE.

32. A particle is moving along the  $y$ -axis. The position of the particle at time  $t \geq 0$  is given by  $y(t) = y^3 - 4y^2 + 7$ . Which of the following expressions gives the total distance that the particle traveled in the first four seconds?

- (A)  $\int_0^{\frac{8}{3}} y^3 - 4y^2 + 7 \, dy - \int_{\frac{8}{3}}^4 y^3 - 4y^2 + 7 \, dy$   
(B)  $-\int_0^{\frac{8}{3}} y^3 - 4y^2 + 7 \, dy + \int_{\frac{8}{3}}^4 y^3 - 4y^2 + 7 \, dy$   
(C)  $\int_0^{\frac{8}{3}} 3y^2 - 8y \, dy - \int_{\frac{8}{3}}^4 3y^2 - 8y \, dy$   
(D)  $-\int_0^{\frac{8}{3}} 3y^2 - 8y \, dy + \int_{\frac{8}{3}}^4 3y^2 - 8y \, dy$

- 
33. If  $\int_0^4 f(x) \, dx = 10$  and  $\int_8^0 f(x) \, dx = -23$ , what is  $\int_4^8 f(x) \, dx$ ?

- (A) -33  
(B) -13  
(C) 13  
(D) 33

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34. If  $f'(x) = 3\sin x + 2\cos x$  and  $f(0) = 5$ , find  $f(x)$ .

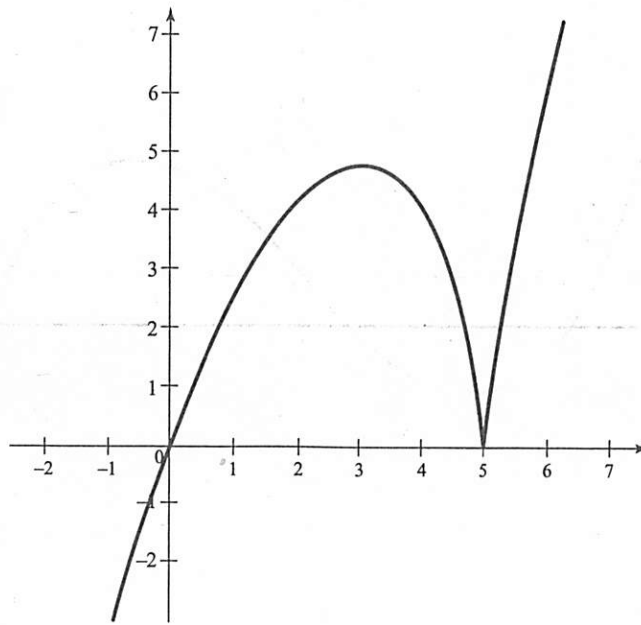
- (A)  $-3\cos x + 2\sin x + 5$   
(B)  $-3\cos x + 2\sin x + 8$   
(C)  $3\cos x - 2\sin x + 2$   
(D)  $3\cos x - 2\sin x + 5$

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35. The graph of  $f$  is shown below. Which of the following is true about the function at  $x = 5$ ?



- (A)  $f'(5)$  is undefined.
- (B)  $f(5)$  is undefined.
- (C)  $f'(5) = 0$
- (D) None of the above is true.

36. The function  $f$  is defined by  $f(x) = e^{-\frac{x^2}{4}}$ . What is the area between  $f$  and the  $x$ -axis from  $x = -1.65$  to  $x = 1.65$ ?

- (A) 0
- (B) 1.341
- (C) 2.682
- (D) 3.950

GO ON TO THE NEXT PAGE.

