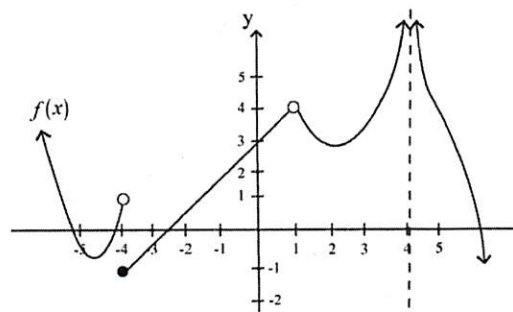


## QUIZ 2.1 - 2.2

1. For the function  $f$  shown, evaluate the following: [4]

- a)  $\lim_{x \rightarrow -4^-} f(x) = \underline{1}$   
 b)  $\lim_{x \rightarrow -4^+} f(x) = \underline{-1}$   
 c)  $\lim_{x \rightarrow 1^-} f(x) = \underline{4}$   
 d)  $\lim_{x \rightarrow 1^+} f(x) = \underline{4}$   
 e)  $\lim_{x \rightarrow 4^-} f(x) = \underline{+\infty}$   
 f)  $\lim_{x \rightarrow 4^+} f(x) = \underline{+\infty}$   
 g)  $\lim_{x \rightarrow -4} f(x) = \underline{DNE}$   
 h)  $\lim_{x \rightarrow 1} f(x) = \underline{4}$   
 i)  $\lim_{x \rightarrow 4} f(x) = \underline{+\infty}$



- j)  $f(-4) = \underline{-1}$   
 k)  $f(1) = \underline{DNE}$   
 l)  $f(4) = \underline{DNE}$

2. Determine the left and right sided limits of the following function at -3.  $f(x) = \begin{cases} \frac{x^2+6x+9}{2x+6}, & x < -3 \\ -2x+5, & x \geq -3 \end{cases}$

$$[2] \lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^-} \frac{x^2+6x+9}{2x+6} = \lim_{x \rightarrow -3^-} \frac{(x+3)^2}{2(x+3)} = 0$$

$$\lim_{x \rightarrow -3^+} f(x) = \lim_{x \rightarrow -3^+} (-2x+5) = 11$$

3. Let  $k$  be a function. Functions  $g$  and  $h$  are twice differentiable functions such that  $g(2) = h(2) = 4$ , and we have  $g(x) \leq k(x) \leq h(x)$  for  $1 < x < 3$ . [2]

Is  $k$  continuous at  $x = 2$ ?

$$\left. \begin{aligned} \lim_{x \rightarrow 2} \frac{x^2-4}{x-2} &= \lim_{x \rightarrow 2} \frac{(x+2)(x-2)}{x-2} = \lim_{x \rightarrow 2} (x+2) = 4 \\ \lim_{x \rightarrow 2} \frac{8 \sin(x-2)}{2x-4} &= \lim_{x \rightarrow 2} \frac{8}{2} \cdot \frac{\sin(x-2)}{x-2} = 4 \end{aligned} \right\} \text{Squeezing theorem}$$

$$\lim_{x \rightarrow 2} f(x) = 4$$

4. Evaluate each of the following limits. Show your work for all questions. [12]

a.  $\lim_{x \rightarrow 2} \frac{x^2+5x+6}{x-3} = \frac{20}{-1}$

$$= \boxed{-20}$$

b.  $\lim_{x \rightarrow +\infty} \frac{3x^2-4}{x^2-1} = \lim_{x \rightarrow +\infty} \frac{3x^2}{x^2}$

$$= \boxed{3}$$

c.  $\lim_{x \rightarrow -5^-} \frac{|5+x|}{x^2-25}$

$$|5+x| = \begin{cases} 5+x & \text{if } x \geq -5 \\ -5-x & \text{if } x < -5 \end{cases}$$

2

$$\lim_{x \rightarrow -5^-} \frac{|5+x|}{x^2-25} = \lim_{x \rightarrow -5^-} \frac{-(5+x)}{(x+5)(x-5)}$$

$$= \boxed{\frac{1}{10}}$$

e)  $\lim_{h \rightarrow 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$

$$= \lim_{h \rightarrow 0} \frac{3 - (3+h)}{3(3+h)h}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{3(3+h)h}$$

$$= \lim_{h \rightarrow 0} \left( -\frac{1}{3(3+h)} \right)$$

$$= \boxed{-\frac{1}{9}}$$

g)  $\lim_{x \rightarrow \infty} \frac{x^2-x+1}{x^3+2}$

$$= \lim_{x \rightarrow \infty} \frac{x^2}{x^3}$$

$$= \boxed{0}$$

i)  $\lim_{x \rightarrow \infty} \frac{3x - \sin 3x}{x}$

$$= \lim_{x \rightarrow \infty} \left( 3 - \frac{\sin(3x)}{x} \right)$$

$$= \boxed{3}$$

d.  $\lim_{x \rightarrow -1^+} \frac{3x^2-4}{x^2-1} = \boxed{+\infty}$

f.  $\lim_{x \rightarrow 0} \frac{\sqrt{3-x}-\sqrt{3}}{x} \cdot \frac{\sqrt{3-x}+\sqrt{3}}{\sqrt{3-x}+\sqrt{3}}$

$$= \lim_{x \rightarrow 0} \frac{3-x-3}{x(\sqrt{3-x}+\sqrt{3})}$$

$$= \lim_{x \rightarrow 0} \left( -\frac{1}{\sqrt{3-x}+\sqrt{3}} \right)$$

$$= \boxed{-\frac{1}{2\sqrt{3}}}$$

h)  $\lim_{x \rightarrow -\infty} \frac{x^3-x+1}{x^2+2}$

$$= \lim_{x \rightarrow -\infty} \frac{x^3}{x^2}$$

$$= \boxed{-\infty}$$

j)  $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin x} = \lim_{x \rightarrow 0} \frac{\cancel{\sin 3x}}{3x} \cdot \frac{x}{\cancel{\sin x}} \cdot 3$

$$= \boxed{3}$$

1. 2. 3. 4. 5. 6. 7. 8. 9. 10.

11. 12. 13. 14. 15. 16. 17. 18. 19. 20.

21. 22. 23. 24. 25. 26. 27. 28. 29. 30.

31. 32. 33. 34. 35. 36. 37. 38. 39. 40.

41. 42. 43. 44. 45. 46. 47. 48. 49. 50.

51. 52. 53. 54. 55. 56. 57. 58. 59. 60.

61. 62. 63. 64. 65. 66. 67. 68. 69. 70.

71. 72. 73. 74. 75. 76. 77. 78. 79. 80.

81. 82. 83. 84. 85. 86. 87. 88. 89. 90.

91. 92. 93. 94. 95. 96. 97. 98. 99. 100.

101. 102. 103. 104. 105. 106. 107. 108. 109. 110.

111. 112. 113. 114. 115. 116. 117. 118. 119. 120.

121. 122. 123. 124. 125. 126. 127. 128. 129. 130.

131. 132. 133. 134. 135. 136. 137. 138. 139. 140.

141. 142. 143. 144. 145. 146. 147. 148. 149. 150.

151. 152. 153. 154. 155. 156. 157. 158. 159. 160.