QUIZ 4.1 – 4.4

1. For what value(s) of c will $y = x^3 + \frac{c}{x}$ have no critical point? $D = \mathbb{R} \setminus \{0\}$

critical points when y' DNE on D or $y' = 0$.

$y' = 3x^2 - \frac{c}{x^2}$ • y' DNE when $x = 0$, but $0 \notin D$

$y' = \frac{3x^4 - c}{x^2}$ • $y' = 0$ when $3x^4 - c = 0$. This equation has no solution when $\boxed{c < 0}$

2. Determine the coordinates of the inflection points of $y = x^4 + \frac{3}{5}x^5$

$y' = 4x^3 + 3x^4$

$y'' = 12x^2 + 12x^3$

$= 12x^2(1+x)$

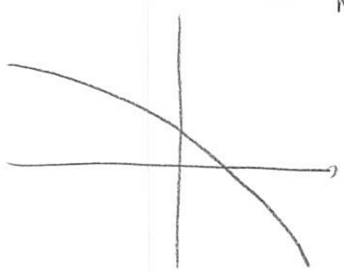
x	$-\infty$		-1		0		$+\infty$
y''		$-$	0	$+$	0	$+$	

point of inflection : $(-1, \frac{2}{5})$ because y'' changes sign.

3. If y is a function of x such that $\frac{dy}{dx} < 0$ for all x and $\frac{d^2y}{dx^2} < 0$ for all x, propose a possible sketch of the function.

decreasing

concave down



4. If $f(x) = x^3e^{-2x}$ Determine the variations of the function.

$D = \mathbb{R}$

$f'(x) = 3x^2 \cdot e^{-2x} - 2e^{-2x} \cdot x^3$

$= x^2 e^{-2x}(3 - 2x)$

x	$-\infty$		0		$3/2$		$+\infty$
$f'(x)$		$+$	0	$+$	0	$-$	
$f(x)$		↗		↗		↘	

f is increasing on $(-\infty, 3/2)$ because f' is positive (and 0 a finite number of times)

f is decreasing on $(3/2, +\infty)$ because f' is negative.

5. If the derivative of the function f is $f'(x) = -3(x + 2)(x + 1)^2(x + 3)^3$ then find the local minimums of the function.

x	$-\infty$		-3		-2		-1		$+\infty$
$-3(x+2)$		$+$		$+$	0	$-$		$-$	
$(x+1)^2$		$+$		$+$		$+$	0	$+$	
$(x+3)^3$		$-$	0	$+$		$+$		$+$	
$f'(x)$		$-$	0	$+$	0	$-$	0	$-$	

local min at -3 because f' changes from $-$ to $+$.

6. Find each value of c in the interval [0, 3] that satisfies the conditions of the Mean Value Theorem for the function $f(x) = x^2 + 2x - 1$. **[2]**

- $\dfrac{f(3)-f(0)}{3-0} = \dfrac{14+1}{3} = 5$
- f continuous & diff. on $[0,3]$

MVT: there exists $c \in (0,3)$ such that $f'(c) = 5$

$f'(x) = 2x+2$

$f'(c) = 5 \iff 2c+2 = 5$

$2c = 3$

$\boxed{c = 3/2}$

7. Determine the variations and the concavity of: $f(x) = x\sqrt{5-x}$. Sketch a rough shape of the graph of f. **[4]**

$D = (-\infty, 5]$

$f'(x) = \sqrt{5-x} + x \cdot \dfrac{1}{2\sqrt{5-x}} \cdot (-1)$

$= \dfrac{2(5-x) - x}{2\sqrt{5-x}}$

$= \dfrac{10-3x}{2\sqrt{5-x}}$

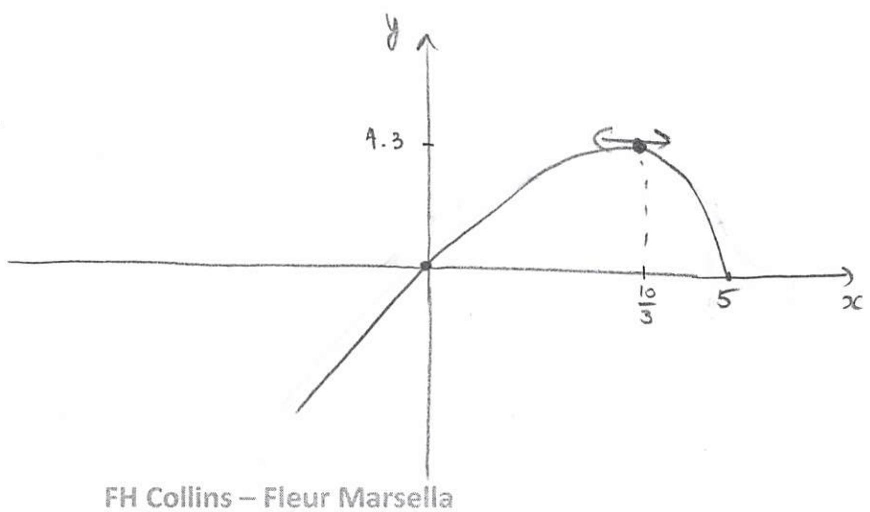
x	$-\infty$		$10/3$		5
$f'(x)$		$+$	0	$-$	$\|$
$f(x)$	$-\infty$	$\nearrow$	4.3	$\searrow$	0