Section I

37. The average value of the function  $f(x) = \ln^2 x$  on the interval  $[2, 4]$  is

- (A) 1.204
- (B) 2.159
- (C) 2.408
- (D) 8.636

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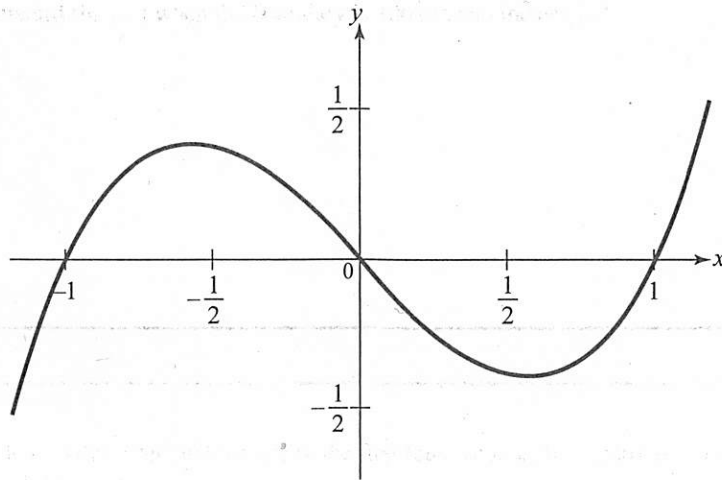
38.  $\frac{d}{dx} \int_0^{3x} \cos(t) dt =$

- (A)  $\sin 3x$
- (B)  $\cos 3x$
- (C)  $3 \sin 3x$
- (D)  $3 \cos 3x$

GO ON TO THE NEXT PAGE

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39. The graph of the function  $f$  shown in the figure below. For which of the following values of  $x$  is  $f'(x)$  positive and decreasing?



- (A)  $a$   
 (B)  $b$   
 (C)  $c$   
 (D)  $d$

40. The radius of a sphere is increasing at a rate proportional to itself. If the radius is 4 initially, and the radius is 10 after two seconds, what will the radius be after three seconds?

- (A) 15.81  
 (B) 16.00  
 (C) 25.00  
 (D) 62.50

GO ON TO THE NEXT PAGE.

## Section I

41. Suppose  $f(x) = \int_0^x (t^3 + t) dt$ . Find  $f'(5)$ .

- (A) 130
  - (B) 120
  - (C) 76
  - (D) 74
- 

42.  $\int \ln 2x \, dx =$

- (A)  $\frac{\ln 2x}{2x} + C$
  - (B)  $x \ln x - x + C$
  - (C)  $x \ln 2x - x + C$
  - (D)  $2x \ln 2x - 2x + C$
- 

43. Given  $f(x) = \begin{cases} ax^2 + 3bx + 14; & x \leq 2 \\ 3ax + 5b; & x > 2 \end{cases}$ , find the values of  $a$  and  $b$  that make  $f$  differentiable for all  $x$ .

- (A)  $a = -6, b = 2$
  - (B)  $a = 6, b = -2$
  - (C)  $a = 2, b = 6$
  - (D)  $a = 2, b = -6$
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GO ON TO THE NEXT PAGE.

GO ON TO THE NEXT PAGE.

44. A circular shock wave is spreading from its point of origin at the rate of  $640 \frac{\text{m}^2}{\text{s}}$ . How fast is a point on the boundary of the shock wave moving outward (in  $\frac{\text{m}}{\text{s}}$ ) when the boundary is 100 m from the origin?
- (A) 1.019
  - (B) 2.037
  - (C) 3.201
  - (D) 6.400

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45. Find the distance traveled (to three decimal places) in the first four seconds, for a particle whose velocity is given by  $v(t) = 7e^{-t^2}$ , where  $t$  stands for time.
- (A) 0.976
  - (B) 6.204
  - (C) 6.359
  - (D) 12.720

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**STOP**  
**END OF PART B, SECTION I**  
**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.**  
**DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.**

SECTION II  
GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example,  $\int_1^5 x^2 dx$  may not be written as  $\text{fnInt}(X^2, X, 1, 5)$ .
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

STOP

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.  
GO ON TO THE NEXT PAGE.

## SECTION II, PART A

Time—30 minutes

Number of problems—2

**A graphing calculator is required for some problems or parts of problems.**

During the timed portion for Part A, you may work only on the problems in Part A.

