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QUIZ 3.1 – 3.3

1. Find the derivative of each of the following. Be sure to simplify when possible. [6]

a) $y = \dfrac{x^2 - 5x}{2x+1}$

$$\frac{dy}{dx} = \frac{(2x-5)(2x+1) - 2(x^2-5x)}{(2x+1)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{2x^2+2x-5}{(2x+1)^2}}$$

$$= \frac{4x^2+2x-10x-5-2x^2+10x}{(2x+1)^2}$$

b) $y = (2x^3 - 7x + 1)(3x - 8)$

$$\frac{dy}{dx} = (6x^2-7)(3x-8) + 3(2x^3-7x+1)$$

$$= 18x^3 - 48x^2 - 21x + 56 + 6x^3 - 21x + 3$$

$$\boxed{\frac{dy}{dx} = 24x^3 - 48x^2 - 42x + 59}$$

c) $y = 3x^5 + \dfrac{8}{x^2} + \sqrt{x} - \pi^2$

$$\frac{dy}{dx} = 15x^4 - 16x^{-3} + \frac{1}{2}x^{-1/2}$$

$$\boxed{\frac{dy}{dx} = 15x^4 - \frac{16}{x^3} + \frac{1}{2\sqrt{x}}}$$

2. Prove that the the function *f* has a zero on the interval [1; 10] without determining it. [2]

$$f(x) = \sqrt{10x} - 15logx$$

- $f(1) = \sqrt{10} > 0$
- $f(10) = 10 - 15 = -5 < 0$
- f is continuous on $[1;10]$

$0 \in [-5; \sqrt{10}]$

IVT $\Rightarrow \exists c \in [1;10] \,/\, f(c) = 0$

3. Let $f(x) = \begin{cases} x^2 & , x \leq 1 \\ 2x & , x > 1 \end{cases}$ [2]

a) Determine $f'(x)$ for $x < 1$.

$f'(x) = 2x$

b) Determine $f'(x)$ for $x > 1$.

$f'(x) = 2$

c) Is *f* differentiable at 1? Explain. *Be careful, there is a trap…*

$\left.\begin{array}{l} f(1) = \lim\limits_{x\to 1^+} f(x) = 1 \\ \lim\limits_{x\to 1^-} f(x) = 2 \end{array}\right\}$ f is not continuous at 1, so it is $\underline{\text{not}}$ differentiable. (even if $\lim\limits_{x\to 1^+} f'(x) = \lim\limits_{x\to 1^-} f'(x) = 2$)

4. Use the definition of the derivative to determine the derivative of $f(x) = \sqrt{x}$. [2]

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \frac{1}{2\sqrt{x}}.$$

$$\Rightarrow \boxed{f'(x) = \frac{1}{2\sqrt{x}}}$$

5. Use the alternate definition of the derivative to determine the derivative of $g(x) = 3x^2$. [2]

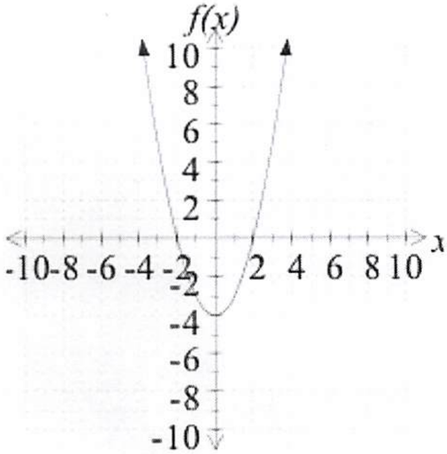
$$\frac{g(c) - g(x)}{c - x} = \frac{3c^2 - 3x^2}{c - x} = \frac{3(c^2 - x^2)}{c - x} = \frac{3(c+x)(c-x)}{c-x}$$

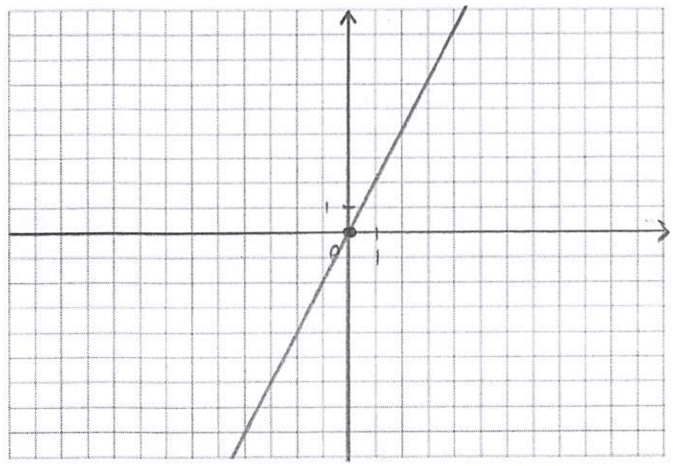
$$= 3(c+x)$$

$$\lim_{c\to x} \frac{g(c) - g(x)}{c - x} = 6x$$

$$\Rightarrow \boxed{g'(x) = 6x}$$

6. Graph the derivative of the following function on the second grid: [1]





7. Find the equation of the tangent line to $y = 5\sqrt{x} - \frac{27}{x}$ at $x = 9$. [2]

$$\frac{dy}{dx} = \frac{5}{2\sqrt{x}} + \frac{27}{x^2}$$

point : $(9; 12)$

$$\left.\frac{dy}{dx}\right|_{x=9} = \frac{7}{6}$$

$$\Rightarrow \boxed{y - 12 = \frac{7}{6}(x - 9)}$$