$f''(x) = \dfrac{-3 \cdot 2\sqrt{5-x} - (10-3x)\cdot \frac{1}{\sqrt{5-x}} \cdot (-1)}{4(5-x)}$

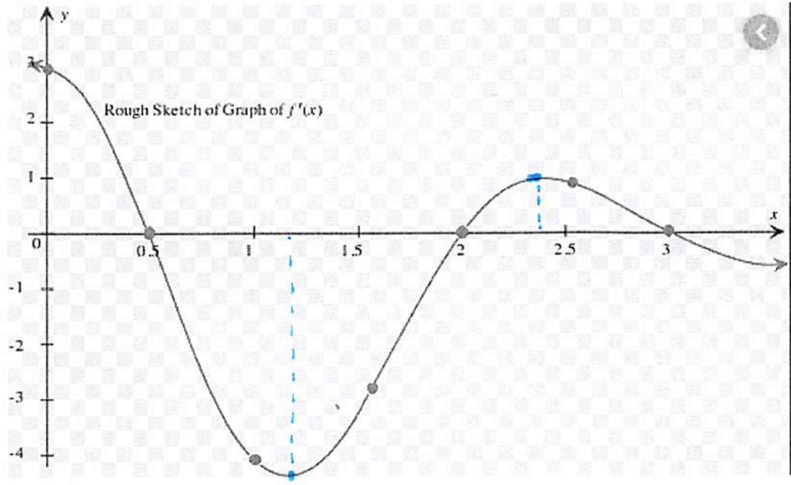
$= \dfrac{-6(5-x) + 10 - 3x}{4(5-x)\sqrt{5-x}}$

$= \dfrac{3x-20}{4(5-x)\sqrt{5-x}}$

x	$-\infty$		5
$f''(x)$		$-$	$\|$



8.



Note: This is the graph of the derivative of *f*, not the graph of *f*.

The figure above shows the graph of f', the derivative of the function f. The domain of f is the set of all real numbers x such that $0 \le x \le 3.5$.

a) For what value(s) of x, $0 < x < 3.5$, does f have a relative maximum? Justify your answer. **[1]**

when f' changes from + to −
i.e when $x = 0.5$ and $x = 3$

b) On what intervals, for $0 < x < 3.5$, is the graph of f increasing? Justify your answer. **[2]**

when $f'(x) > 0$
i.e when $x \in (0, 0.5)$ and $x \in (2, 3)$

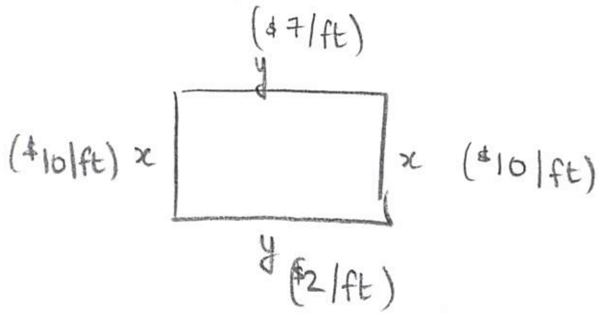
c) On what intervals, for $0 < x < 3.5$, is the graph of f concave up? Justify your answer. **[2]**

when f' is increasing.
i.e when $x \in (1.2, 2.3)$ approx.

d) For what value(s) of x, $0 < x < 3.5$, does f have a point of inflection? Justify your answer. **[2]**

when f' changes direction.
i.e when $x \simeq 1.2$ and $x \simeq 2.3$

8. We are going to fence in a rectangular field. If we look at the field from above the cost of the vertical sides are \$10/ft, the cost of the bottom is \$2/ft and the cost of the top is \$7/ft. If we have \$700 determine the dimensions of the field that will maximize the enclosed area. **[4]**



$A = x \cdot y$

$700 = 20x + 9y$

$y = \dfrac{700 - 20x}{9}$

$A = x\left(\dfrac{700-20x}{9}\right)$

$= -\dfrac{20}{9}x^2 + \dfrac{700}{9}x$

$D = [0, 35]$

$A'(x) = -\dfrac{40}{9}x + \dfrac{700}{9}$

$= \dfrac{-40x+700}{9}$

x	0		$35/2$		35
$A'(x)$		+	0	−	
$A(x)$		↗		↘	

A is max at $x = 35/2$ because A' changes from + to −

Dimensions: $x = \dfrac{35}{2}$ and $y = \dfrac{700 - 20 \times 35/2}{9}$

$y = \dfrac{350}{9}$