Name: Solutions 18 AP Calc

QUIZ 7.1 – 7.2

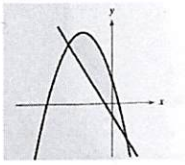
NO Calculator Section

Multiple Choices:

1. What is the area of the region bounded by the graphs of $x = y^2 - 2y$ and $y = -x + 2$?

A) $\frac{3}{2}$ B) $\frac{10}{3}$ C) $\frac{9}{2}$ D) $\frac{31}{6}$

2. The figure below shows the graphs of $y = 2 - 4x - x^2$ and $y = -2x - 1$.
What is the area of the shaded region?



A) $\frac{5}{3}$ B) 10 C) $\frac{32}{3}$ D) $\frac{86}{3}$

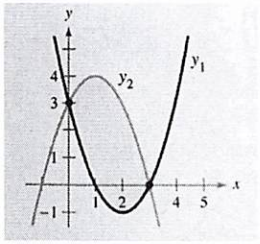
3

3. Which integral gives the area A of the region bounded by the graph of $f(x) = x^3 - 2x$ and the tangent line to the graph of f at $(-1,1)$?

A) $A = \int_{-1}^{2}(x^3 + 3x + 2)dx$ B) $A = \int_{-1}^{2}(x^3 - 3x - 2)dx$ C) $A = \int_{-1}^{4}(-x^3 + 3x + 2)dx$ D) $A = \int_{-1}^{2}(-x^3 + 3x + 2)dx$

Free Response Questions:

4. Determine the area of the shaded region:



$y_1 = (x-2)^2 - 1 \quad y_1 = x^2 - 4x + 3$

$y_2 = -(x-1)^2 + 4 \quad y_2 = -x^2 + 2x + 3$

$$\int_0^3 (y_2 - y_1)dx = \int_0^3 \left(-x^2 + 2x + 3 - (x^2 - 4x + 3)\right)dx$$

$$= \int_0^3 (-2x^2 + 6x)dx$$

$$= -\frac{2}{3}x^3 + 3x^2 \Big]_0^3 = -18 + 27 = \boxed{9}$$

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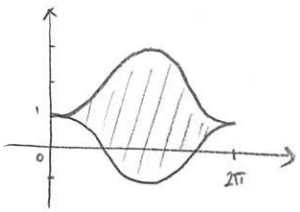
5. Sketch the region bounded by the graphs of $f(x) = \cos x$ and $g(x) = 2 - \cos x$ for $0 \le x \le 2\pi$ and determine an integral that represents the area of the enclosed region (without evaluating it).



$$A = \int_0^{2\pi} 2(1 - \cos x)dx$$

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Calculator section

6. A tank contains 125 gallons of heating oil at time $t = 0$.
During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate:
$$H(t) = 2 + \frac{10}{1+\ln(t+1)} \text{ gallons per hour.}$$
During the same time interval, heating oil is removed from the tank at the rate:
$$R(t) = 12\sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

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$\int_0^{12} H(t)\,dt \approx 70.571$ gallons

b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Justify.

$H(t) - R(t)$ is the rate of change of the amount of oil.

2

$H(6) - R(6) \approx -2.924 < 0$

Therefore: the level is falling at time $t = 6$ h.

c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

2

$125 + \int_0^{12} (H(t) - R(t))\,dt \approx 122.026$ gallons

d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

I'm looking at local mins for $125 + \int_0^{12} (H(t) - R(t))\,dt$.

The critical points are where $H(t) - R(t) = 0$

On the calculator, I get $t_1 \approx 4.790$ and $t_2 \approx 11.318$

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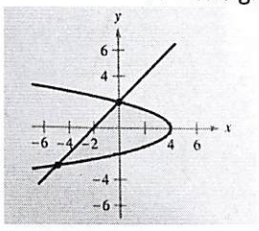
t	0	t_1	t_2	12
$H(t) - R(t)$	+	0	−	0	+
$Q(t)$	125	↗	↘		↗

$Q(t_2)$ is a local min

$Q(t_2) = 125 + \int_0^{11.318} (H(t) - R(t))\,dt \approx 120.738$

$Q(t_2)$ is the absolute min. $\boxed{t \approx 11.318}$

7. The graphs of $x = 4 - y^2$ and $x = y - 2$ are given. Determine the area of the shaded region:



$$\int_{-3}^{2} (4 - y^2 - (y-2))\,dy = \int_{-3}^{2} (-y^2 - y + 6)\,dy$$

$$= -\frac{1}{3}y^3 - \frac{1}{2}y^2 + 6y\Big]_{-3}^{2} = -\frac{8}{3} - 2 + 12 - \left(9 - \frac{9}{2} - 18\right)$$

2

$$= \boxed{\frac{125}{6}}$$