QUIZ 3.1 – 3.6

1. Find the derivative of each of the following. Be sure to simplify when possible. [12]

a) $y = \tan^2(3-\theta^2)$

$$\frac{dy}{dx} = 2\tan(3-\theta^2).\ \sec^2(3-\theta^2).(-2\theta)$$

$$= \boxed{-4\theta.\tan(3-\theta^2).\sec^2(3-\theta^2)}$$

b) $y = (4x+1)^3(5-x)^{-3}$

$$\frac{dy}{dx} = 3(4x+1)^2(4)(5-x)^{-3} - 3(4x+1)^3(5-x)^{-4}(-1)$$

$$= 12(4x+1)^2(5-x)^{-3} + 3(4x+1)^3(5-x)^{-4}$$

$$= \frac{3(4x+1)^2}{(5-x)^4}\left(\underbrace{4(5-x)+4x+1}_{=21}\right)$$

$$\boxed{\frac{dy}{dx} = \frac{63(4x+1)^2}{(5-x)^4}}$$

c) $y = \tan x(\csc x + 1)$

$$\frac{dy}{dx} = \sec^2x(\csc x+1) - \csc x.\cancel{\cot x}.\cancel{\tan x}$$

$$= \boxed{\sec^2x.\csc x + \sec^2x - \csc x.}$$

$$= \frac{1+\sin x - \cos^2x}{\cos^2x.\sin x}$$

d) $y = 4x^6(x^2+3x+5)^5$

$$\frac{dy}{dx} = 24x^5(x^2+3x+5)^5 + 20x^6(x^2+3x+5)^4 \times (2x+3)$$

$$= 4x^5(x^2+3x+5)^4\left(6(x^2+3x+5)+5x(2x+3)\right)$$

$$= \boxed{4x^5(x^2+3x+5)^4(16x^2+33x+30)}$$

e) $k(x) = \sqrt{\dfrac{1+x}{4-x}}$

$$k'(x) = \frac{1}{2}\sqrt{\frac{4-x}{1+x}} \times \frac{4-x+1+x}{(4-x)^2}$$

$$= \boxed{\frac{5\sqrt{4-x}}{2(4-x)^2\sqrt{1+x}}}$$

f) $f(x) = \sqrt{1+\sqrt{1+x}}$

$$f'(x) = \frac{1}{2}\left(1+\sqrt{1+x}\right)^{-1/2} \cdot \frac{1}{2}(1+x)^{-1/2}$$

$$= \frac{1}{4\sqrt{1+x}\sqrt{1+\sqrt{1+x}}}$$

2. Find the equation of the tangent line to $f(x) = \dfrac{x}{\tan x}$ at the point where $x = \dfrac{\pi}{4}$. [3]

$$f'(x) = \frac{\tan x - \sec^2 x \cdot x}{\tan^2 x}$$

$$f'\left(\frac{\pi}{4}\right) = \frac{1 - 2 \times \frac{\pi}{4}}{1}$$

$$= 1 - \frac{\pi}{2}$$

Point: $\left(\frac{\pi}{4} ; \frac{\pi}{4}\right)$

$$\boxed{y - \frac{\pi}{4} = \left(1 - \frac{\pi}{2}\right)\left(x - \frac{\pi}{4}\right)}$$

3. A particle's position along a number line is given by $s(t) = 4t^2 - 16t$, for $t \geq 0$ [5]

a) Find the particle's velocity and acceleration functions.

2

$v(t) = s'(t) = 8t - 16$

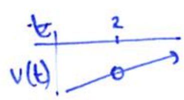
$a(t) = s''(t) = 8$

b) When does the particle change direction?

1

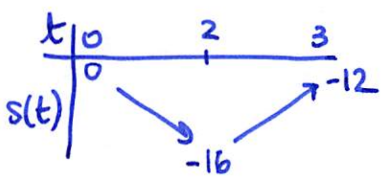
when v changes sign.

when $\boxed{t = 2}$



c) Find the total distance travelled by the particle in the first 3 seconds.

2



$D = 16 + 4 = \underline{20}$