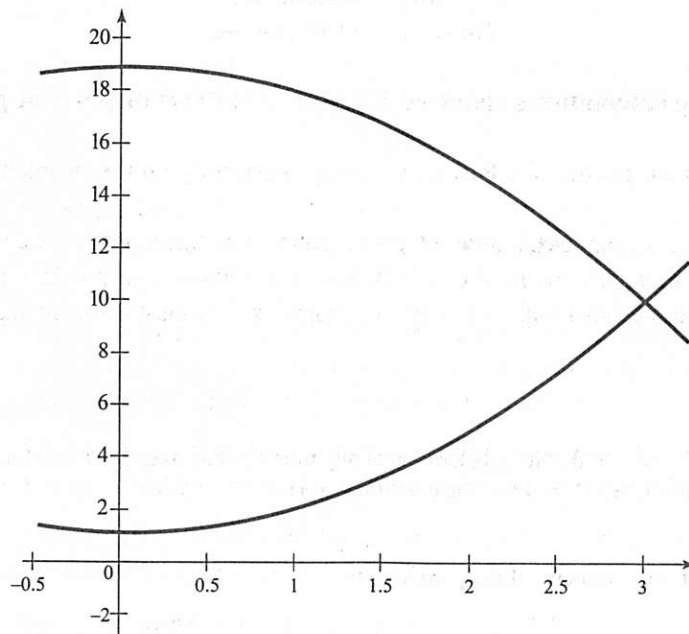
On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

1. A particle moves along the  $x$ -axis so that its acceleration at any time  $t > 0$  is given by  $a(t) = 12t - 18$ . At time  $t = 1$ , the velocity of the particle is  $v(1) = 0$  and the position is  $x(1) = 9$ .
    - (a) Write an expression for the velocity of the particle  $v(t)$ .
    - (b) At what values of  $t$  does the particle change direction?
    - (c) Write an expression for the position  $x(t)$  of the particle.
    - (d) Find the total distance traveled by the particle from  $t = \frac{3}{2}$  to  $t = 6$ .
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**GO ON TO THE NEXT PAGE.**

Section II

2. Let  $R$  be the region in the first quadrant bounded from above by  $g(x) = 19 - x^2$  and from below by  $f(x) = x^2 + 1$ .



- Find the area of  $R$ .
- A solid is formed by revolving  $R$  around the  $x$ -axis. Find the volume of the solid.
- A solid has its base as the region  $R$ , whose cross-sections perpendicular to the  $x$ -axis are squares. Find the volume of the solid.

GO ON TO THE NEXT PAGE.



## SECTION II, PART B

Time—1 hour

Number of problems—4

No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

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3. Consider the equation  $x^2 - 2xy + 4y^2 = 84$ .

(a) Write an expression for the slope of the curve at any point  $(x, y)$ .

(b) Find the equation of the tangent lines to the curve at the point  $x = 2$ .

(c) Find  $\frac{d^2y}{dx^2}$  at  $(0, \sqrt{21})$ .

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4. Water is draining at the rate of  $48\pi$  ft<sup>3</sup>/second from the vertex at the bottom of a conical tank whose diameter at its base is 40 feet and whose height is 60 feet.

(a) Find an expression for the volume of water in the tank, in terms of its radius, at the surface of the water.

(b) At what rate is the radius of the water in the tank shrinking when the radius is 16 feet?

(c) How fast is the height of the water in the tank dropping at the instant that the radius is 16 feet?

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GO ON TO THE NEXT PAGE.

## Section II

5. Let  $f$  be the function given by  $f(x) = 2x^4 - 4x^2 + 1$ .
- Find an equation of the line tangent to the graph at  $(-2, 17)$ .
  - Find the  $x$ - and  $y$ -coordinates of the relative maxima and relative minima. Verify your answer.
  - Find the  $x$ - and  $y$ -coordinates of the points of inflection. Verify your answer.
- 

6. Grain is being loaded into a silo at the rate of  $G(t) = 400e^{-\frac{t^2}{4}}$  ft<sup>3</sup>/hr, where  $t$  is the number of hours that it is being loaded,  $0 \leq t \leq 8$ . At time  $t = 0$ , there is 100 ft<sup>3</sup> of grain in the silo. Grain is also being removed through the base of the silo at the following rates, where  $R(t)$  is the amount of grain being removed in ft<sup>3</sup>/hour,  $0 \leq t \leq 8$ :

$t$	0	2	5	7	8
$R(t)$	60	90	110	120	125

- Estimate the total amount of grain removed from the silo at  $t = 8$  hrs, using a left-hand Riemann Sum and 4 subintervals.
  - Estimate the amount of grain in the silo at the end of 8 hours, using your answer from part (a).
  - Estimate  $R'(5)$ , showing your work. Indicate the units of measure.
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**STOP**

**END OF EXAM**

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**GO ON TO THE NEXT PAGE.**