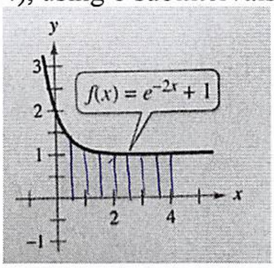
Name: Solutions 24 AP Calc

QUIZ 5.1 – 5.3

Multiple Choices **[4]**

1. Find the MRAM approximation for the area of the shaded region (under the curve between 0 and 4), using 8 subintervals of equal width.



$\frac{1}{2}\left(f(0.25)+f(0.75)+f(1.25)+f(1.75)+f(2.25)+f(2.75)+f(3.25)+f(3.75)\right)$

A) 4.425 B) 4.480 C) 6.719 D) 8.959

2. The expression $\frac{1}{10}\left[\left(\frac{1}{10}\right)^2+\left(\frac{2}{10}\right)^2+\left(\frac{3}{10}\right)^2+\cdots+\left(\frac{10}{10}\right)^2\right]$ is a Riemann sum approximation for

A) $\int_0^1 x^2dx$ B) $\frac{1}{10}\int_0^1 x^2dx$ C) $\int_1^{10} x^2dx$ D) $\int_0^{10} x^2dx$

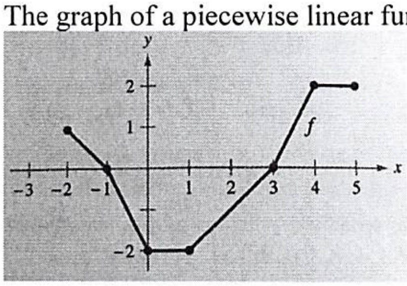
3. The table shows selected values for a continuous function f over the interval [2,8].

x	2	3	5	8
$f(x)$	8	22	72	142

Using the subintervals [2,3]., [3,5] and [5,8]., what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

A) 268 B) 338 C) 430 D) 592

4. The graph of a piecewise linear function f is shown below.



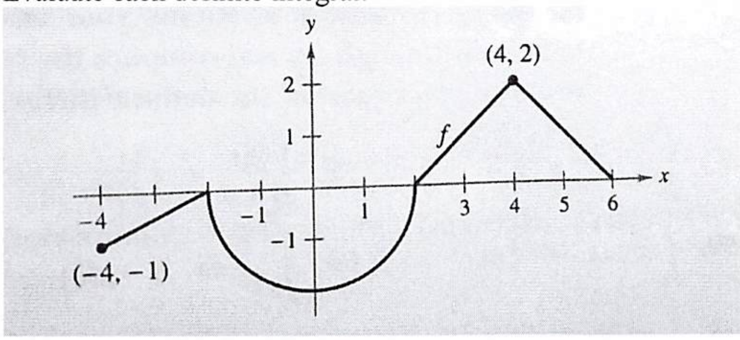
If g is the function defined by $g(x)=\int_4^x f(t)dt$, determine $g(-1)$.

A) -6 B) -4 C) 4 D) 6

4

Free Response Questions:

5. The graph of f below consists of line segments and a semi-circle. Evaluate each definite integral. [4]



a) $\int_0^4 f(x)dx$ $= -\frac{1}{4}\pi.2^2 + \frac{1}{2}2\times2 = 2-\pi$

b) $\int_0^6 |f(x)|dx$ $= \frac{1}{4}\pi.2^2 + \frac{1}{2}2\times4 = \pi+4$

4

c) $\int_4^{-2} f(x)dx$ $= -\int_{-2}^{4} f(x)dx = \frac{1}{2}\pi.2^2 - \frac{1}{2}.2\times2 = 2\pi - 2$

d) $\int_0^2 (f(x)+2)dx$ $= \int_0^2 f(x)dx + 2\times2 = -\frac{1}{4}\pi.2^2 + 4 = 4-\pi$

6. a) Write the following integral as the limit of a Riemann sum: $\int_0^2 e^x dx$ [1.5]

$$\int_0^2 e^x dx = \lim_{n\to+\infty} \sum_{k=1}^{n} \frac{2}{n} e^{\frac{2k}{n}}$$

b) Write the following limit as an integral: [1.5]

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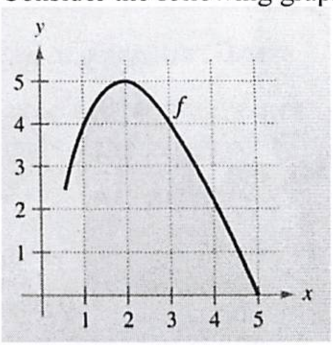
$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{1}{n}\right)\left[8\left(\frac{i}{n}\right)+3\right]$$

$$= \int_0^1 (8x+3)dx$$

$=$

$= \int_{-2}^{0} f(x)\,dx +$

7. Consider the following graph: [3]



Using 4 rectangles, determine an approximation of the shaded region (under the curve between 1 and 5) with the given method:

a) LRAM

$4+5+4+2=15$

3

b) RRAM

$5+4+2+0=11$

8. Use your calculator to determine $\int_1^3 \ln x\, dx$ to the nearest thousandth. [1]

$\int_1^3 \ln x\, dx \doteq 1.296$

1

9. Given $\int_0^5 f(x)dx = 10$ and $\int_5^7 f(x)dx = 3$, evaluate: [4]

a) $\int_0^7 f(x)dx = \int_0^5 f(x)dx + \int_5^7 f(x)dx = \boxed{13}$

b) $\int_5^5 f(x)dx = \boxed{0}$

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c) $\int_5^0 f(x)dx = -\int_0^5 f(x)dx = \boxed{-10}$

d) $\int_0^5 3f(x)dx = 3 \cdot \int_0^5 f(x)dx = \boxed{30}$

10. Given $\int_2^6 f(x)dx = 10$ and $\int_2^6 g(x)dx = -2$, evaluate: [3]

a) $\int_2^6 [f(x) + g(x)]dx = \int_2^6 f(x)dx + \int_2^6 g(x)dx = \boxed{8}$

b) $\int_2^6 [g(x) - f(x)]dx = \int_2^6 g(x)dx - \int_2^6 f(x)dx = -2 - 10 = \boxed{-12}$

3

c) $\int_2^6 [f(x) + 3]dx = \int_2^6 f(x)dx + 4 \times 3 = 10 + 12 = \boxed{22}$

11. Determine the exact value of the average value of the following function on the interval [1,4]:

$$f(x) = 3 - x$$

[2]

$$\frac{1}{3}\int_1^4 (3-x)dx = \frac{1}{3}\left(\frac{1}{2}\times 2\times 2 - \frac{1}{2}\times 1\times 1\right)$$

$$= \frac{1}{3}\left(2 - \frac{1}{2}\right)$$

$$= \boxed{\frac{1}{2}}$$

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